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Similarity Conditions for Comparing Closed-Loop Vehicle Roll and Pitch Dynamics

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***Abstract*— Previous work has demonstrated that dynamic matching and similitude between controllers can be achieved for planar dynamic motion by careful construction of the test vehicle and testing under conditions that match selected parameters. Central to this analysis was the use of the Buckingham Pi theorem as applied to the planar vehicle dynamics. Increasingly, scale vehicle testbeds are being used to investigate *non-planar* dynamics such as roll, pitch, and jackknifing for tractor-trailer systems, but little investigation has been given to conditions of dynamic similarity in these cases. Additionally, some authors have questioned how to scale non-kinematic dynamics that also influence chassis behavior, for instance ABS subsystems or actuator dynamics. This work presents solutions to these questions by presenting methods to derive conditions for similitude for non-planar and non-kinematic dynamic motion.**

***Index Terms*—Vehicle, scaling, similitude, dimensional analysis, roll, pitch.**

I. Introduction

Most standard methods of automating and analyzing vehicle chassis dynamics initially focus on a simplified, linear planar vehicle model known as the bicycle model [1,2], historically explained in detail by Dugoff, Fancher, and Segel [2]. Although the bicycle model is relatively simple, many investigations have verified that it remains a good approximation for full-size vehicle dynamics as long as accelerations are limited to 0.3 g's [3]. While this is suitable for vehicle control when the control requirements are not too demanding, increasing focus is on the analysis of roll and pitch behavior of vehicles in order to better control chassis motion in more demanding and emergency-type maneuvers. Bicycle model dynamics may not be suitable by themselves for controller design in these scenarios.

Previous work by the author examining the planar bicycle model vehicle dynamics revealed that vehicle size, mass, and velocity could be normalized in a manner that greatly facilitated vehicle-to-vehicle comparisons of dynamic behavior. This research identified average vehicle characteristics, a well-defined distribution and range in normalized vehicle parameters, and a duality between velocity and tire-force gain-scheduling methods that were previously overlooked [4-6].

The use of a scale-vehicle testbed (Fig. 1) originally motivated research into methods of normalizing the planar chassis behavior in a size-independent manner. Scale testbeds provide several advantages over full-scale vehicle testing. First, the availability of scale components makes construction faster and cheaper: a new vehicle/test design of moderate complexity can be built from scratch in about 100 person-hours for less than $2000. The vehicles are extremely durable and small enough to allow physical intervention during an impending accident. Roadway scheduling and public safety is not issue, and the simulated roadway surface can be varied quickly and easily to emulate different road surfaces and conditions. Testing is generally very repeatable: the vehicle can intentionally be crashed, spun, nudged, lifted, and/or otherwise destabilized in an easy manner. Finally, hobbyists in the radio-controlled vehicle community have given considerable design attention to creating scale-similar components, which greatly aids the controls engineer in ensuring dynamic similarity between scale and full-sized vehicles.

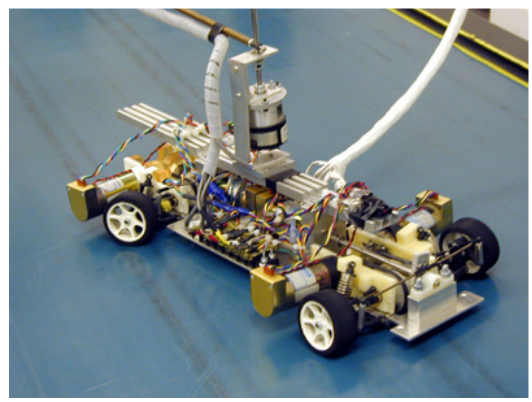

Fig. 1. The use of a scale vehicle testbed originally motivated a size-independent, comparative study of vehicle chassis dynamic parameters .

A second-generation scale-vehicle and roadway testbed is being built to evaluate dynamics and control of large size heavy-vehicles. Naturally, an analysis of similarity conditions for non-planar and roll behavior is of particular interest.

II. Similarity Criteria for Planar Dynamics

In order to develop similarity conditions for roll and pitch dynamics, the conditions for similarity of planar dynamics

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must first be understood, as these dynamics generally act to excite off-planar motion. The planar motion bicycle model can be described by equations of the form:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \qquad (1)$$
$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)$$

with

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{C_{\alpha f}+C_{\alpha r}}{mU} & \frac{C_{\alpha f}+C_{\alpha r}}{m} & \frac{b\cdot C_{\alpha r}-a\cdot C_{\alpha f}}{mU} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{b\cdot C_{\alpha r}-a\cdot C_{\alpha f}}{I_z\cdot U} & \frac{a\cdot C_{\alpha f}-b\cdot C_{\alpha r}}{I_z} & -\frac{a^2\cdot C_{\alpha f}+b^2\cdot C_{\alpha r}}{I_z\cdot U} \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ \frac{C_{\alpha f}}{m} \\ 0 \\ \frac{a\cdot C_{\alpha f}}{I_z} \end{bmatrix} \qquad (2)$$

The C matrix depends on available measurements and the D matrix is zero. The parameters are given by:

- m = vehicle mass
- I_z = vehicle moment of inertia
- U = vehicle longitudinal velocity
- a = distance from C.G. to front axle
- b = distance from C.G. to rear axle
- L = vehicle length, a + b
- $C_{\alpha f}$ = cornering stiffness of front 2 tires
- $C_{\alpha r}$ = cornering stiffness of rear 2 tires

The above parameters are listed below with their dimensional *exponents* in columns, where each row stresses the physical dependence on the standard dimensional units:

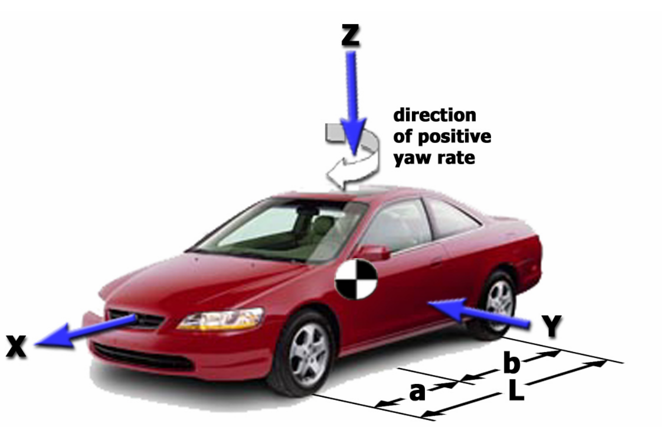

Fig. 2. Vehicle coordinates and parameters

(3)

	kilograms	*meters*	*seconds*	m	L	U	a	b	$C_{\alpha f}$	$C_{\alpha r}$	I_z
kilograms	1	0	0	1	0	0	1	1	1	1	1
meters	0	1	0	0	1	1	0	0	1	1	2
seconds	0	0	1	0	0	−1	0	0	−2	−2	0

As an example, the I_z term has units of kg-m^2, so the dimensional exponents are 1, 2, and 0 for kg, m, and s, respectively. One insight formalized in the Buckingham-Pi Theorem is that the use of the external parameters, *meters-kilograms–seconds* is a design choice. Instead of using the SI system to measure dimensions, an alternative parameter dimension set could used as long as it involves any three parameters that dimensionally span the mass, length, and time unit dimensions. For instance, we may choose to use the parameter set of vehicle mass, *m*, vehicle length, *L*, and vehicle velocity, *U*, to create a *'vehicle-mLU'* system. Under this definition, we have a dimensional basis given by the three column vectors from the matrix in Eq. (3):

$$B_{m\times m} = \begin{array}{c} \begin{matrix} m & L & U \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix} \end{array} \qquad (4)$$

and a remaining parameter set (dropping the unnecessary external parameters of meters, kilograms, and seconds) given by:

$$P_{m\times n-m} = \begin{array}{c} \begin{matrix} a & b & C_{\alpha f} & C_{\alpha r} & I_z \end{matrix} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & -2 & -2 & 0 \end{bmatrix} \end{array} \qquad (5)$$

If we calculate $B^{-1}{}_{m\times m}\cdot P_{m\times n-m}$, we obtain the conversion of dimensional powers from SI to the *vehicle-mLU* dimensions:

$$B^{-1}{}_{m\times m}P_{m\times n-m} = \begin{array}{c} \begin{matrix} a & b & C_{\alpha f} & C_{\alpha r} & I_z \end{matrix} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & -1 & -1 & 2 \\ 0 & 0 & 2 & 2 & 0 \end{bmatrix} \end{array} \qquad (6)$$

The new columns now represent a new exponent basis in the *vehicle-mLU* dimensions. For instance, the term $C_{\alpha f}$ has dimensions: *m-L*$^{-1}$-*U*^{2}. Division of $C_{\alpha f}$ by *m-L*$^{-1}$-*U*^{2} will produce a dimensionless parameter.

The above operation is equivalent to measuring model parameters along each new dimensional space. For instance, instead of measuring a ride-height, h, as 0.5 m, it would be equally appropriate to measure it as 0.25 *L*, where *L* is the length of the vehicle. And rather than report all measurements in terms of vehicle variables such as *L*, it is easier to form dimensionless parameters (pi parameters), so that only the ratio is needed for descriptive measurement. Dimensionally, this is achieved by multiplications and divisions of each parameter by powers of *m, L,* and *U* to obtain a dimensionless pi parameter. In the planar-chassis dynamics example, the five associated parameters are numerically evaluated as:

$$\pi_1 = \frac{a}{L}, \pi_2 = \frac{b}{L}, \pi_3 = \frac{C_{\alpha f}\cdot L}{mU^2}, \pi_4 = \frac{C_{\alpha r}\cdot L}{mU^2}, \pi_5 = \frac{I_z}{mL^2} \qquad (7)$$

An important result is that these parameters are now scale invariant with regard to measurement systems outside the *vehicle-MLU* dimension space. In other words, these parameters will have the same numerical value regardless of the unit system used (SI, English, etc.). Traditionally, such nondimensional or dimensionless parameters are denoted as pi-parameters in honor of the notation first used by Buckingham.

Plant-to-plant comparisons and similarity in dynamics between systems relies on numerical comparisons between different normalized, dimensionless parameters, an approach proven via the Buckingham-Pi Theorem nearly 100 years ago [7]. The conditions of the theorem are straightforward and independent of the governing laws: if two systems have equivalent pi-parameters and these parameters are the only ones present in the description of the system dynamics, then the systems are said to be dimensionally similar. The 'closeness' of two systems with regard to their pi-parameters is a very fundamental measure of the similarity of two systems.

With regard to roll dynamics, an apparent method to establish roll similarity would be to repeat the procedure of planar dynamics using instead a dynamic model for roll and pitch to determine associated parameters. While this lengthy approach would give valid results, a single 'roll' model is not as agreed upon as with the planar bicycle model: each control application may have different requirements and preferences for model complexity and therefore different order of governing differential equations. Some researchers lump vehicle roll behavior into a modified bicycle model formulation to create a 3rd-order system [8], while others independently model the suspension subsystems and thereby obtain overall roll/pitch behavior through dissimilar suspension displacements [9]. For this reason, focus is given in this work on general methods of utilizing dimensional analysis, rather than specific model formulation.

Prior to determining a level of model complexity, the dominant model parameters must be ascertained. Exact matching of these parameters is not necessary for dynamic similarity, i.e. for valid conclusions to be made on one vehicle using results from another. Rather, the pi parameters must be equal. Examining the prior work on comparing bicycle model behavior, the pi parameters of Eq. 7 do not contain information regarding the order or formulation of the governing system differential equations (Eq. 1), except that it is assumed that these pi-parameters are the only ones present in the governing equation. If there is disagreement over the order and nature of the governing equations, but agreement on what physical parameters are involved with these equations, then comparison of the associated pi-parameters will still guarantee similarity between vehicles.

The realization that system similarity arguments are independent of the governing equations is one of the key insights and benefits of the Buckingham Pi theorem.

III. Classification of Similarity Criteria

Because two systems will never be exactly similar, it is important to classify types of similarity in order to evaluate areas of expected dynamic matching between two systems. The physical relevance of each pi parameter in Eq. 7 provides insight into conditions for appropriate comparison of dynamics. The first two parameters, $\pi_1 = a/L$ and $\pi_2 = b/L$, both represent geometric similarity conditions, namely that the size ratios, for instance that Center of Gravity (C.G.) locations must be in equivalent relative locations. For geometric equivalence of two systems, the relevant lengths of one system must be a constant geometric ratio of the other. Relevance is determined by assessing, usually from the governing dynamics, which lengths participate in the dynamic and/or kinematic description of the system.

The last pi parameter $\pi_5 = I_z / m \cdot L^2$ represents similarity in mass distribution in the vehicle. If two vehicle systems are geometrically similar and have similar mass distributions, then their static forces will be equivalent. Hence they will have dynamic similarity in non-accelerating reference frames, a condition sometimes referred to as kinematic similarity.

The third and fourth parameters, $\pi_3 = C_{\alpha f} \cdot L / m \cdot U^2$ and $\pi_4 = C_{\alpha r} \cdot L / m \cdot U^2$ represent ratios of system forces and moments (i.e. tire behavior) to inertial forces (the vehicle's kinetic energy). If these parameters are similar between two systems and if these two systems are already geometrically and kinematically similar, then the behavior of the two systems should be similar when dynamic forces are being generated.

In summary, for appropriate comparison of model dynamics between different vehicles the following conditions must be met:

- Geometric equivalence: ratios of lengths in governing model must be equivalent
- Kinematic equivalence: mass distribution must be such that equivalent ratios of mass-moments are obtained.
- Dynamic equivalence: ratios of potential/kinetic energies must be maintained such that frequency ratios and/or? damping ratios are equivalent.

Unfortunately, a clear distinction between types of equivalence is not always appropriate. For instance, the last pi parameter $\pi_5 = I_z / m \cdot L^2$ is inherently descriptive of mass distribution (kinematic equivalence) but is also necessary to describe similarity of rotational acceleration (i.e. dynamic equivalence).

IV. Geometric Similarity Criteria for Roll and Pitch

Geometric equivalence requires that the geometry of the vehicle must satisfy geometrically similar ratios. With specific regard to the vehicle height, h, and track width, T, the following additional pi-parameters may be formed:

$$\pi_6 = \frac{h}{L}, \pi_7 = \frac{T}{L} \tag{8}$$

As an example of the utility of these parameters, consider first the forces produced on the front and rear axle due to combined steady-state longitudinal acceleration, a_x:

$$F_{front} = mg\frac{b}{L} - ma_x\frac{h}{L} \qquad F_{rear} = mg\frac{a}{L} + ma_x\frac{h}{L} \tag{9}$$

The axle forces can be derived by force/moment balance on the vehicle as shown in Fig. 3. The participating parameters are listed below with their column-wise dimensional dependence similar to that used in Eq. (3).

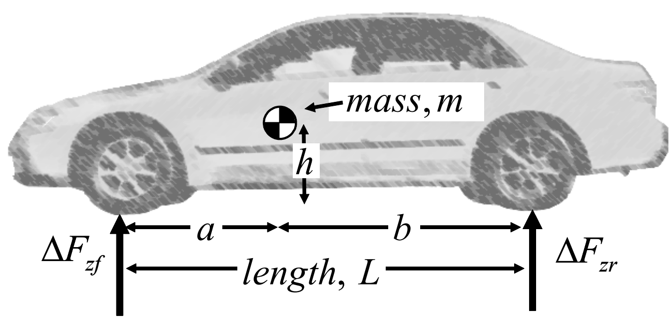

Fig. 3. Vehicle diagram illustrating dimension analysis of pitch behavior.

$$\begin{matrix} & a,b & h & g & a_x & F_{f,r} \end{matrix}$$
$$P_{m\times n-m} = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & -2 & -2 & -2 \end{bmatrix} \qquad (10)$$

Repeating the calculation similar to Eq. (6), we obtain the parameters as measured in the *vehicle-mLU* dimensions:

$$\begin{matrix} & a,b & h & g & a_x & F_{f,r} \end{matrix}$$
$$B^{-1}_{m\times m} P_{m\times n-m} = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & -1 & -1 \\ 0 & 0 & 2 & 2 & 2 \end{bmatrix} \qquad (11)$$

This predicts the following pi parameters:

$$\frac{a}{L}, \frac{b}{L}, \frac{h}{L}, \frac{g\cdot L}{U^2}, \frac{a_x \cdot L}{U^2}, \frac{F_{f,r}\cdot L}{m\cdot U^2} \qquad (12)$$

If similarity of the bicycle model and similarity of the tire forces is already established, then the parameters $a/L, b/L, g\cdot L/U^2, a_x \cdot L/U^2$ will be equivalent. Since the parameter $F_{f,r}\cdot L/m\cdot U^2$ represents how the output measurement of the tire forces will scale, the parameter $\pi_6 = h/L$ is the only additional parameter that must be satisfied for similarity. This analysis suggests that Eq. (9) can be rewritten as:

$$\frac{F_{front}\cdot L}{m\cdot U^2} = \frac{g\cdot L}{U^2}\cdot \pi_2 - \frac{a_x \cdot L}{U^2}\cdot \pi_6 \qquad (13)$$
$$\frac{F_{rear}\cdot L}{m\cdot U^2} = \frac{g\cdot L}{U^2}\cdot \pi_1 + \frac{a_x \cdot L}{U^2}\cdot \pi_6$$

Matching the pi-parameters guarantees similarity between the predictions of tire normal forces for this simple example. A similar static analysis of lateral weight transfer would also require an additional pi-parameter for track-width, T: $\pi_7 = T/L$.

Hereafter it is assumed that the roll axis and pitch axis for two vehicles under comparison will be geometrically similar. However, roll-axis and pitch-axis geometry is very strongly dependent on suspension and vehicle setup, and consequently depends on a great number of geometric parameters that may be tuned by the driver or researcher. A detailed comparison of these aspects is beyond this introductory presentation, but the dimensional analysis of such geometry follows the same analysis as presented here.

In order to compare the pi-parameters of an experimental vehicle to those of a production vehicle, one must ascertain the pi parameters of vehicles in production. One of the most extensive public databases of vehicle parameters is published by the National Highway Transportation Safety Administration [11], and using this data one can generate distributions of pi parameters that serve as guidelines toward new vehicle design. For illustration, the distribution of heights (Fig. 4) is compared to the distribution of the corresponding pi parameter (Fig. 5). The data is grouped as SUV's with Vans with Trucks (hereafter referred to collectively as SUV's) versus all other vehicles. This grouping is presented to illustrate the differences and overlap between these two types of vehicles.

In Fig. 4, it would appear that the C.G. heights of SUV's show a distribution that is too dissimilar from other vehicles to allow dynamic analysis and controller design for one vehicle type to be beneficial to the other. However, the *pi parameters* show a large portion overlap between the two classes of vehicles, suggesting that some roll/pitch controller designs for SUV's might be shared with non-SUV's and vice versa. It is also clear from Fig. 5 that the *range* in plant variation of SUV's is much larger than that of other vehicles, suggesting that controller robustness might be more difficult to establish on SUV's versus other vehicles.

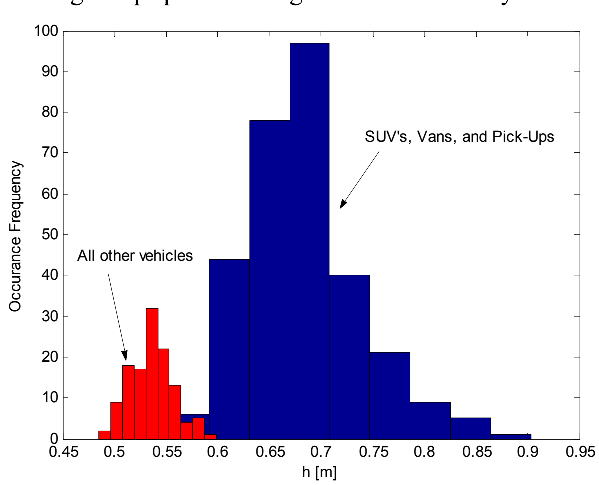

Fig. 4. Histogram of the parameter h, comparing different vehicle types

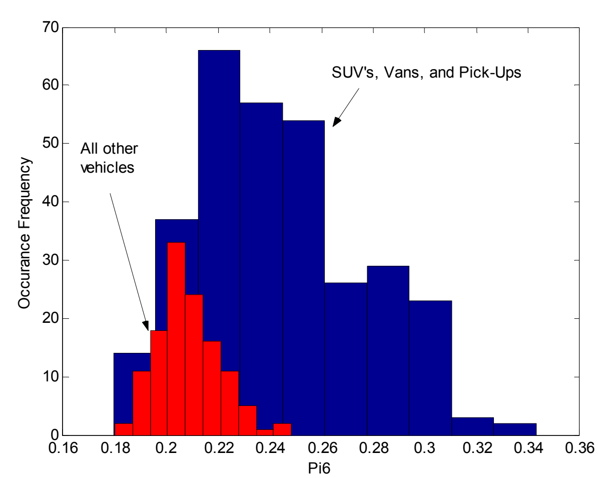

Fig. 5. Histogram of C.G. height pi-parameter, $\pi_6 = h_{CG}/L$

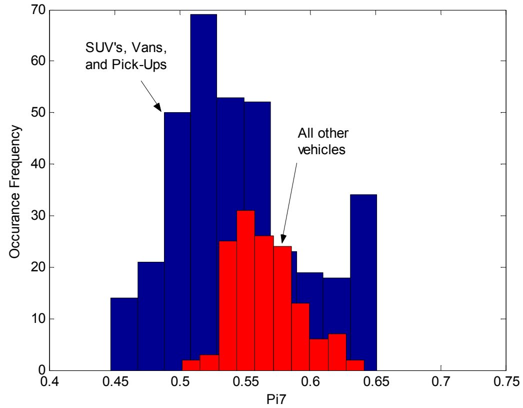

Fig. 6. Histogram of track pi-parameter, π_7 = T/L

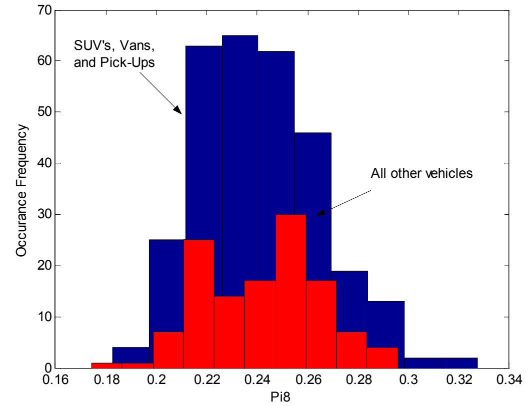

Fig. 7. Histogram of Iy pi-parameter, π_8 = Iy/mL^2

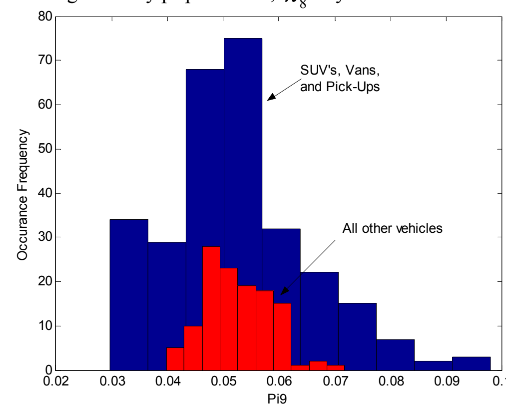

Fig. 8. Histogram of Ix pi-parameter, π_9 = Ix/mL^2

A similar distribution of the pi-parameter for vehicle track is shown in Fig. 6. The pi-parameters for track for SUV’s show a very similar distribution to that of non-SUV’s, but again with a wider variation. This suggests that roll or pitch controller robustness may be more difficult to establish on SUV’s versus non-SUV vehicles.

V. Dynamic Similarity Criteria for Roll and Pitch

In addition to the conditions for *geometric* similarity, additional conditions exist for *kinematic* and *dynamic* similarity in roll and pitch dynamics. First, the inertias along the roll and pitch dynamic modes must be dynamically similar, i.e., the pi parameters:

$$\pi_8 = \frac{I_y}{m \cdot L^2}, \pi_9 = \frac{I_x}{m \cdot L^2} \tag{14}$$

must match. Additionally, the tire forces, suspension forces, and roll/sway-bar stiffness must also be dimensionally similar.

Distributions of the π_8 and π_9 parameters are shown in Figs. 7 and 8 respectively. Both exhibit remarkably similar distributions between SUV’s and non-SUV’s, again with the remark that there appears to be more scatter in the SUV parameters. A summary of the pi-parameters is shown in Table 1, including parameters from previous work not discussed in detail here.

VI. Subsystem Equivalence: Similarity of Suspension Systems, Steering Actuators, and Control Systems

Previous discussion focused on macroscopic vehicle characteristics – mass, inertia, lengths – yet it does not address how to establish similarity of subsystems such as suspension, steering actuators, or the control algorithm itself. The nature of suspension similarity is especially important: For a vehicle to roll, deflection of the vehicle is required out of plane and this deflection occurs in vehicles almost solely through the action of suspension system. To address dimensional similarity of subsystems, one simply continues the dimensional normalization process discussed earlier, being careful to continue using the previous *vehicle-MLU* dimension space. As the following examples illustrate, the resulting pi parameters will often involve a need to preserve frequency ratios.

Table 1. Summary of pi ranges observed in production vehicles

		3 σ range	
	Mean	Low	High
π_1 = a/L	0.4484	0.2783	0.6184
π_2 = b/L	0.5516	0.2783	0.6184
$\pi_3 \cdot U^2 = C_{\alpha f} L/m$ [1]	145.6771	-22.3561	313.7104
$\pi_4 \cdot U^2 = C_{\alpha r} L/m$ [1]	159.9095	-9.1050	328.9240
$\pi_5 = I_z/m \cdot L^2$	0.2487	0.1749	0.3225

	SUV	3 σ range		NonSUV	3 σ range	
	Mean	Low	High	Mean	Low	High
π_6 = h/L	0.2436	0.1462	0.3409	0.2082	0.1708	0.2456
π_7 = T/L	0.5452	0.3962	0.6942	0.5649	0.4855	0.6443
$\pi_8 = Iy/m \cdot L^2$	0.2414	0.1687	0.3141	0.2417	0.1690	0.3144
$\pi_9 = I_x/m \cdot L^2$	0.0525	0.0141	0.0909	0.0525	0.0141	0.0909

[1] with U measured in m/s

Consider approximating a vehicle’s suspension system as a mass-spring-damper system, with external driving force as the input, u(t). The input/output behavior of mass-spring damper system depends on the parameters m_s, b_s, and k_s, which are the suspension mass, damping, and spring constant respectively. The dynamic equations can also be written in terms of natural frequency and damping ratio in the standard second-order form with a constant gain, K:

$$\frac{Y(s)}{U(s)} = \frac{K}{s^2 + 2 \cdot \xi \cdot \omega_n \cdot s + \omega_n^2} \tag{15}$$

To find the pi-values for the suspension subsystem model, the dimensional span of each of the parameters is written to create the P matrix as shown in Eq. (5):

$$P_{m\times n-m} = \begin{array}{c} \begin{matrix} m_s & b_s & k_s & \zeta & \omega_n & K \end{matrix} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -2 & 0 & -1 & 0 \end{bmatrix} \end{array} \quad (16)$$

(Note that the Laplace s-parameter, while omitted, has dimensions that are equivalent to frequency as well). By calculating $B^{-1}{}_{m\times m} \cdot P_{m\times n-m}$, the conversion of dimensional powers from SI to the *vehicle-mLU* dimensions is obtained:

$$B^{-1}{}_{m\times m} P_{m\times n-m} = \begin{array}{c} \begin{matrix} m_s & b_s & k_s & \zeta & \omega_n & K \end{matrix} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & -1 \\ 0 & -1 & -2 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \end{bmatrix} \end{array} \quad (17)$$

Suggesting pi-values:

$$\frac{m_s}{m}, \ \frac{b_s \cdot L}{mU}, \ \frac{k_s \cdot L^2}{m \cdot U^2}, \ \zeta, \ \frac{\omega_n \cdot L}{U}, \ K \cdot m \quad (18)$$

The first three of the above parameters correspond to maintaining mass ratios, dissipation rates, and the ratio of spring potential energy to vehicle kinetic energy. The next two pi-values suggest that the damping ratio must be preserved (an intuitive result) and that the frequency ratios must be preserved. That is, if two systems are to be dynamically similar, then the ratio of frequencies involved with chassis dynamics and any subsystems must satisfy constant geometric ratios:

$$\left.\frac{\omega_{n,c} \cdot L}{U}\right|_{system\,A} = \left.\frac{\omega_{n,c} \cdot L}{U}\right|_{system\,B} \quad (19)$$

or, more directly, $\omega_{n,c,A} / \omega_{n,c,B} = constant$. For instance, if two systems, system 1 and system 2, are dynamically similar and have suspension dynamics with natural frequencies of 5 and 10 rad/sec respectively, then the pole locations of the bicycle model for system 2 must be twice those of system 1, the steering actuator must have twice the bandwidth, the roll-mode must have twice the natural frequency, etc.

Especially with regard to actuator dynamics and/or control laws, the notion of 'fast' or 'slow' dynamics is therefore a relative term. With regard to dimensional analysis, the creation of pi-parameters for actuators or control laws follows the same procedure earlier: (i) determine a list of relevant parameters – control gains for instance; (ii) determine the dimensional span of each parameter; and (iii) transform the parameter measure into a new dimensional basis to generate a set of pi-parameters. The result will necessarily depend on the *ratio* of bandwidth between the two systems rather than the *absolute* bandwidth. A 4 Hz bandwidth steering actuator that is determined to be sufficient for actuating the steering linkage of an autonomous Buick sedan (large vehicle with slow chassis dynamics) may be unsuitable for autonomous control of a Ford Focus (small vehicle with fast chassis dynamics). Most steering actuators encountered in the literature have bandwidths between 3 and 5 Hz. Research is ongoing to compile a sufficient sample of actuator dynamics to create useful comparison histograms as shown earlier. Theoretically, comparative studies of roll controllers are being initiated using this dimensional analysis technique. Experimentally, research is also ongoing to physically measure roll dynamics of passenger SUV vehicles, commercial heavy vehicles (tractor-trailers), and busses at the Pennsylvania Transportation Institute.

VII. Conclusions

Dimensional scaling concepts developed earlier for planar vehicle behavior were extended to determine conditions for roll and pitch similarity. A methodology for deriving chassis pi parameters was demonstrated by derivation of several key roll and pitch parameters. Distributions of these parameters were also given comparing large vehicle behavior – SUV's, Vans, and Trucks – with all other vehicles. In some parameters, very distinct differences can be seen. A distinguishing characteristic in general is that large vehicles have a greater variation in vehicle parameters compared to other vehicles.

The dimensional scaling analysis was then extended to allow comparison of subsystems including suspension, control laws, and actuator dynamics. In addition to requiring similar damping ratios, it was shown that the ratio of natural frequencies between chassis dynamics (roll, pitch, yaw) and subsystems, whether they are actuators, controllers, or higher-order dynamics, should be preserved.

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