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Control and Adaptation of TDMA in Wireless Networks

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***Abstract*— Time-Division Multiple-Access (TDMA) is a widely used technique for simultaneous utilization of a single channel by multiple users. In its traditional form, TDMA suffers from a lack of power-efficiency, which is particularly damaging in wireless communications. This paper develops two controlled versions of TDMA, which lead to considerable power-efficiency improvements without a loss in average throughput for all users. The first one provides at least $11\,\mathrm{dB}$ improvement in comparison with traditional TDMA but lacks location-fairness. The second, which is an adaptive version of the first, provides at least $5\,\mathrm{dB}$ improvement along with excellent location-fairness.**

I. INTRODUCTION

Time-Division Multiple-Access (TDMA), whereby each user is assigned a particular time slot to transmit its information packet, is often used in wireline and wireless networks as a means to utilize a single channel by multiple users [1]. Although quite simple in implementation, TDMA suffers from two problems. First, it is throughput-deficient, since the user, assigned to a particular slot, may have no packet to transmit. Second, it is power-deficient, since the user, selected for transmission, may have bad (randomly fluctuating) channel conditions and, thus, would have to expend substantial power to attain the necessary signal-to-noise ratio (SNR). The latter is particularly damaging in wireless networks where users often have limited local power supply.

Throughput-efficient versions of TDMA that are based on control-theoretic approach have been introduced in [2], [3]. In the current paper, we develop two controlled versions of TDMA, which are power-efficient. The first one, referred to as *Ranking TDMA* (R-TDMA), selects for transmission the user with the best current channel conditions. We show that R-TDMA is extremely power-efficient, reducing power consumption by at least $11\,\mathrm{dB}$ in comparison with uncontrolled TDMA. However, as our analysis shows, R-TDMA has a deficiency of its own: it lacks short-term location-fairness, which implies that mobile users positioned in bad locations *vis-à-vis* the base station would have smaller average throughput than others. To overcome this problem, we propose *Adaptive Ranking TDMA* (AR-TDMA), whereby all users are adapted (or "virtually" transferred) to a single location, and then the user with the best current channel conditions is selected for transmission. We prove that AR-TDMA is both power-efficient (at least $5\,\mathrm{dB}$ reduction) and location-fair.

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Control and adaptation of TDMA in wireless networks are not new ideas; they have been explored in the literature within the context of transmission scheduling [4]–[8]. Specifically, in [4], a fair scheduling algorithm for wireline networks is extended to wireless networks. It allows users lagging in information flow to catch up with leading users, thus ensuring both long- and short-term fairness. Opportunistic transmission scheduling policies, which at each time slot pick a single user for transmission, are presented and proved to be optimal under various constraints in [5]–[7]. Stochastic approximation is employed to make these policies long-term fair, and a modification is suggested to make the policy in [5] short-term fair. Another policy, which also schedules users one-at-a-time and is long- and short-term fair, is the Proportional Fair algorithm proposed and implemented by QualComm for 3G1X EVDO downlink [8].

The existing literature clearly exhibits the benefit of control and adaptation of TDMA in wireless networks. However, several issues remain to be addressed. For instance, the short-term performance has not been thoroughly analyzed. In this paper, we introduce several novel metrics, which we believe are important for quantifying short-term behavior. They include: conditional expectation of short-term average throughput given location, variance of short-term average throughput, and expected number of consecutive time slots without, and with, a transmission. These metrics are referred to as *location-fairness*, *throughput variability*, *downtime*, and *uptime*, respectively. Another issue is the lack of analytical methods for performance evaluation available in the literature. Since such methods are valuable in gaining insights as well as for design purposes, deriving them is of interest. In this paper, they are derived and utilized to evaluate the above-mentioned performance measures.

The outline of this paper is as follows: In Section II, the TDMA model is described. Performance measures are introduced in Section III. In Section IV, R-TDMA and AR-TDMA are developed. Their analysis and comparisons with traditional TDMA and with an existing approach proposed in the literature are carried out in Section V. Finally, the conclusions are formulated in Section VI. The proofs can be found in [9].

II. MODELING

The TDMA system considered in this paper consists of N mobile users, a channel, and a base station. Assumptions on each of these elements are as follows:

Users

At each time slot $k \in \mathbf{Z}$, one of the N users is selected to send an information packet to the base station. If user i is selected, it transmits with power $p_i(k) > 0$. Otherwise, it remains silent with $p_i(k) = 0$. To focus the analysis on

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power-efficiency issues, we assume that the users always have packets to transmit.

Channel

The channel affects the transmissions so that the received SNR of user i at time slot k, $r_i(k)$, is given by

$$r_i(k) = e^{x_i(k)} p_i(k), \quad k \in \mathbf{Z},\ i = 1, 2, \ldots, N, \tag{1}$$

where $e^{x_i(k)}$ is the *channel gain* of user i, which combines path loss, shadowing, thermal noise power, and other radio-wave propagation effects, and $x_i(k) \in \mathbf{R}$ is the *log-channel gain*.

The sequences of log-channel gains, $\{x_i(k), k \in \mathbf{Z}\}$, $i = 1, 2, \ldots, N$, are assumed to be random processes. For performance analysis and comparison purposes, the following assumption is imposed:

Assumption 1. Processes $\{x_i(k), k \in \mathbf{Z}\}$, $i = 1, 2, \ldots, N$, are independent. For each i, process $\{x_i(k), k \in \mathbf{Z}\}$ is a wide-sense stationary (WSS) Gaussian random process with mean $\mathrm{E}\{x_i(k)\} = \mu_x$ and autocovariance function

$$\mathrm{E}\{(x_i(k+\ell) - \mu_x)(x_i(k) - \mu_x)\} = \sigma_x^2 \rho_x(\ell), \quad \ell \in \mathbf{Z},$$

where $\mu_x \in \mathbf{R}$, $\sigma_x > 0$, $\rho_x(0) = 1$, $\rho_x(\ell) = \rho_x(-\ell)$, $|\rho_x(\ell)| < 1\ \forall \ell \neq 0$, and $\lim_{\ell \to \infty} \rho_x(\ell) = 0$, i.e., $\mathrm{E}\{(x_i(k) - \mu_x)^2\} = \sigma_x^2$ is the variance of $x_i(k)$ and $\rho_x(\ell)$ is the correlation coefficient of $x_i(k+\ell)$ and $x_i(k)$.

Base Station

At each time slot k, the base station attempts to decode the packet sent by the selected user. The (normalized) throughput of user i at time slot k, $t_i(k)$, is assumed to be a function of $r_i(k)$,

$$t_i(k) = \Phi(r_i(k)), \quad k \in \mathbf{Z},\ i = 1, 2, \ldots, N, \tag{2}$$

where $\Phi : [0, \infty) \to [0, 1)$ depends on the modulation, demodulation, and coding schemes employed, as well as the channel.

The following assumption on Φ is introduced for performance analysis and comparisons:

Assumption 2. Function $\Phi : [0, \infty) \to [0, 1)$ is strictly increasing and satisfies $\Phi(0) = 0$.

To summarize, the TDMA system considered in this paper is modeled by (1), (2), with $x_i(k)$ specified by Assumption 1 and Φ by Assumption 2.

III. PERFORMANCE MEASURES

Typically, performance metrics considered in wireless networks are the average throughput and the average transmit power, defined on the *infinite* time interval. Unfortunately, these averages may be deficient in delay-sensitive applications. The reason is that, even if, for example, the average throughput is high, it does not imply that a reliable communication has taken place at every relatively short time interval. To account for this deficiency, in this work we consider averages defined on *finite* time intervals: the finite-time average transmit power of user i,

$$\bar{p}_i(k_1, k_2) = \tfrac{1}{k_2 - k_1 + 1} \sum_{k=k_1}^{k_2} p_i(k), \quad k_1, k_2 \in \mathbf{Z},\ k_1 \leq k_2, \tag{3}$$

and the finite-time average throughput of user i,

$$\bar{t}_i(k_1, k_2) = \tfrac{1}{k_2 - k_1 + 1} \sum_{k=k_1}^{k_2} t_i(k), \quad k_1, k_2 \in \mathbf{Z},\ k_1 \leq k_2. \tag{4}$$

For the sake of brevity, we omit below the term “finite-time”. The averages (3), (4) are random variables. In this work, a number of their statistical properties are examined and treated as performance metrics:

Performance Measure P1. Mean of $\bar{p}_i(k_1, k_2)$, i.e., $\mathrm{E}\{\bar{p}_i(k_1, k_2)\}$.

Performance Measure P2. Mean of $\bar{t}_i(k_1, k_2)$, i.e., $\mathrm{E}\{\bar{t}_i(k_1, k_2)\}$.

Under the assumption of ergodicity, P1 and P2 coincide with the infinite-time averages. Measure P2 reflects only the “average” behavior of $\bar{t}_i(k_1, k_2)$. It does not tell how $\bar{t}_i(k_1, k_2)$ would depend on the location of user i relative to the base station, nor does it describe the variability of $\bar{t}_i(k_1, k_2)$. As a result of the latter, little can be said about the probability that $\bar{t}_i(k_1, k_2)$ exceeds a certain level and, hence, about whether communications are reliable. These shortcomings are alleviated by the following metrics:

Performance Measure P3. Conditional mean of $\bar{t}_i(k_1, k_2)$ given $x_i(k_1) = x_o \in \mathbf{R}$, i.e., $\mathrm{E}\{\bar{t}_i(k_1, k_2) | x_i(k_1) = x_o\}$.

Since a large (small) $x_i(k_1)$ typically corresponds to user i being in a good (bad) location at time slot k_1, P3 expresses the dependency of $\bar{t}_i(k_1, k_2)$ on the location of user i and, thus, characterizes short-term *location-fairness* of the system.

Performance Measure P4. Variance of $\bar{t}_i(k_1, k_2)$, i.e., $\mathrm{var}\{\bar{t}_i(k_1, k_2)\}$.

Measure P4 represents *throughput variability* and, together with $\mathrm{E}\{\bar{t}_i(k_1, k_2)\}$, provides bounds on the probability that $\bar{t}_i(k_1, k_2)$ exceeds a certain level (via the Chebyshev inequality).

Two additional measures, intended to describe the regularity of transmissions, are the number of consecutive time slots without, and with, a transmission by a certain user, referred to as the *downtime* and *uptime*, respectively. To formalize these measures, let $\mathbf{Z}_+$ denote the set of positive integers and let

$$d_i(k) = \begin{cases} \min\{\ell \in \mathbf{Z}_+ : p_i(k+\ell) > 0\}, & \text{if } p_i(k-1) > 0,\ p_i(k) = 0, \\ 0, & \text{otherwise,} \end{cases}$$

$$u_i(k) = \begin{cases} \min\{\ell \in \mathbf{Z}_+ : p_i(k+\ell) = 0\}, & \text{if } p_i(k-1) = 0,\ p_i(k) > 0, \\ 0, & \text{otherwise.} \end{cases}$$

Then, whenever $d_i(k) > 0$, a period without a transmission by user i begins at time slot k and lasts for $d_i(k)$ time slots, i.e., the downtime of user i is $d_i(k)$; analogously, whenever $u_i(k) > 0$, a period with consecutive transmissions by user i begins at time slot k and lasts for $u_i(k)$ time slots, i.e., the uptime of user i is $u_i(k)$. Here, we are interested in:

Performance Measure P5. Mean downtime of user i, i.e., $\mathrm{E}\{d_i(k)|d_i(k) > 0\}$.

Performance Measure P6. Mean uptime of user i, i.e., $\mathrm{E}\{u_i(k)|u_i(k) > 0\}$.

IV. TRADITIONAL, RANKING, AND ADAPTIVE RANKING TDMA

A. Traditional TDMA

In traditional TDMA, users take turn to transmit one-at-a-time in a pre-defined manner. For instance, user 1 is scheduled for transmission at time slots $1, N+1, 2N+1, \ldots$, user 2 at time slots $2, N+2, 2N+2, \ldots$, and so on. Often, conventional power control [10]–[14] is also employed to ensure that the users transmit with power such that some desired SNRs are achieved. Although, in general, these SNRs may be different for different users, assuming that all of them are the same facilitates the comparison of traditional TDMA with those developed below. Therefore, we assume that the transmit power $p_i(k)$ of user i at time slot k is given by

$$p_i(k) = \begin{cases} r_d e^{-x_i(k)}, & \text{if } \frac{k-i}{N} \in \mathbf{Z}, \\ 0, & \text{otherwise}, \end{cases} \quad k \in \mathbf{Z}, \qquad (5)$$

where $r_d > 0$ is the desired SNR. Equation (5), along with (1), implies that user i would transmit and maintain its SNR at r_d at time slots $i, N+i, 2N+i, \ldots$, and remain silent at others.

B. Ranking TDMA

Traditional TDMA schedules users for transmission in advance, irrespective of their locations and channel conditions. Hence, it may be *power-inefficient*, since poorly located users are forced to transmit packets that could be sent with less power when channel conditions improve. To increase power-efficiency, consider the following transmission scheduling scheme:

$$p_i(k) = \begin{cases} r_d e^{-x_i(k)}, & \text{if } x_i(k) > x_j(k)\ \forall j \neq i, \\ 0, & \text{otherwise}, \end{cases} \quad k \in \mathbf{Z}. \qquad (6)$$

Equation (6) ensures that the user with the largest channel gain is always selected for transmission. Thus, we refer to (6) as *Ranking TDMA* (R-TDMA). Clearly, R-TDMA maximizes power-efficiency because at any time slot the power consumed by the transmitting user is minimal.

C. Adaptive Ranking TDMA

Although R-TDMA optimizes power-efficiency, it is biased against poorly located users, whose channel gains are small, making them unlikely to be selected for transmission for a long time. To compensate for this "built-in" unfairness, the selection criterion should be modified to account for the users' locations. This may be accomplished by introducing variables $\tilde{x}_i(k) \in \mathbf{R}$, $i = 1, 2, \ldots, N$, defined as

$$\tilde{x}_i(k) = x_i(k) - \bar{x}_i(k-L, k-1), \quad k \in \mathbf{Z}, \qquad (7)$$

where $\bar{x}_i(k-L, k-1)$ is the moving average of the log-channel gain of user i over the past $L \in \mathbf{Z}_+$ time slots, i.e.,

$$\bar{x}_i(k-L, k-1) = \frac{1}{L} \sum_{\ell=1}^{L} x_i(k-\ell), \quad k \in \mathbf{Z}. \qquad (8)$$

When $L \to \infty$, the averaging in (8) eliminates the location-dependence of $\bar{x}_i(k-L, k-1)$; when $L = 1$, there is no averaging. Therefore, there must be an L^* such that fading dips are averaged out but the location-dependence of $\bar{x}_i(k-L, k-1)$ is preserved. With this L^*, the variable $\tilde{x}_i(k)$ in (7) accounts for the current channel conditions of user i and is less dependent on its location. Thus, if the user with the largest $\tilde{x}(k)$ is selected for transmission, i.e.,

$$p_i(k) = \begin{cases} r_d e^{-x_i(k)}, & \text{if } \tilde{x}_i(k) > \tilde{x}_j(k)\ \forall j \neq i, \\ 0, & \text{otherwise}, \end{cases} \quad k \in \mathbf{Z}, \qquad (9)$$

then one can expect the resulting system to be more location-fair. We refer to (7)–(9) as *Adaptive Ranking TDMA* (AR-TDMA), since it adapts (or "virtually" transfers) all users to a single location, and then let them compete for transmission with equal opportunities.

V. PERFORMANCE ANALYSIS AND COMPARISONS

A. Performance Analysis

The following theorems characterize the performance of traditional TDMA, R-TDMA, and AR-TDMA.

Theorem 1. *Consider the traditional TDMA described by* (1), (2), (5)*. Let $x_i(k)$ be specified by Assumption 1 and Φ by Assumption 2. Then, for any $k_1, k_2 \in \mathbf{Z}$, with $\frac{k_2-k_1+1}{N} \in \mathbf{Z}_+$, any $k \in \mathbf{Z}$, and $i = 1, 2, \ldots, N$, Performance Measures P1–P6 are given by*

$$\begin{aligned}
&\mathrm{E}\{\bar{p}_i(k_1, k_2)\} = \frac{1}{N} r_d e^{\frac{\sigma_x^2}{2} - \mu_x}, \\
&\mathrm{E}\{\bar{t}_i(k_1, k_2)\} = \frac{1}{N} \Phi(r_d), \\
&\mathrm{E}\{\bar{t}_i(k_1, k_2)|x_i(k_1) = x_o\} = \frac{1}{N} \Phi(r_d), \\
&\mathrm{var}\{\bar{t}_i(k_1, k_2)\} = 0, \\
&\mathrm{E}\{d_i(k)|d_i(k) > 0\} = N - 1, \\
&\mathrm{E}\{u_i(k)|u_i(k) > 0\} = 1.
\end{aligned}$$

Theorem 2. *Consider the R-TDMA described by* (1), (2), (6)*. Let $x_i(k)$ be specified by Assumption 1 and Φ by Assumption 2. Then, for any $k_1, k_2 \in \mathbf{Z}$, $k_1 \leq k_2$, any $k \in \mathbf{Z}$, and $i = 1, 2, \ldots, N$, Performance Measures P1–P6 are given by*

$$\mathrm{E}\{\bar{p}_i(k_1, k_2)\} = r_d e^{\frac{\sigma_x^2}{2} - \mu_x} \int_{-\infty}^{\infty} \frac{e^{-\frac{v^2}{2}}}{\sqrt{2\pi}} Q_1^{N-1}(v + \sigma_x)\, dv,$$

$$\mathrm{E}\{\bar{t}_i(k_1,k_2)\} = \frac{1}{N}\Phi(r_d),$$

$$\mathrm{E}\{\bar{t}_i(k_1,k_2)|x_i(k_1)=x_o\} = \frac{\Phi(r_d)}{K}\int_{-\infty}^{\infty}\frac{e^{-\frac{v^2}{2}}}{\sqrt{2\pi}}\Big(\sum_{\ell=0}^{K-1} Q_1^{N-1}\Big(\sqrt{1-\rho_x^2(\ell)}v-\rho_x(\ell)\frac{x_o-\mu_x}{\sigma_x}\Big)\Big)\,dv,$$

$$\mathrm{var}\{\bar{t}_i(k_1,k_2)\} = \frac{\Phi^2(r_d)}{K}\Big[\frac{1}{N}-\frac{1}{N^2}+2\sum_{\ell=1}^{K-1}\Big(1-\frac{\ell}{K}\Big)\times\Big(\Omega(N,\rho_x(\ell))-\frac{1}{N^2}\Big)\Big],$$

$$\mathrm{E}\{d_i(k)|d_i(k)>0\} = \frac{N-1}{1-N\Omega(N,\rho_x(1))},$$

$$\mathrm{E}\{u_i(k)|u_i(k)>0\} = \frac{1}{1-N\Omega(N,\rho_x(1))},$$

where $K=k_2-k_1+1$,

$$Q_1(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-\frac{v^2}{2}}\,dv, \tag{10}$$

$$\Omega(N,\rho)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(v,w;\rho)Q_2^{N-1}(v,w;\rho)\,dw\,dv, \tag{11}$$

$$Q_2(x,y;\rho) = \int_x^{\infty}\int_y^{\infty} f(v,w;\rho)\,dw\,dv,$$

$$f(x,y;\rho) = \frac{1}{2\pi\sqrt{1-\rho^2}}e^{-\frac{x^2+y^2-2\rho xy}{2(1-\rho^2)}}.$$

In Theorem 2, K is the number of time slots between k_1 and k_2. Furthermore, Q_1 is the standard Q-function and Q_2 is its two-dimensional counterpart, i.e., if X and Y are standard, jointly Gaussian random variables with correlation coefficient ρ, then $Q_1(x)=\mathrm{P}\{X>x\}$ and $Q_2(x,y;\rho)=\mathrm{P}\{X>x,Y>y\}$.

Theorem 3. *Consider the AR-TDMA described by* (1), (2), (7)–(9). *Let* $x_i(k)$ *be specified by Assumption 1 and* Φ *by Assumption 2. Then, for any* $k_1,k_2\in\mathbf{Z}$, $k_1\le k_2$, *any* $k\in\mathbf{Z}$, *and* $i=1,2,\ldots,N$, *Performance Measures P1–P6 are given by*

$$\mathrm{E}\{\bar{p}_i(k_1,k_2)\}=r_d e^{\frac{\sigma_x^2}{2}-\mu_x}\int_{-\infty}^{\infty}\frac{e^{-\frac{v^2}{2}}}{\sqrt{2\pi}}Q_1^{N-1}(v+\sigma_x\rho_{x\tilde{x}}(0))\,dv,$$

$$\mathrm{E}\{\bar{t}_i(k_1,k_2)\} = \frac{1}{N}\Phi(r_d),$$

$$\mathrm{E}\{\bar{t}_i(k_1,k_2)|x_i(k_1)=x_o\} = \frac{\Phi(r_d)}{K}\int_{-\infty}^{\infty}\frac{e^{-\frac{v^2}{2}}}{\sqrt{2\pi}}\Big(\sum_{\ell=0}^{K-1} Q_1^{N-1}\Big(\sqrt{1-\rho_{x\tilde{x}}^2(-\ell)}v-\rho_{x\tilde{x}}(-\ell)\frac{x_o-\mu_x}{\sigma_x}\Big)\Big)\,dv,$$

$$\mathrm{var}\{\bar{t}_i(k_1,k_2)\} = \frac{\Phi^2(r_d)}{K}\Big[\frac{1}{N}-\frac{1}{N^2}+2\sum_{\ell=1}^{K-1}\Big(1-\frac{\ell}{K}\Big)\times\Big(\Omega(N,\rho_{\tilde{x}}(\ell))-\frac{1}{N^2}\Big)\Big],$$

$$\mathrm{E}\{d_i(k)|d_i(k)>0\} = \frac{N-1}{1-N\Omega(N,\rho_{\tilde{x}}(1))},$$

$$\mathrm{E}\{u_i(k)|u_i(k)>0\} = \frac{1}{1-N\Omega(N,\rho_{\tilde{x}}(1))},$$

where $K=k_2-k_1+1$, Q_1 *is given in* (10), Ω *is given in* (11),

$$\rho_{\tilde{x}}(\ell) = \frac{(1+\frac{1}{L})\rho_x(\ell)-\frac{1}{L^2}\sum_{\ell_1=-L}^{L}|\ell_1|\rho_x(\ell+\ell_1)}{1+\frac{1}{L}-\frac{2}{L^2}\sum_{\ell_1=1}^{L}\ell_1\rho_x(\ell_1)},$$

$$\rho_{x\tilde{x}}(\ell) = \frac{\rho_x(\ell)-\frac{1}{L}\sum_{\ell_1=1}^{L}\rho_x(\ell+\ell_1)}{\sqrt{1+\frac{1}{L}-\frac{2}{L^2}\sum_{\ell_1=1}^{L}\ell_1\rho_x(\ell_1)}}.$$

Note that in Theorems 1–3, both $\mathrm{E}\{\bar{p}_i(k_1,k_2)\}$ and $\mathrm{E}\{\bar{t}_i(k_1,k_2)\}$ are independent of k_1, k_2, and i; both $\mathrm{E}\{\bar{t}_i(k_1,k_2)|x_i(k_1)=x_o\}$ and $\mathrm{var}\{\bar{t}_i(k_1,k_2)\}$ are dependent on K and independent of i; and both $\mathrm{E}\{d_i(k)|d_i(k)>0\}$ and $\mathrm{E}\{u_i(k)|u_i(k)>0\}$ are independent of k and i. Hence, below we write them as $\mathrm{E}\{\bar{p}\}$, $\mathrm{E}\{\bar{t}\}$, $\mathrm{E}\{\bar{t}_K|x_o\}$, $\mathrm{var}\{\bar{t}_K\}$, $\mathrm{E}\{d|d>0\}$, and $\mathrm{E}\{u|u>0\}$, respectively.

B. Performance Comparisons

Theorems 1–3 are now utilized to compare the performance of traditional TDMA, R-TDMA, and AR-TDMA. For comparison purposes, we adopt special cases of Assumptions 1 and 2. Namely, $\mu_x=2$, $\sigma_x=2.5$, and $\rho_x(\ell)=0.6(0.99999)^{|\ell|}+0.4(0.98)^{\ell^2}$ (we select this expression for $\rho_x(\ell)$ in order to account for the fast- and slow-changing components of the channel gains). In addition, we assume that Φ is defined by

$$\Phi(r)=\max_{q\in\{2,4,\ldots,32\}}\frac{q}{32}\sum_{j=0}^{\lfloor\frac{32-q}{2}\rfloor}\binom{32}{j}(1-(1-\tfrac{1}{2+r})^5)^j(1-\tfrac{1}{2+r})^{5(32-j)}$$

for $r>0$ and $\Phi(0)=0$. This Φ corresponds to the TDMA system operating in a Rayleigh fading channel using binary frequency shift keying (BFSK) modulation, noncoherent demodulation, and Reed-Solomon codes (see [15] for more details). Finally, we set $\mathrm{E}\{\bar{t}\}=0.08$ (i.e., the long-term average throughput for each of the N users is 0.08) and $L=50$ for AR-TDMA.

Figure 1 shows the $\mathrm{E}\{\bar{p}\}$ needed by every user so that each of them has $\mathrm{E}\{\bar{t}\}=0.08$, as a function of N (typically, $3\le N\le 8$ in TDMA systems). Analyzing Figure 1, we observe that:

- R-TDMA yields at least $11\,\mathrm{dB}$ power-efficiency improvement over traditional TDMA. Moreover, the improvement increases as N increases. For $N=10$, the improvement is as large as $26\,\mathrm{dB}$, i.e., 398 times!
- AR-TDMA is not as power-efficient as R-TDMA, but still offers at least $5\,\mathrm{dB}$ power-efficiency improvement over traditional TDMA. Similarly, the improvement increases with N. For $N=10$, it reaches $12\,\mathrm{dB}$, i.e., 15 times.

Figure 2 represents $\mathrm{E}\{\bar{t}_K|x_o\}$ as a function of x_o, for $K=20$, i.e., the mean of a user's average throughput over $K=20$ time slots, given the user's log-channel gain at the beginning of these time slots. The quantity x_o, which depends strongly on the user's location, is normally distributed with mean $\mu_x=2$ and standard deviation $\sigma_x=2.5$. The dashdot and solid curves correspond to R-TDMA and AR-TDMA, respectively, with each curve corresponding to some N. From Figure 2, we see that:

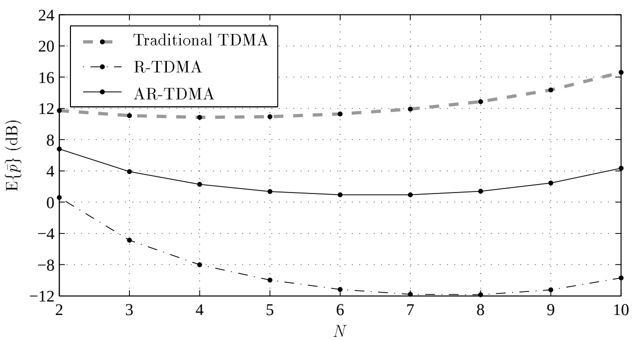

Fig. 1. Power-efficiency comparison

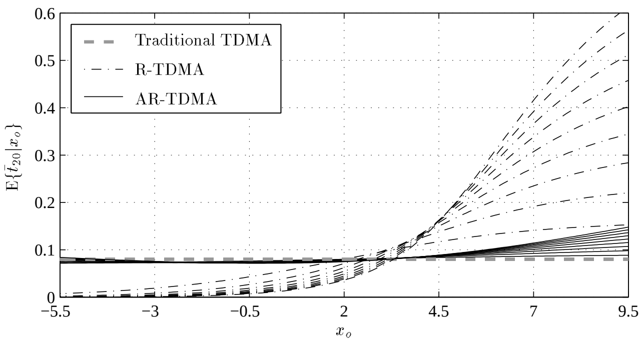

Fig. 2. Location-fairness comparison

- Traditional TDMA achieves ideal location-fairness, as evident by the flatness of the thick dashed line.
- R-TDMA leads to $\mathrm{E}\{\bar{t}_{20}|x_o\}$ that increases significantly as x_o increases, implying that a user can expect a much larger short-term average throughput while in a good location (i.e., large x_o) than in a bad one (i.e., small x_o). Thus, R-TDMA is location-unfair.
- AR-TDMA results in $\mathrm{E}\{\bar{t}_{20}|x_o\}$ that changes only slightly with x_o and stays above or at most 10% below that of traditional TDMA. Hence, AR-TDMA provides excellent location-fairness.

Figure 3 shows the $\mathrm{var}\{\bar{t}_K\}$ (again for $K = 20$), $\mathrm{E}\{d|d > 0\}$, and $\mathrm{E}\{u|u > 0\}$ experienced by every user, due to achieving $\mathrm{E}\{\bar{t}\} = 0.08$, as a function of N. From Figure 3, we observe that:

- As expected, traditional TDMA ensures zero throughput variability, downtime of $N - 1$, and uptime of 1.
- R-TDMA causes large throughput variability, long downtime, and long uptime, relative to traditional TDMA.
- AR-TDMA yields throughput variability, downtime, and uptime that are between those of traditional TDMA and R-TDMA.

As it follows from the above, although R-TDMA is dramatically more power-efficient than traditional TDMA, it is location-unfair and causes large throughput variability and long downtime. Thus, it may be suitable only for delay-insensitive applications. On the other hand, AR-TDMA results in a substantial power-efficiency improvement over traditional TDMA, while ensuring excellent location-fairness, small throughput variability, and short downtime. Hence, it may be recommended for both delay-sensitive and delay-insensitive applications.

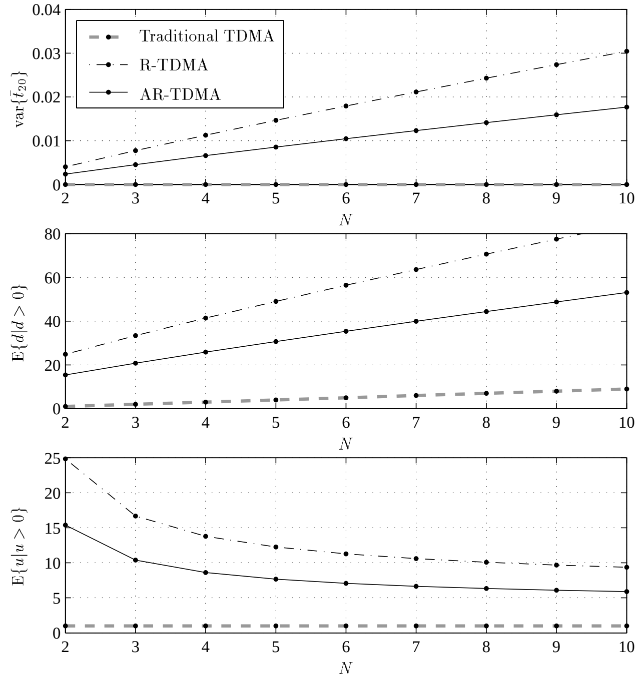

Fig. 3. Throughput variability, mean downtime, and mean uptime comparisons

C. Comparison with an Existing Approach

The literature offers a different approach for improving power-efficiency of TDMA systems without jeopardizing short-term fairness [4], [5], [8]. This approach is based on the idea of allowing users, lagging in information flow, to catch up with leading users. Although a straightforward comparison of this approach with AR-TDMA is impossible due to differences in assumptions, we modify it in such a way that its main idea is preserved and a comparison with AR-TDMA becomes meaningful.

In this framework, the approach of [4], [5], [8] can be described as follows. Let $s_i(k) = 1$ imply that user i is selected to transmit at time slot k, and $s_i(k) = 0$ imply the opposite. Assume that $s_i(k)$ is determined by the following procedure:

$$s_i(k) = \begin{cases} 1, & \text{if } x_i(k)+y_i(k) > x_j(k)+y_j(k) \ \forall j \neq i, \\ 0, & \text{otherwise,} \end{cases} \quad k \in \mathbf{Z}, \tag{12}$$

where $y_i(k)$, $i = 1, 2, \ldots, N$, are updated according to

$$y_i(k) = y_i(k-1) + \varepsilon\Big(\frac{1}{N} - \bar{s}_i(k-M, k-1)\Big), \quad k \in \mathbf{Z}, \tag{13}$$

with $\varepsilon > 0$. In (13), $\bar{s}_i(k-M, k-1)$ is the fraction of time user i was selected to transmit during the past $M \in \mathbf{Z}_+$

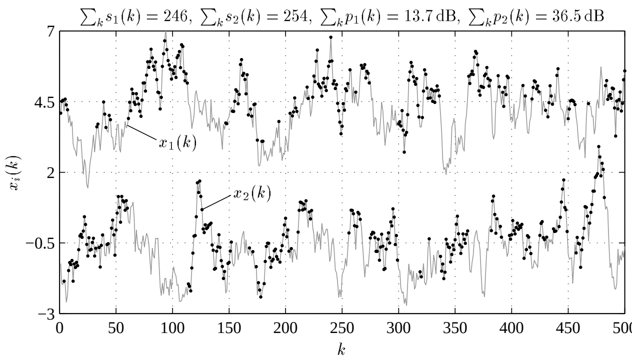

Fig. 4. Behavior of AR-TDMA

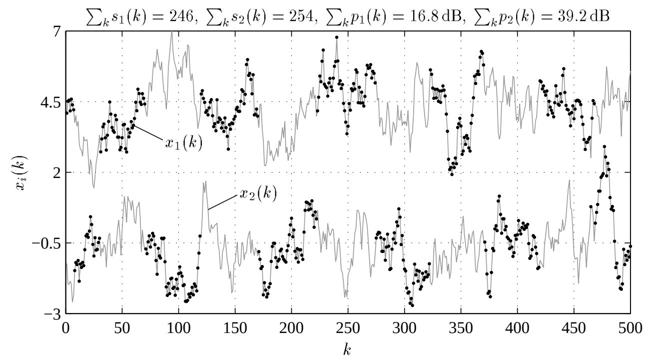

Fig. 5. Behavior of procedure (12)–(14) with $\varepsilon = 1$ and $M = 50$

time slots, i.e.,

$$\bar{s}_i(k-M, k-1) = \frac{1}{M}\sum_{\ell=1}^{M} s_i(k-\ell), \quad k \in \mathbf{Z}, \tag{14}$$

and the quantity $\frac{1}{N}$ is the desired fraction of time. Note that when $\bar{s}_i(k-M, k-1) < \frac{1}{N}$, i.e., when user i is lagging, $y_i(k)$ gradually increases. As a result of (12), the probability of user i being selected to transmit increases. Similarly, when user i is leading, its probability of being selected decreases. Therefore, procedure (12)–(14) preserves the idea of the approach proposed in [4], [5], [8].

To compare procedure (12)–(14) with AR-TDMA, consider a TDMA system with $N = 2$ users. Suppose users 1 and 2 are, respectively, in good and bad locations, so that $x_1(k)$ and $x_2(k)$ are, respectively, large and small on the average. Figures 4 and 5 show identical realizations of $x_1(k)$ and $x_2(k)$ for 500 time slots. The black dots in these figures indicate which user is transmitting at a particular time slot. The numbers above each of these figures represent the total number of time slots assigned to each user and the total power consumed by each. Figure 4 illustrates the behavior of AR-TDMA, while Figure 5 does the same for procedure (12)–(14) with $\varepsilon = 1$ and $M = 50$. Comparing these figures, we conclude that:

- Both AR-TDMA and procedure (12)–(14) are short-term fair, as the total number of time slots assigned to users 1 and 2 are roughly equal, i.e., $\sum_k s_1(k) \approx \sum_k s_2(k)$.
- AR-TDMA always selects the user with the best "local" channel conditions for transmission (see, e.g., $k \approx 50, 100, 240, 350$ of Figure 4). In contrast, procedure (12)–(14) often selects the user, who is experiencing deep fades, for transmission because the user has been lagging (see, e.g., $k \approx 50, 100, 310, 350$ of Figure 5).
- AR-TDMA is more power-efficient than procedure (12)–(14). Indeed, users 1 and 2, respectively, consumed $3.1\,\mathrm{dB}$ and $2.7\,\mathrm{dB}$ less total power under AR-TDMA than under procedure (12)–(14).

VI. CONCLUSIONS

This paper shows that using control of transmission in TDMA (i.e., R-TDMA) leads to a dramatic improvement in power-efficiency without a loss in average throughput for all users. Using both control and adaptation (i.e., AR-TDMA) results in both power-efficiency and location-fairness.

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