Proceeding of the 2004 American Control Conference
Boston, Massachusetts June 30 - July 2, 2004

Algorithms for Air Traffic Flow Management under Stochastic Environments

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***Abstract*— A major portion of the delay in the Air Traffic Management Systems (ATMS) in US arises from the stochastic disturbances such as convective weather. However, in the current practice, the predicted storm zones are completely avoided as if they are deterministic obstacles. As a result, the current strategy is too conservative and incurs a high delay. In this paper, we seek to reduce the system delay through explicitly modelling the dynamic and stochastic nature of the storms and adding recourse in the routing and the flow management problem. We address the *multi-aircraft* flow management problem using a stochastic dynamic programming algorithm, where the evolution of the weather is modelled as a stationary Markov chain. Our solution provides a dynamic routing strategy for *"N-aircraft"* that minimizes the *expected delay* of the overall system while taking into consideration the constraints obtained by *the sector capacities*, as well as avoidance of *conflicts* among the aircraft. Our simulation suggests that a significant improvement in delay can be obtained by using our methods over the existing methods.**

I. INTRODUCTION

Air traffic delay due to convective weather has grown rapidly over the last few years. According to the FAA 2002, flight delays have increased by more than 58 percent since 1995, cancellations by 68 percent. The airspace capacity reduces drastically with the presence of convective weather. The drastic reduction of airspace capacity interrupts traffic flows and causes delays that ripple through the system. Consequently, weather related delays, which are stochastic in nature, contribute to around 80% of the total delay in most of the years in US since 1995.

There has been a major effort to address delay in the traffic flow management problem in the deterministic setting [3], [1], [2], [4], where demand and capacities are considered deterministic. In these works, various traffic flow management algorithms are proposed in order to reduce the system delay, given that the system capacity is exactly known. However, the major contributor of delay is weather, which is probabilistic in nature and cannot be addressed in this framework. In our previous work [5], we incorporated recourse in the planning process where we addressed *the single aircraft* problem using Markov decision processes (where the weather processes is modelled as a stationary Markov chain) and a dynamic programming algorithm. Our approach provides a set of optimal decisions to *a single aircraft* that starts moving towards the destination along a certain path, with the recourse option of choosing a new path whenever new information is obtained, such that the expected delay is minimized. As we addressed the problem in the stochastic framework, we obtain "the best policy".

In this paper, we extend our model for *multiple aircraft*. The problem of routing under convective weather becomes much more complex in a congested airspace because both aircraft conflicts and traffic flow management issues must be resolved at the same time. In this work, we provide a dynamic routing strategy for multiple aircraft that minimizes the expected delay of the overall system while satisfying the consideration of the constraints obtained by the sector capacity, as well as avoidance of conflicts among the aircraft. Moreover, we have used a more general weather dynamic model where the predicted zones can have more than two different states.

II. WEATHER UNCERTAINTY MODEL

Various weather teams (Center for Civil Force Protection (CCFP), Integrated Terminal Weather System (ITWS) etc) produce predictions that some zones in the airspace may be unusable in certain time interval and their predictions are dynamically updated at every $T = 15$ minutes. The later an event is from the prediction time, the more unreliable it becomes. It is reasonable to assume that we have a deterministic knowledge of the weather in the time interval of $0-15$ minutes in future. Hence, each aircraft has a perfect knowledge about the weather in the regions that are 15 minutes (15 times the velocity of the aircraft provides the distance) away from it.

We discretize time as $1,2,...,n$ *stages* according to the weather update. Stage 1 corresponds to $0-15$ minutes from the current time, stage 2 corresponds to $15-30$ minutes from the current time. We choose n that accommodates the worst case routing of the aircraft. Let there be m storms that are predicted to take place at the region $K_1, K_2, ...K_m$. For each K_i, there can be multiple outcomes. Depending on the coverage area and the intensity of the prediction, we allow to have different realizations. Let l is the number of possible outcomes of the prediction in each region. For an example, $K_{11}, K_{12}, \ldots, K_{1l}$, are the possible outcome regions in K_1 ($K_{1i} \subset K_1 \forall i < l$ and $K_{il} = K_1$). "0" corresponds to the situation where there is no storm in the region K_1, "1" corresponds to the state where there is a storm, but only materialized in the region K_{11}, and similarly "l" corresponds to the situation where there is a storm, but only materialized in the region K_{11}. "l" corresponds to the worst possible

Research supported in part by Eurocontrol-014692, and DARPA-F33615-01-C-3150.

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outcome when K_{1l} or the whole region K_1 has been affected by the storm. For each storm, there can be $(l+1)$ different outcomes. As there are m storms , the Markov chain is a $(l+1)^m$ state Markov chain. For $l=2$, and $m=2$: $[0\ 0]^T, [1\ 0]^T, [2\ 0]^T, [1\ 0]^T, [1\ 1]^T, [1\ 2]^T, [2\ 0]^T, [2\ 1]^T$, and $[2\ 2]^T$ form the state space. We define p_{ij} as the probability of the storm state to be j in the next stage if the current state is i.

III. Problem Formulation

We consider a two dimensional flight plan of multiple aircraft whose nominal paths are obstructed by predicted convective weathers. All the aircraft considered here are in the TMA/En-route portion of their flights. Hence, the velocities of all the aircraft considered are constant.

We use a rectangular gridding system to represent the airspace where we consider each grid point as a way point. There are N aircraft currently positioned at $O^1, O^2, \ldots, O^N$ and the destination points of the aircraft are $D^1, D_2, \ldots, D^N$ (Figure 3). $O^i = [O^i(x), O^i(y)]^T \in \mathbf{R}^2$, where $O^i(x)$ and $O^i(y)$ are respectively the x and y coordinates of the origin of aircraft i. Similarly, $D^i = [D^i(x), D^i(y)]^T \in \mathbf{R}^2$, where $D^i(x)$ and $D^i(y)$ are respectively the x and y coordinates of the destination of aircraft i. Without the presence of convective weather, aircraft will try to follow the straight line connecting the origin and the destination, if those paths don't result in conflicts. There is a prediction that there can be m storms located at $K_1, \ldots, K_m$ places such that those zones might be unusable at certain time. $w \in W$ are the weather states and $|W| = (l+1)^m$. The airspace that is considered here is confined in f sectors, and the capacities of the sectors are $C_1, \ldots, C_f$.

The predictions are dynamically updated with time. In the current practice, these stochastic convective zones are assumed to be completely unusable, and solution proceeds as if they are deterministic constraints. As those zones were just predicted to be of unusable with a certain probability, it often turns out that the zones were perfectly usable. As the routing strategies do not use these resources, airspace resources are under-utilized, leading to congestion in the remaining airspace through ripple effect. In our proposed

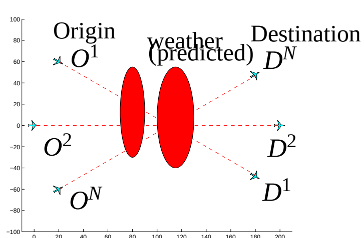

Fig. 1. A 2-D view of the problem.

model, we will not exclude the zones which are predicted to be unusable (with some probability) at a certain time and we will take into consideration the fact that there will more updates with the course of flight and recourse will be applied accordingly. We take a less conservative route in avoiding the bad weather zone where we take a risk in delay to attain a better expected delay instead of avoiding the bad weather zones deterministically. We consider the following two schemes,

1) All of the N aircraft have the same priority.
2) Every aircraft has a different priority. Without the loss of generality, we assume that (priority of aircraft 1)>(priority of aircraft 2)> … >(priority of aircraft N).

Scheme 1: All of the N aircraft have equal priority.

At each stage (15 minutes time span, before the next update), the storm state is assumed to stay constant. $X^1 \in \mathbf{R}^2, X^2 \in \mathbf{R}^2, \ldots, X^N \in \mathbf{R}^2$ represent the locations of aircraft. If the state of the convective weather is w (as section II), we define a state of the system as $s = (w, X^1, \ldots, X^N) \in \mathbf{R}^{2N+1}$, which represents the positions of all the aircraft and the storm situation . Furthermore, we define S as the set of all possible states s. At any stage t, we want to choose an action (the directions of all the N aircraft to follow) from the set of allowable actions in state s, A_s. Actions are the directions of all the N aircraft to follow in each stage with different realizations of the weather. We can obtain the action A_s, that is the directions of the aircraft to follow if we calculate the points that can be reached by each aircraft in the next 15 minutes, given their current positions. This can be approximately calculated if we draw an annular region with $15 \times v_{AC} \pm \varepsilon$ as radii, with a predefined angle θ and checking which grid points fall in the region. Let, $A = \cup_{s \in S} A_s$. We assume that S and A do not vary with time.

If we decide to choose an action (the directions of all the N aircraft to follow) $a \in A_s$ in state s at the stage t, we pay a cost $c_t(s,a)$, which is the sum of distances travelled by N aircraft with the action a for a time interval of 15 minutes. For notational simplicity, we assume that the velocities of all the aircraft are equal (v_{AC}). Hence, if we minimize the expected distance travelled, we minimize the expected delay. (Exactly the same optimization formulation holds even if the velocities of the aircraft are different from each other: where we need to multiply the cost functions with appropriate multiplying factors). Furthermore, we define an indicator function $I_{kj}(X^k)$, whose value is 1 if X^k is in the sector j, and 0 otherwise. In our optimization problem, we will like to minimize the expected sum of the distances travelled by N aircraft, while resolving all the potential conflicts and satisfying the sector capacity constraints. The optimization problem can be written as,

$$\min_{a \in A} \mathbf{E}_s \left(\sum_{t=T}^{nT} c_t(s,a) \right)$$
$$\text{s.t.} \|X^i(t,w) - X^{j \neq i}(t,w)\|_2 > r,\ \forall i, j\ \forall\ t \forall\ w,$$
$$\sum_k I_{kj} \le C_j\ \forall\ 0 \le j \le f,$$

where r is the minimum permissible separation distance between two aircraft, and $\|.\|_2$ is the euclidian norm.

Scheme 2: (priority of aircraft 1)$>$(priority of aircraft 2)$> \ldots >$(priority of aircraft N)

This is a sequential optimization problem and the steps are as follows,

Step 1**:** In this step, we assume that there is only one aircraft in the airspace and that is aircraft 1, which has the highest priority. The state of the optimization problem is defined as $s_1 = (w, X^1) \in S_1$, where w is the storm state and X^1 is the position of the aircraft 1. If we decide to choose an action $a_1 \in A_{s1}$ in state s_1 at the stage t, we pay a cost $c_t^1(s_1, a_1)$, which is the distance travelled by aircraft 1 with the action a_1. We optimize the following problem,

$$\min_{a_1 \in A_{s1}} \mathbf{E}_{s_1} \left(\sum_{t=T}^{nT} c_t^1(s_1, a_1) \right)$$

If we solve this optimization problem, we obtain the optimal policy a_1^o which provides us $X_{opt}^1(t, w)$, the optimal position of aircraft 1 for all time t and for all the storm state w.

Step 2**:** In this step, we only consider aircraft 2, which has the second highest priority. The state of the optimization problem $s_2 = (w, X^2) \in S_2$. At any stage t, we want to choose an action (the direction aircraft 2 to follow) from the set of allowable actions in state s_2, A_{s2}. Let, $A_2 = \cup_{s_2 \in S_2} A_{s2}$. For the aircraft 2, we solve the following optimization problem,

$$\min_{a_2 \in A_{s2}} \mathbf{E}_{s_2} \left(\sum_{t=T}^{nT} c_t^2(s_2, a_2) \right)$$
$$\|X^2(t,w) - X_{opt}^1(t,w)\|_2 > r, \ \forall \ \forall \ t \ \forall \ w,$$

and obtain $X_{opt}^2(t, w)$.

Step 3- **Step** N**:** We keep following the same procedure till we have solved for all N aircraft. For the Nth aircraft,which has the lowest priority, the optimization problem is the following,

$$\min_{a_N \in A_{sN}} \mathbf{E}_{s_N} \left(\sum_{t=T}^{nT} c_t^N(s_N, a_N) \right)$$
$$\text{s.t.} \|X^N(t,w) - X_{opt}^1(t,w)\|_2 > r, \ \forall \ t \ \forall \ w,$$
$$\|X^N(t,w) - X_{opt}^2(t,w)\|_2 > r, \ \forall \ t \ \forall \ w,$$
$$\ldots$$
$$\|X^N(t,w) - X_{opt}^{(N-1)}(t,w)\|_2 > r, \ \forall \ t \ \forall \ w,$$
$$\sum_k I_{kj} \leq C_j \ \forall \ 0 \leq j \leq f,$$

where $X_{opt}^1, \ldots, X_{opt}^{(N-1)}$ are obtained from previous iterations.

In both of the schemes, we look for the "best policy". Determining the "best policy" is to decide where to go next given the currently available information. We consider the set of decisions facing all of the aircraft that start moving towards the destination along a certain path, with the recourse option of choosing a new path whenever a new information is obtained.

IV. Markov Decision Process

Let a system has the finite state space S, and the finite action set A. Moreover, we denote by $P = (P^a)_{a \in A}$ the collection of transition matrices, and by $c_t(i_t, a_t)$ the cost corresponding to state i_t and action a_t at time t. If we are trying to solve the optimization problem, which is to minimize the expected cost over a finite horizon:

$$\min_{a \in A} \mathbf{E}_i \left(\sum_{t=T}^{NT} c_t(i_t, a_t) \right)$$

the value function can be computed via the Bellman recursion

$$V_t(i) = \min_{a \in A} \left(c_t(i, a) + \sum_j P^a(i, j) V_{t+1}(j) \right),$$

which provides the optimal action at each stage [6].

V. Solution of Scheme 1

We propose a Markov Decision Process algorithm to solve the traffic flow management where each of the N aircraft has same priority. The steps of the algorithm are as follows,

Step 1**: Preliminary calculations**

The state of the Markov Decision Process (MDP) for this problem is $s = (w, X^1, X^2, \ldots, X^N)$ ($(2N+1)$ tuple vector) and $s \in S$ (defined in the section III). If we discritize the airspace by D number of nodes, $|S| = D^{2N}(l+1)^m$. There are n stages in this MDP (obtained in section II). In addition, we need to calculate the action set A of the MDP as described in the previous section. Once we have the set of all possible controls, we readily obtain P^a from the weather data.

Step 2**: Assigning appropriate costs**

We assign costs in such a way that our algorithm provides paths that include going through the zones in the absence of storms while avoid it if there is a storm. Furthermore, it should also make sure that there is no conflict among N aircraft. We define $c(w, X^1, \ldots, X^N, (X^{1i}), \ldots, (X^{Ni}))$ as the sum of all $1 \leq k \leq N$ costs obtained by aircraft k to go to X^{Ki} from X^k.

Provision 1: Avoid if storm, otherwise take a shortcut

We introduce a function $PROV_1 : \mathbf{R}^{4N+1} \to \{0, 1\}$ in order to provide us the provision of avoiding a zone if there is a storm, otherwise taking a shortcut.

Let the storm state w corresponds to the fact that $K_{w^1 1} \subseteq K_1, \ldots, K_{w^N N} \subseteq K_N$ are the zones that will be affected by the convective weather. We further define $K_w = \cup_i K_{w^i i}$.

If there exists a k $(1 \leq k \leq N)$ such that $(X^{ki} \in K_w)$ or (there exists a $0 \leq \lambda \leq 1$ such that $\lambda X^{ki} + (1-\lambda) X^k \in K_w$, which means that the line connecting the points X^k, X^{ki} cut any of the predicted storm zone)

$PROV_1(w, X^1, \ldots, X^N, X^{1i}, \ldots, X^{Ni}) = 1$,

else

$PROV_1(w, X^1, \ldots, X^N, X^{1i}, \ldots, X^{Ni}) = 0$

endif.

Provision 2: Avoid conflict among each other

First, we introduce a function $CF_{v_1v_2}: \mathbf{R}^2 \times \mathbf{R}^2 \times \mathbf{R}^2 \times \mathbf{R}^2 \rightarrow \{0,1\}$, where it takes the origin and destination points of two aircraft with velocities v_1 and v_2 and provides "$1''$ if they are in conflict and "$0''$ otherwise. We will demonstrate how to obtain $CF_{v_1v_2}(I_1,F_1,I_2,F_2)$, where I_j is the initial point and F_j is the final point of the aircraft j. At time t, the positions of aircraft 1 and 2 are $I_1 + \frac{F_1-I_1}{\|F_1-I_1\|_2}v_1t$ and $I_2 + \frac{F_2-I_2}{\|F_2-I_2\|_2}v_2t$ respectively. The distance between them at time t, $d(t) = \|\Delta I - (\Delta W)t\|_2$, where $U_1 = \frac{F_1-I_1}{\|F_1-I_1\|_2}$, $U_1 = \frac{F_2-I_2}{\|F_2-I_2\|_2}$, $\Delta I = I_2 - I_1$, and $\Delta W = (v_1U_1 - v_2U_2)$. $\arg\min_t d(t) = \arg\min_t (\Delta I - t\Delta W)^T(\Delta I - t\Delta W)$. In order to find the optimal time t^* at which two aircraft come to the closest point, we set $\frac{\partial}{\partial t}(\Delta I - \Delta W t)^T(\Delta I - \Delta W t) = 0$. Solving this, we obtain $t^* = \frac{\Delta W^T \Delta I}{\Delta W^T \Delta W}$. If $t^* < 0$, the two aircraft are diverging, hence $CF_{v_1v_2}(I_1,F_1,I_2,F_2) = 0$. Also, if $\|\Delta I + t^*\Delta W\|_2 > r$, $CF_{v_1v_2}(I_1,F_1,I_2,F_2) = 0$, else $CF_{v_1v_2}(I_1,F_1,I_2,F_2) = 1$.

In this problem, as we have assumed that all the aircraft are flying at the same speed, we can write $CF(.,.,.,.)$ instead of $CF_{v_1v_2}(.,.,.,.)$. In addition, we introduce a function $PROV_2 : \mathbf{R}^{4N} \rightarrow \{0,1\}$ that provides us the provision of conflict avoidance.

If $\forall l,k \; l \neq k$, $\|X^{li} - X^{ki}\|_2 > r$ or $CF(X^l, X^{li}, X^k, X^{ki}) = 0$,
$PROV_2(X^1,\ldots,X^N,X^{1i},\ldots,X^{Ni}) = 0$,
else
$PROV_2(X^1,\ldots,X^N,X^{1i},\ldots,X^{Ni}) = 1$
endif.

Finally, the cost function is defined as following,
if $PROV_1(w,X^1,\ldots,X^N,X^{1i},\ldots,X^{Ni}) = 1$ or $PROV_2(X^1,\ldots,X^N,X^{1i},\ldots,X^{Ni}) = 1$,
$c(w,X^1,\ldots,X^N,X^{1i},\ldots,X^{Ni}) =$ A very high value,
else
$c(w,X^1,\ldots,X^N,X^{1i},\ldots,X^{Ni}) = \sum_{k=1}^{N} \|X^{ki} - X^k\|_2$
endif.

Step 3: Assigning appropriate Value function

We define $V_t(s)$ as the value function which is the expected minimum distance to go if the current state is s and the current stage is t. We need to add the following provisions in the value function in order to obtain the correct solution.

Provision 1: Reach the destination points

The value function should have boundary conditions such that we obtain a complete path (path stating at the origin and ending at the destination) as a solution. For the destination points $D^1,\ldots,D^N$, the conditions below would guarantee that the solution will provide a complete path.

For any state weather state w and the last stage n,
if $\{X^1 = D^1\},\ldots,\{X^N = D^N\}$
$V_n(w,X^1,\ldots,X^N) = 0$,
else
$V_n(w,X^1,\ldots,X^N) =$ a very high value
endif.

Provision 2: Direct cost at the end of the flight

We assign the boundary values to the value function for the states which corresponds to the aircraft locations that are less than $15v_{AC}$ apart from the destination points. Let the state corresponds to the aircraft location of $X^1,\ldots,X^N$ and $(\|(X^i - D^i\|_2 \leq 15v_{AC} \; \forall i)$ and the stage $t > 1$.

If (w corresponds to the storm state such that no storm zone intersects the straight line $\{\lambda X^k + (1-\lambda)D^k$ and $0 \leq \lambda \leq 1\}$)
$V_t(w,X^1,\ldots,X^N) = \sum_k \|X^k - D^k\|_2$, for any $t > 1$,
else
$V_t(w,X^1,\ldots,X^N) = \infty$
endif.

Provision 3: Sector Capacity

In order to ensure that the total number of aircraft in a sector at any time does not exceed the sector capacity, we assign value function appropriately. For a state $s = (w,X^1,\ldots,X^N)$, if there exists at least one j such that $\sum_k I_{kj} > Cj$, then $V_t = \infty$.

Step 4: Implementing the recursive equations

The recursive equation that solves the problem is as follows,

$$V_t(s) = \min_{a \in A}\{c(s,a) + \sum_{s'} P^a(s,s')V_{t+1}(s')\}.$$

We use the backward dynamic programming technique to solve these equations. We start with the final stage and go back iteratively to the first stage and obtain solutions for every stage and for every state. At the first stage, the solution is readily obtained as we know the current state. The aircraft will continue flying according to the solution until a new update is obtained. At the next stage, we will receive a new update, which corresponds to a new state. As we have already calculated all the optimal control for all possible states, we just check the vector $V_2(.)$ and obtain the control. The aircraft will proceed in this way till they reach the destination points (checking the vector $V_n(.)$). In this way, we compute a routing strategy that provides the minimum expected delay.

VI. SOLUTION OF SCHEME 2

In this scheme, we assume that (priority of aircraft 1)>(priority of aircraft 2)> $\ldots$ >(priority of aircraft N) (as described in the section III).

Step 1: Optimal route for aircraft 1

In this step, we find the optimal route for aircraft 1, which has the highest priority. In a sense, we assume that there is no aircraft in the airspace. The MDP state $s_1 = (w,X^1) \in \mathbf{R}^3$ and $s_1 \in S_1$ (defined in the section III). We discretize the airspace and time in a same way as described in section V and section II, which yields $|S_1| = D^2(l+1)^m$, and n stages in this MDP. Actions of this MDP are the directions of aircraft 1 to follow in each stage with different realizations of the weather. As described in section V, we obtain the action a_1, that is the directions of aircraft 1 to follow if we calculate the points that can be reached by aircraft 1 in the next 15 minutes, given its current position. We define $c_1(w,X^1,X^{1i})$ as the cost to go if the aircraft 1 goes from X^1 to X^{1i} in a stage. For assigning the appropriate value, we only need to add provision 1 (calculation of which is same as described in section V: step 2: provision 1). As there is only one aircraft, there is no need to add the provision for conflict avoidance. The value function for the MDP is defined as $V_t^1(s_1)$, which is the expected minimum distance to go if the current state is s_1 and the current stage is t. In the value function, we add the the first two provisions (the calculation is same as described in section V: step 4: provision 1and 2). As there is no other aircraft in the airspace, we do not need to add any provision that require satisfying the sector capacities. Once we have assigned all the boundary values properly, we can solve the following recursion,

$$V_t^1(s_1) = \min_{a_1 \in A_1} \{c_1(s_1,a_1) + \sum_{s_1'} P^a(s_1,s_1')V_{t+1}^1(s_1')\},$$

and we obtain the solution of the recursion which provides us $X_{opt}^1(t,w)$.

Step $2-N$: Optimal route for aircraft $2-N$

We follow the same procedure in the next steps, except we add the provisions that prohibit conflicts and satisfy the sector capacity constraints. In the the second step, we add the conflict avoidance provision in the cost function, where $X_{opt}^1(t,w)\ \forall\ t\ \forall w$ are considered "NO-GO" zones. We use the same procedure described in section V: step 3: provision 2, where we assign high cost for violating these constraints. In each iteration, we keep a record of I_{kj} and assign a very a high value to the value function if the sector capacity constraints are violated. When we solve the appropriate recursion, these additional features will guarantee the conflict avoidance and satisfy the sector capacity constraints. For the Nth aircraft, which has the lowest priority, the provision 2 for cost function will be as follows, if $\forall 1 \leq k \leq N-1$,
$\|X^{Ni} - X^{ki}\|_2 > r$ or $CF(X^N, X^{Ni}, X_{opt}^k, X_{opt}^{ki}) = 0$,
$PROV_2^N(X^N, X^{Ni}) = 0$,
else
$PROV_2^N(X^N, X^{Ni}) = 1$
endif.

The cost function will be defined in the same way (section V) using these two provisions. Also, $V_t^N(.)$ is very high for the states which violates the sector capacity constraints. With this provisions in the cost and value functions, we can solve the recursion and obtain optimal strategy for all of the N aircraft that minimize the delay, given the above priority scheme. This scheme avoids combinatorial explosion as $|S_1 = |S_2| = \ldots = |S_N| = D^2(l+1)^m$. On the other hand, as this is a more constrained optimization problem, it yields higher delays than the scheme 1.

VII. Simulation

In this section, we discuss the results of the implementation of both algorithms in various scenarios involving dynamic routing and traffic flow management of multiple aircraft under uncertainty.

We have implemented both algorithms in MATLAB and we ran our experiment on a standard PC. In the first experiment, there are two aircraft with origins at $O^1 = [0,96]^T$, and $O^2 = [0,-96]^T$, and destinations at $D^1 = [312,-96]^T$ and $D^2 = [312,96]^T$ (all the units in n.mi.). The velocities of the aircraft are 480 n.mi/hour. There is a prediction of a convective weather. The storm zone is a rectangle whose corner points are $[168,96]^T$, $[168,-96]^T$, $[192,-96]^T$, $[192,96]^T$, which may obstruct the nominal flight path of the aircraft. Moreover, there is a critical airspace within this zone which will definitely be affected if the the storm takes place. The critical zone is assumed a rectangle with corner points $[168,60]^T$, $[168,-60]^T$, $[192,-60]^T$, $[192,60]^T$ (the shaded zone in figure 2). We assume that the weather information of the portion of the airspace that can be reached in 15 minutes is deterministic and the probability of the storm propagates in a Markovian fashion with time. Also, the minimum separation distance between two aircraft, $r = 5$(n.mi) in this example. The weather update is received once every 15 minutes. We discritize the time in 15 minutes time intervals (stages). We define "0" as the state when there is no storm, "1" as the state when only the critical zone is affected by the storm, and "2" as the state when the whole predicted zone has been affected by the storm. The prediction matrix is a follows,

$$P = \begin{pmatrix} 0.4 & 0.4 & 0.2 \\ 0.33 & 0.33 & 0.33 \\ 0.33 & 0.33 & 0.33 \end{pmatrix}.$$

$P(i+1,j+1)$ corresponds to the probability that the storm state will be j in the next stage, if the current storm state is i; i.e., $P(2,1) = 0.3$ means that the probability that the storm state will be 1 (no storm) in the next stage is 0.3 given the current storm state is 0 (only critical zone is affected by the storm).

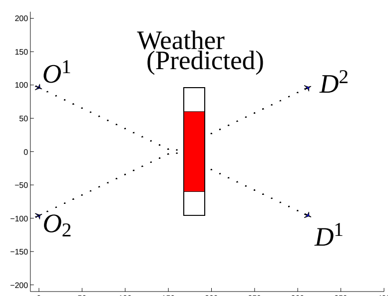

Fig. 2. Routing of two aircraft.

In this scenario, we implemented both of our strategies (scheme 1 and 2) and compared their performances with the traditional strategy (TS) where the convective zone is avoided as if it is a deterministic obstacle while avoiding conflicts. We define " Delay Measure (DM_i^A)" as the extra flight path required by aircraft i in a strategy A in excess of the nominal flight path (which is the Euclidean distance between the origin and the destination; 366.34 (n.mi.) in this problem); $DM_{ij}^A = DM_i^A + DM_j^A$. Furthermore, we introduce a performance metric "Improvement Measure ($IM_i^{A/B}$)" of Strategy 'A' over 'B', which is the percentage of the maximum possible improvement gained for aircraft i by using strategy 'A' instead of using strategy 'B'; $IM_i^{A/B} = 100 \times \frac{DM_i^B - DM_i^A}{DM_i^B}$, and $IM_{ij}^{A/B} = 100 \times \frac{DM_{ij}^B - DM_{ij}^A}{DM_{ij}^B}$. Higher IM corresponds to better delay. In TS, if we resolve the conflict, aircraft 1 and 2 follow paths with a length of 455.12n.mi). $DM_1^{TS} = DM_2^{TS} = 455.12 - 366.34 = 88.78$, and $DM_{12}^{TS} = DM_1^{TS} + DM_2^{TS} = 177.56$.

Using the scheme 1, where both aircraft have equal priority, aircraft 1 and 2 will initially follow a path with an angle of 30.96^0 and -30.96^0 respectively till they get the next update. Both of them will avoid the storm zone when there is a storm and take a direct route if there is no storm and the solution of the strategy is conflict free. In this way, both the aircraft follow a flight path that yield a expected delay of 398.67 (n.mi.). $DM_1^1 = DM_2^1 = 32.33$, $IM_1^{1/TS} = IM_2^{1/TS} = IM_{12}^{1/TS} = 100 \times \frac{88.78-32.33}{74.78} = 63.58\%$. Similarly if we use scheme 2, where aircraft 1 has higher priority over the aircraft 2, aircraft 1 and 2 initially fly at an angle -36.86^0 and 53.13^0 till they get the next update. Similar to the scheme 1, both of them will avoid the storm zone when there is a storm and take a direct route if there is no storm and the solution of the strategy is conflict free. The expected distance travelled by the aircraft are 382.08 (n.mi.) and 426.24(n.mi.) respectively; $IM_1^{2/TS} = 78.95\%$, $IM_2^{2/TS} = 40.18\%$, and $IM_{12}^{2/TS} = 56.76\%$. The summary of the result is presented in I. We observe that we obtain a

	IM of Scheme 1 over TS	IM of Scheme 2 over TS
Aircraft 1	56.71%	78.95 %
Aircraft 2	67.72%	40.18%
System	63.03 %	56.76%

TABLE I

IMPROVEMENT COMPARISONS

better system delay in case of scheme 1. However, in real life, depending upon the aircraft type, size, and hub and spoke network, it might be more reasonable to prioritize the routing strategy. Moreover, the computation time for scheme 2 is 8.31 seconds, which is much faster than the computation time for scheme 1 (7.46 minutes). We can avoid a combinatorial explosion in case of scheme 2 and can handle large number of aircraft. There is no significant computation cost in adding an extra aircraft. In order to illustrate this point, if we add one more aircraft in the system with $O^3 = [0,0]^T$ and $D^3 = [360,0]^T$ (figure 3), our algorithm gives the routing strategy for aircraft 3 with an additional 4.84 secs. Aircraft 3 should have an initial angle of 14.036^0, which yields $IM_3^{2/TS} = 34.62\%$ (for aircraft 3) and $IM_{123}^{2/TS} = 51.23\%$ (for the system). In addition, we ran an experiment for the same weather prediction where a platoon of aircraft are positioned at $O^1 = [48,48]^T$, $O^2 = [24,24]^T$, and $O^3 = [0,0]^T$. The destination point for all of the them is $[360, 0^T]$ and (priority of aircraft 1)>(priority of aircraft 2)>(priority of aircraft 3). Initial vector provided by our algorithms for the aircraft are 33.61^0, 36.65^0, and 38.65^0. The system level IM obtained by using scheme 2

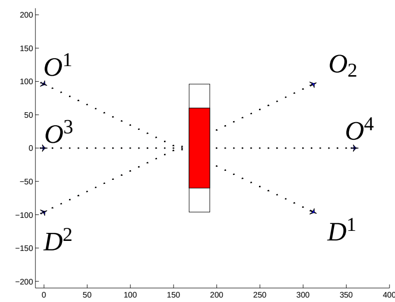

Fig. 3. Routing of three aircraft.

over TS is 58.35% (figure 4).

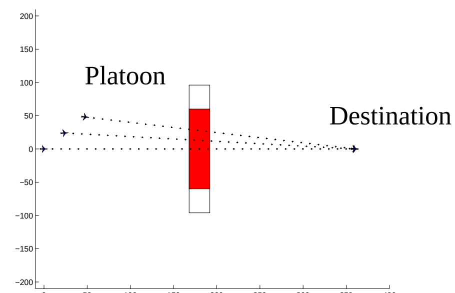

Fig. 4. Routing of a platoon of aircraft.

VIII. CONCLUSION

We provide a traffic flow management tool that can assist the Air Traffic Controllers and the Airline Dispatchers in managing traffic flow dynamically and routing multiple aircraft under weather uncertainty. Our proposed strategies deliver a less circuitous route for an aircraft whose nominal path is potentially obstructed by weather. Moreover, they inhibit the overloading of aircraft in the neighboring sectors of the predicted storm zones, thus the ripple effect of delay due to convective weather is restricted. As a result, our algorithms provide a traffic flow management scheme that minimizes the expected delay of the system.

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