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Unknown Inputs Proportional Integral Observers for Descriptor Systems with Multiple Delays and Unknown Inputs

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***Abstract*— Based on the unknown input (UI) proportional observers [4], a systematic full-order unknown input proportional integral observers (UIPIO) is derived for a descriptor system with delayed state and UI. We show that the proportional integral observers proposed, gives robust state estimation face of UI and uncertain parameters. Sufficient conditions for the existence of the observer are given and proved.**

I. INTRODUCTION

Descriptor systems may present impulsive responses. For engineering systems, impulse may deteriorate performances, damage components, or even destroy the system. Therefore, impulsive behavior is undesirable and its elimination is an important issue in both control system and observer design. In addition since descriptor systems are very sensitive to slight input changes, the presence of UI should be considered when designing observers. However few results have been presented for observer design of descriptor systems with UI [2], [4], [6] and very few in presence of time delay [8] and [1].

In [4] a method based on the concept of matrix generalized inverse to design a reduced-order proportional UI observer has been proposed. Following a recent result [5] where only proportional integral observer for biais UI descriptor systems was derived, the design a full-order UIPIO for UI discrete time-delay descriptor systems is proposed using a matrix generalized inverse approach. The main contribution lie in the fact that multiple delays and UI are present in both the state and measurement variables.

II. PROBLEM FORMULATION

In this section, using two regular transformations an equivalent descriptor system without delay and with sub-vector UI decoupled measurements is derived for the purpose of the observer design.
Let us consider the UI discrete-time descriptor system with multiple delays of the form

$$\begin{array}{l} E^* x_{k+1} = \sum_{i=0}^{s} A_i^* x_{k-i} + B^* u_k + F_w^* w_k \\ y_k^* = \sum_{i=0}^{s} C_i^* x_{k-i} + D^* u_k + G_w^* w_k \end{array} \quad (1)$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^k$, $w \in \mathbb{R}^q$ and $y^* \in \mathbb{R}^p$ denote the state vector, the control input vector, the unknown input vector and the output vector, respectively. E^*, $A_i^* \in \mathbb{R}^{m\times n}$, $B^* \in \mathbb{R}^{m\times k}$, $F_w^* \in \mathbb{R}^{m\times q}$, $C_i^* \in \mathbb{R}^{p\times n}$, $D^* \in \mathbb{R}^{p\times k}$ and $G_w^* \in \mathbb{R}^{p\times q}$ are known constant matrices. The integer $s \geq 0$ represents the maximal delay assumed to be known. Let $r := rankE^* < n$ and assume without loss of generality $\begin{bmatrix} C_0^* & \dots & C_s^* & G_w^* \end{bmatrix} = p$.
Since $rankE^* = r$, there exists a regular matrix P such that

$$PE^* = \begin{bmatrix} E \\ 0 \end{bmatrix},\ PB^* = \begin{bmatrix} B \\ B_0 \end{bmatrix},\ PF_w^* = \begin{bmatrix} F_w \\ F_{w_0} \end{bmatrix} \text{ and } PA_i^* = \begin{bmatrix} A_i \\ A_{0_i} \end{bmatrix}$$

for $i = 0, \dots, s$, with $E \in \mathbb{R}^{r\times n}$ and $rankE = r$. Following [1] and [7] (1) can be rewritten as

$$\begin{array}{l} \bar{E}\bar{x}_{k+1} = A\bar{x}_k + \bar{B}u_k + \bar{F}_w w_k \\ y_k = C\bar{x}_k + G_w w_k \end{array} \quad (2)$$

$$\text{where } y_k = \begin{bmatrix} -B_0 u_k \\ y_k^* - D^* u_k \end{bmatrix} \in \mathbb{R}^t,\ \bar{x}_k = \begin{bmatrix} x_k \\ x_{k-1} \\ \vdots \\ x_{k-s} \end{bmatrix} \in \mathbb{R}^{n_a},$$

$$\bar{E} = \begin{bmatrix} E & 0 \\ 0 & I_{ns} \end{bmatrix} \in \mathbb{R}^{r_a \times n_a},\ \bar{B} = \begin{bmatrix} B \\ 0 \\ \dots \\ 0 \end{bmatrix},$$

$$\bar{F}_w = \begin{bmatrix} F_w \\ 0 \\ \dots \\ 0 \end{bmatrix},\ A = \begin{bmatrix} A_0 & A_1 & \dots & A_{s-1} & A_s \\ I_n & 0 & \dots & 0 & 0 \\ 0 & I_n & \dots. & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & I_n & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} A_{0_0} & A_{0_1} & \dots & A_{0_s} \\ C_0^* & C_1^* & \dots & C_s^* \end{bmatrix},\ G_w = \begin{bmatrix} F_{w0} \\ G_w^* \end{bmatrix}$$

with $n_a = n(s+1)$, $r_a = ns + r$ and $t = p + m - r$. Let $rankG_w = q_1 \leq q$, then there exist two regular matrices U and V such that $UG_wV = \begin{bmatrix} I_{q_1} & 0 \\ 0 & 0 \end{bmatrix}$. System (2) can now be written as

$$\bar{E}\bar{x}_{k+1} = \Phi\bar{x}_k + \bar{B}u_k + F_1 y_{1_k} + F_2 w_{2_k} \quad (3)$$

$$y_{1_k} = C_1\bar{x}_k + w_{1_k} \quad (4)$$

$$y_{2_k} = C_2\bar{x}_k \quad (5)$$

where $\Phi = A - F_1 C_1$ with $F_w V = \begin{bmatrix} F_1 & F_2 \end{bmatrix}$,

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = Uy,\ y_2 \in \mathbb{R}^{p_1},\ \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = UC \text{ and } w = V \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

with $p_1 = t - q_1$ and $rank C_2 = p_1$ (since $\begin{bmatrix} C_0^* & \dots & C_s^* & G_w^* \end{bmatrix} = p$, [4])

We aim to find matrices π, K_{p_1}, T, N, K_{p_2} and the gain K_I of the following full order (i.e. $n_a + k$) UIPIO

$$z_{k+1} = \pi z_k + K_{p_1} y_{1k} + K_{p_2} y_{2k} + T\bar{B}u_k + T\bar{B}v_k \quad (6)$$
$$v_{k+1} = v_k + K_I \left(y_{2k} - C_2 \hat{\bar{x}}_k \right) \quad (7)$$
$$\hat{\bar{x}}_k = z_k + N y_{2k} \quad (8)$$
$$\hat{x}_k = \begin{bmatrix} I_n & 0 \end{bmatrix} \hat{\bar{x}}_k \quad (9)$$

where z_k is the state of the observer, such that the state estimation error satisfies (10) without knowing the UI w.

$$\lim_{t\to\infty} e_k = \hat{x}_k - x_k = 0, \quad \forall\ z_0,\ v_0,\ x_0,\ x_{-1}, \dots\ x_{-s} \quad (10)$$

Remark 1: From (2) it can be seen that for a large value of s, the order of matrix A may become very high leading to a high-order state observer. In this case, [7] propose to derive the dynamics of the observer from a set of unstable and/or poorly damped eigenvalues of the system.

III. Observer design

By (8), (9) and (5) the estimation error (10) becomes

$$e_k = \begin{bmatrix} I_n & 0 \end{bmatrix} \left(z_k - \left(I_{n_a} - NC_2 \right) \bar{x}_k \right) \quad (11)$$

If $rank \begin{bmatrix} \bar{E}^T & C_2^T \end{bmatrix} = n_a$, then there exists a full row rank matrix $\begin{bmatrix} T & N \end{bmatrix}$ such that

$$\begin{bmatrix} T & N \end{bmatrix} \begin{bmatrix} \bar{E} \\ C_2 \end{bmatrix} = I_{n_a} \quad (12)$$

and (11) reduces to $e_k = \begin{bmatrix} I_n & 0 \end{bmatrix} (z_k - T\bar{E}\bar{x}_k)$. Let $\tilde{z}_{k+1} = z_{k+1} - T\bar{E}\bar{x}_{k+1}$. One can deduce:

$$\begin{aligned} \tilde{z}_{k+1} &= \pi \tilde{z}_k + \left(\pi T\bar{E} + K_{p_2} C_2 - T\Phi \right) \bar{x}_k \\ &\quad + \left(K_{p_1} - TF_1 \right) y_{1k} + T\bar{B}v_k - TF_2 w_{2_k} \\ v_{k+1} &= v_k - K_I C_2 \tilde{z}_k \end{aligned}$$

If the following constraints hold true

$$\pi T\bar{E} + K_{p_2} C_2 - T\Phi = 0 \quad (13)$$
$$K_{p_1} - TF_1 = 0 \quad (14)$$
$$TF_2 = 0 \quad (15)$$

the difference state estimation error system becomes

$$\begin{aligned} \tilde{z}_{k+1} &= \pi \tilde{z}_k + T\bar{B}v_k \\ v_{k+1} &= v_k - K_I C_2 \tilde{z}_k \end{aligned}$$

Let $K_{p_2} = \pi N + K_p$. By (13) and (12) we obtain

$$\pi = T\Phi - K_p C_2 \quad (16)$$

The state estimation error system is then described by

$$\begin{aligned} \begin{pmatrix} \tilde{z}_{k+1} \\ v_{k+1} \end{pmatrix} &= A_{zv} \begin{pmatrix} \tilde{z}_k \\ v_k \end{pmatrix} \\ e_k &= \begin{bmatrix} I_n & 0 \end{bmatrix} \tilde{z}_k \end{aligned} \quad (17)$$

where $A_{zv} = \left(\begin{bmatrix} T\Phi & T\bar{B} \\ 0 & I \end{bmatrix} - \begin{bmatrix} K_p \\ K_I \end{bmatrix} \begin{bmatrix} C_2 & 0 \end{bmatrix} \right)$.

The main result is the following.

Theorem 1: The full-order UIPIO (6:9) exists if and only if

$$rank \begin{bmatrix} \bar{E} & F_2 \\ C_2 & 0 \end{bmatrix} = n_a + q - q_1 \quad (18)$$

$$\begin{aligned} rank &\begin{bmatrix} zI_{n_a} - T\Phi & -T\bar{B} \\ 0 & zI_k - I_k \\ C_2 & 0 \end{bmatrix} \\ &= n_a + k \quad \forall\ \mathbb{R}(z) \in C, \quad |z| \geq 1 \end{aligned} \quad (19)$$

Proof: With $K_{p_2} = \pi N + K_p$ and $K_{p_1} = TF_1$ the convergence of the UIPIO is proved when A_{zv} is a stability matrix and constraints (12) and (15) hold. First, the stability of A_{zv} is ensured if and only if the pair $\left(\begin{bmatrix} T\Phi & T\bar{B} \\ 0 & I_k \end{bmatrix}, \begin{bmatrix} C_2 & 0 \end{bmatrix} \right)$ is detectable. Now conditions (12) and (15) can be rewritten as

$$\begin{bmatrix} T & N \end{bmatrix} \beta = \begin{bmatrix} I_{n_a} & 0 \end{bmatrix} \quad (20)$$

where $\beta = \begin{bmatrix} \bar{E} & F_2 \\ C_2 & 0 \end{bmatrix}$. A solution $\begin{bmatrix} T & N \end{bmatrix}$ exists if and only if the matrix β is of full column rank [3]. If β is of full column rank., i.e. $rank\beta = n_a + q - q_1$, then a particular solution of (20) is given by ([3] for Y=0)

$$\begin{bmatrix} T & N \end{bmatrix} = \begin{bmatrix} I_{n_a} & 0 \end{bmatrix} \beta^+ \quad (21)$$

where β^+ is the generalized inverse matrix of β, given by $\beta^+ = \left(\beta^T \beta \right)^{-1} \beta^T$. ■

IV. Conclusion

The design and existence conditions of a full-order UIPIO for discrete-time descriptor systems with multiple delays and UI are given. Note that, if large delays may lead to a high order for the observer, a solution may consist to extend the work of [1] where the order of the state observer is independent of the delays. This remains an open problem.

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