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Decentralized Two-Time-Scale Motions Control using Generalized Sampled-Data Hold Functions

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***Abstract*— This paper presents a decentralized variation of the two-time-scale motions controller for linear time invariant systems. A method is proposed to change the structure of the system using discretization and generalized sampled-data hold functions, so that distinct local discrete-time controllers can be applied to each input-output agent. The resultant output feedback decentralized periodic controller has, in fact, a linear time-varying structure. Conditions under which the desired structure modification can be accomplished are given and the simulation results are also included.**

I. Introduction

Most of the existing results on the output feedback control of decentralized linear systems are of a descriptive nature, i.e., they focus on the analysis of the properties of such systems. Thus, the available literature in the field provides a quite complete understanding of the conditions under which linear systems can be controlled by means of a decentralized strategy. However, few design methods exist and some of them involve long numerical procedures. Even of more importance, some of them have limited application because they cannot overcome the limitations imposed by the structural properties of the plant.

The purpose of this work is to provide a simple solution to the problem of stabilizing a controllable linear time-invariant plant with structural properties that prevent the use of time-invariant output feedback.

Consider the output controllable system given by:

$$\begin{cases} \dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t) \\ \mathbf{y}(t) = C\mathbf{x}(t), \end{cases} \tag{1}$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ and $\mathbf{u}(t), \mathbf{y}(t) \in \mathbb{R}^m$, and assume that decentralized output control is required, i.e., the controller must have the structure:

$$u_i(t) = k_i(t, y_i([0,t))), \quad i = 1, ..m. \tag{2}$$

Notation: In what follows, scalars, matrices, and vectors will be denoted by means of small, capital, and bold symbols, respectively.

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The system (1) may have unstable decentralized fixed modes (DFM) [1] and therefore its internal dynamics may be unstable. DFM's can be classified as either being "structured" or "unstructured" [2], [3]. Structured DFMs, are the modes that remain "fixed" with respect to any type of decentralized output feedback, e.g. nonlinear or time-varying control. Unstructured DFM's (UDFM) on the other hand, are the modes that can be eliminated by applying appropriate time-varying controllers. Several time-varying decentralized controllers using vibrational feedback [4], [5], and periodic feedback [6], [7], [8], [9] have been introduced for this purpose. Systems with unstable structured DFMs are not stabilizable with respect to any type of nonlinear or time-varying decentralized controller.

By modifying the structure of the plant [10], sampling can remove the nonzero and distinct UDFM's from the discrete time equivalent system for almost all sampling rates [3]. A discrete-time controller can then be used to stabilize the system. The present development proposes a decentralized periodic controller capable of stabilizing (1) in the presence of unstable UDFM's and unstable internal dynamics. The resulting control law is a modified form of the controller introduced in [11] for a class of nonlinear systems. It will be shown that generalized sampled-data hold functions (GSHF's) can significantly enlarge the class of linear systems to which that control law is applicable.

The paper is organized as follows. Section II provides the basic background related to the material presented later. Subsection II-A briefly overviews the fundamentals of the two-time-scale motions (TTSM) controller and II-B introduces the basics of GSHF's. The main development is presented in Section III. First, the model of the system is sampled with GSHF's in Subsection III-A. Next, Subsection III-B develops the equations of the desired output. The control law appears then in Subsection III-C. Finally, Section III-D extends these results to the case of decentralized control and provides conditions for the existence of the decentralized TTSM controller for linear time-invariant systems. An example and its simulation results are shown in Section IV. Conclusions and comments are expressed in Section V.

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II. Preliminaries

A. The two-time-scale motions controller

The TTSM controller presented in [11] solves the problem of output tracking for multiple-input multiple-output (MIMO) nonlinear systems in the presence of perturbations and parameter variations. Instead of dealing with the original continuous-time system, this design method focuses on a discrete-time approximation whose state is the output of the original system. Such approximation follows from the repeated differentiation of the outputs of the system and the discretization of the resulting equations which directly relate inputs to the output derivatives. In order to allow the discretization of these nonlinear equations, the method introduces a new time scale so that the nonlinear terms can be rejected due to the fast motions in the closed-loop system. The realization error is then defined as the difference between the output of the discrete approximation and the desired output. The method uses the realization error to drive the control input dynamics which, due to the time scale, evolve faster than the output dynamics. Thus, the closed-loop system is composed of a fast and a slow motion subsystems–fast-motion subsystem and slow-motion subsystem, respectively.

One limitation of the TTSM controller presented in [11] is that it was developed for systems with stable internal dynamics.

Another limitation appears after the repeated differentiation of the outputs of the system. In order to clarify this, consider a system with the following structure:

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t)) + \mathbf{g}(\mathbf{x}(t))\mathbf{u}(t) \\ \mathbf{y}(t) = \mathbf{h}(\mathbf{x}(t)), \end{cases}$$

and relative degrees $\alpha_1, \dots \alpha_m$ with respect to each output. A direct relationship between the inputs $\mathbf{u} = [u_1, \dots, u_m]^T$ and the output $\mathbf{y} = [y_1, \dots, y_m]^T$ derivatives is given by:

$$\mathbf{y}^*(t) = \mathbf{h}^*(\mathbf{x}(t)) + \mathbf{g}^*(\mathbf{x}(t))\mathbf{u}(t), \tag{3}$$

with:

$$\mathbf{y}^*(t) = \left[\frac{d^{\alpha_1} y_1(t)}{dt^{\alpha_1}} \dots \frac{d^{\alpha_m} y_m(t)}{dt^{\alpha_m}} \right]^T.$$

In the particular case of the linear systems of the form (1), $\mathbf{h}^*$ and $\mathbf{g}^*$ are given by:

$$\begin{aligned} \mathbf{h}^* &= \begin{bmatrix} \mathbf{c}_1 A^{\alpha_1} \\ \vdots \\ \mathbf{c}_m A^{\alpha_m} \end{bmatrix} \\ \mathbf{g}^* &= \begin{bmatrix} \mathbf{c}_1 A^{\alpha_1 - 1} B \\ \vdots \\ \mathbf{c}_m A^{\alpha_m - 1} B \end{bmatrix}, \end{aligned}$$

where $\mathbf{c}_i$ is the i-th row of the output matrix C.

As presented in [11], the TTSM controller is only suited to systems for which $\mathbf{g}^*$ is invertible. However, for linear time-invariant systems and under some conditions, the use of GSHF's can relax this restriction, as shown next.

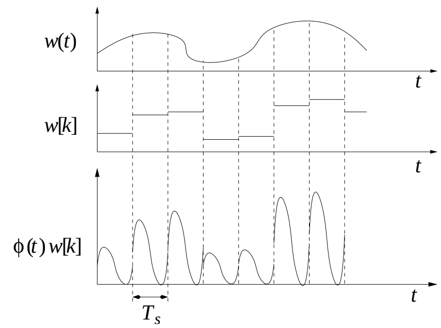

Fig. 1. A sampled arbitrary signal $w(t)$ followed by a zero-order hold (ZOH), generating $w[k]$, and multiplied by the GSHF $\phi(t) = 1 + \sin(\frac{2\pi}{T_s} t)$ to generate $\phi(t) w[k]$.

B. Plant restructure using GSHF's

Consider a zero-order hold with the sampling period T_s. GSHF's are functions with period T_s that are multiplied linear time-invariant by the output of the zero-order hold so that the sampled signal is not constant during the sampling period, i.e., if a GSHF $\phi(t)$ is applied to the signal $\tilde{u}(t)$, the resulting linear time-invariant sampled signal is then given by $u(t) = \phi(t)\tilde{u}[k]$, where $\tilde{u}[k]$ is a signal sampled at $t = kT$, and $\phi(t)$ is the sampled-data hold function which has the property that $\phi(t) = \phi(t + T_s)$ (refer to the example in Fig. 1).

Let (C, A_d, B_d) denote the discrete-time system with input $\tilde{\mathbf{u}}$ obtained by sampling the input of the system (C, A, B) with the GSHF's $\phi(t)$ and sampling period T_s. The matrices A_d and B_d are given by:

$$A_d = e^{AT_s} \quad B_d = \int_0^{T_s} e^{A(T_s - t)} B \phi(t) dt.$$

It is important to note that different choices of $\phi(t)$ generate different matrices B_d. Consequently, under some conditions the function $\phi(t)$ can be chosen so that the matrix B_d has the desired structure [12], [13]. One can achieve more flexibility in the design of the matrix B_d by using different sampling functions $\phi_i(t)$ for each control input (see Fig. 2) so that:

$$u_i(t) = \phi_i \tilde{u}_i[k], \quad kT_s \le t < (k+1)T_s$$

for $i \in \bar{m} = \{1, \dots, m\}$.

In this case, the discrete-time system matrices are given by:

$$A_d = e^{AT_s} \quad B_d = [\mathbf{b}_{d1}, \dots, \mathbf{b}_{dm}], \tag{4}$$

$$\mathbf{b}_{di} = \int_0^{T_s} e^{A(T_s - t)} \mathbf{b}_i \phi_i(t) dt. \tag{5}$$

In fact, the only constraint in the design of B_d is that each of its columns $\mathbf{b}_{di}$ must belong to the controllable subspace of $(A, \mathbf{b}_i)$.

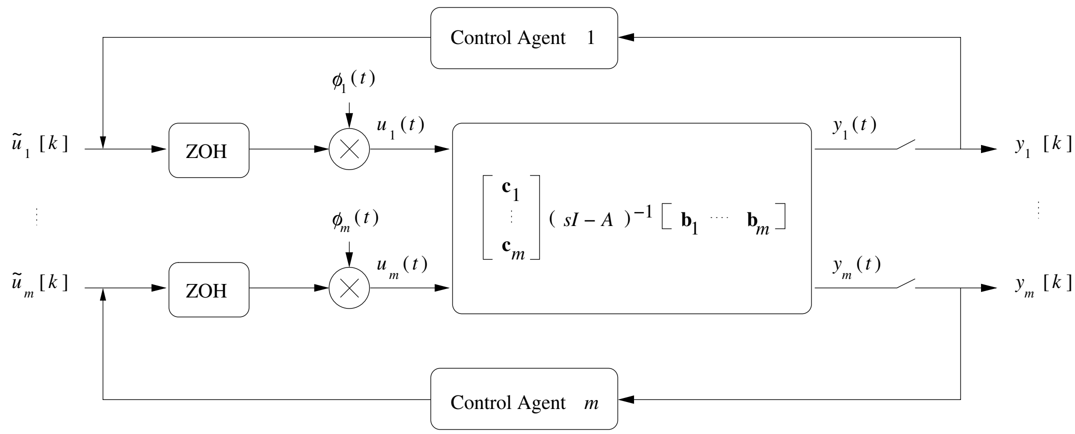

Fig. 2. Closed-loop system with feedback applied through zero-order holds and the GSHF's $\phi_1(t)\dots\phi_m(t)$ at the inputs $u_1 \dots u_m$

III. The TTSM controller for linear time-invariant systems

A. Equations of the sampled system

Consider the system (1) and let $\mathcal{C}$ be its controllable space. If $\exists \mathbf{d}_i \in \mathcal{C}, i \in \bar{m}$ such that $\mathbf{d}_i \not\perp \mathbf{c}_i$, then the matrix B_d can be chosen such that none of the rows of:

$$G \triangleq CB_d \tag{6}$$

is zero, implying that the system given by the triple (C, A_d, B_d) has a relative degree equal to 1 with respect to all of its outputs. The following procedure applies to systems that satisfy these conditions.

The control law design starts with deriving an expression for the output-dynamics in discrete time. To this end, we do not differentiate the output as the original TTSM method proposed in [11] does. Instead, we first obtain the exact discrete-time representation of the original system:

$$\begin{aligned}\mathbf{x}[k] =& e^{AT_s}\mathbf{x}[k-1] \\ &+ T_s\left[\frac{1}{T_s}\int_0^{T_s} e^{A(T_s-t)}B\Phi(t)dt\right]\tilde{\mathbf{u}}[k-1] \\ =& A_d\mathbf{x}[k-1] + T_s\tilde{B}_d\tilde{\mathbf{u}}[k-1].\end{aligned}$$

Hence, we have:

$$\begin{aligned}\mathbf{y}[k] =& CA_d\mathbf{x}[k-1] \\ &+ T_sC\left[\frac{1}{T_s}\int_0^{T_s} e^{A(T_s-t)}B\Phi(t)dt\right]\tilde{\mathbf{u}}[k-1] \\ =& CA_d\mathbf{x}[k-1] + T_sC\tilde{B}_d\tilde{\mathbf{u}}[k-1],\end{aligned} \tag{7}$$

where:

$$\begin{aligned}\tilde{B}_d =& \frac{1}{T_s}\int_0^{T_s} e^{A(T_s-t)}B\Phi(t)dt \\ =& \frac{1}{T_s}\int_0^{T_s}\left[\sum_{j=0}^{\infty}\frac{(T_s-t)^j}{j!}A^j\right]B\Phi(t)dt,\end{aligned} \tag{8}$$

and $\Phi(t)$ is a matrix of sampling functions. In terms of decentralized systems, the introduction of individual GSHF's $\phi_i(t)$ for the input channel $u_i(t)$, $i \in \bar{m}$, provides more flexibility on the design of B_d. Accordingly:

$$\mathbf{u}(t) = \Phi(t)\tilde{\mathbf{u}}[k],$$

where $\Phi(t) = \mathrm{diag}[\phi_1(t), \dots, \phi_m(t)]$.

Note that:

$$\lim_{T_s\to 0}\|\tilde{B}_d(T_s)\| \leq const < \infty$$

and, by Cayley-Hamilton theorem, $\tilde{B}_d$ can be rewritten as:

$$\tilde{B}_d = \frac{1}{T_s}\int_0^{T_s}\left[\sum_{j=0}^{\infty}\psi_j(T_s-t)A^j\right]B\Phi(t)dt$$

where $\psi_j(T_s-t)$, $j \in \mathbb{Z}^+$, are functions of $T_s - t$.

Next, $\Phi(t)$ can be chosen in such a way that the matrix G is invertible or, furthermore, diagonal.

The equations of the resulting linear time-invariant sampled system (7) can be expressed as:

$$\begin{aligned}\mathbf{y}[k] =& Ce^{AT_s}\mathbf{x}[k-1] + T_sC\tilde{B}_d\tilde{\mathbf{u}}[k-1] \\ =& C\left(I+M\right)\mathbf{x}[k-1] + T_sC\tilde{B}_d\tilde{\mathbf{u}}[k-1] \\ =& \mathbf{y}[k-1] \\ &+ T_s\left\{CM\mathbf{x}[k-1] + C\tilde{B}_d\tilde{\mathbf{u}}[k-1]\right\},\end{aligned} \tag{9}$$

where the matrices A_d and B_d are given by (4) and (8), respectively, and the matrix M given by:

$$M = \sum_{j=1}^{\infty}\frac{T_s^{j-1}}{j!}A^j,$$

which has the property that:

$$\lim_{T_s\to 0} M = A$$

If the functions $\phi_i(t)$, $i \in \bar{m}$ are chosen such that the matrix G given by (6) is invertible, the feedback transformation:

$$\tilde{\mathbf{u}}[k] = T_sG^{-1}\mathbf{v}[k] \tag{10}$$

with $\mathbf{v} = [v_i, \ldots, v_m]^T$, applied to (9) yields:

$$\mathbf{y}[k] = \mathbf{y}[k-1] + T_s \left\{ CM\mathbf{x}[k-1] + \mathbf{v}[k-1] \right\}. \quad (11)$$

Thus, (11) can be decoupled into individual components $y_i[k]$, $i \in \bar{m}$, as:

$$y_i[k] = y_i[k-1] + T_s \left\{ \mathbf{c}_i M\mathbf{x}[k-1] + v_i[k-1] \right\}. \quad (12)$$

B. Desired output equations

Assume that the desired outputs of the system are given by the stable transfer functions:

$$\frac{Y_i^*(s)}{R_i(s)} = \frac{\theta_i}{s+\theta_i}, \quad i \in \bar{m} \quad (13)$$

where $Y_i^*(s)$ is the i-th desired output, $R_i(s)$ is the i-th reference signal, and $\theta_i > 0$, $i \in \bar{m}$, are design parameters that determine the settling time and damping ratio.

Ideally the outputs $y_i(t)$ of the system should exactly match those given by the differential equations corresponding to (13), i.e.:

$$\dot{y}_i^*(t) = -\theta_i \left\{ y_i^*(t) - r_i(t) \right\}, \quad i \in \bar{m}.$$

From (13), the i-th desired pulse transfer function is:

$$\begin{aligned} H_i^*(z) &= \frac{z-1}{z} \mathcal{Z} \left\{ \frac{\theta_i}{s(s+\theta_i)} \right\} \\ &= \frac{1-e^{-\theta_i T_s}}{z - e^{-\theta_i T_s}}, \quad i \in \bar{m}. \end{aligned} \quad (14)$$

If $H_i^*(z)$ given above governs the actual output, then $y_i[k]$ is given by:

$$\begin{aligned} y_i[k] =& e^{-\theta_i T_s} y_i[k-1] + r_i[k-1] \left(1 - e^{-\theta_i T_s}\right) \\ =& y_i[k-1] \\ &+ T_s \left(\frac{1-e^{-\theta_i T_s}}{T_s} \right) (r_i[k-1] - y_i[k-1]) \\ \triangleq& F_i[k], \quad i \in \bar{m}, \end{aligned} \quad (15)$$

which defines $F_i[k]$ as the i-th desired output in terms of the values of the previous samples of the i-th actual output and reference.

C. The control law

Define the realization error as:

$$\begin{aligned} e_i^F[k] \triangleq& F_i[k] - y_i[k] \\ =& T_s \left(\frac{1-e^{-\theta_i T_s}}{T_s} \right) (r_i[k-1] - y_i[k-1]) \\ & - T_s \mathbf{c}_i M\mathbf{x}[k-1] - T_s v_i[k-1], \quad i \in \bar{m}, \end{aligned}$$

and the control input as:

$$v_i[k] = v_i[k-1] + \lambda_i(T_s) e_i^F[k], \quad i \in \bar{m}, \quad (16)$$

where $\lambda_i \neq 0$ and $\tilde{\lambda}_i \neq 0$ are constants that satisfy $\lambda_i(T_s) = T_s^{-1}\tilde{\lambda}_i$.

In order to see how the control law defined by (16) allow us to attain the control objective $e_i^F[k] = 0$, note that the closed-loop system is given by (12) and:

$$\begin{aligned} v_i[k] =& \left(1 - \tilde{\lambda}_i\right) v_i[k-1] - \frac{\tilde{\lambda}_i}{T_s} \mathbf{c}_i M\mathbf{x}[k-1] \\ &+ \frac{\tilde{\lambda}_i}{T_s} \left(1 - e^{-\theta T_s}\right) (r_i[k-1] - y_i[k-1]), \end{aligned} \quad (17)$$

for $i \in \bar{m}$.

These last equations show that, for small enough values of T_s, the closed-loop system consists of a slow motion subsystem with state variables y_i, $i \in \bar{m}$, and a fast motion subsystem with state variables v_i, $i \in \bar{m}$,. From (17), the characteristic polynomials of the fast-motion subsystem read as $P_i = z - 1 + \tilde{\lambda}_i$, $i \in \bar{m}$. Therefore, in order to stabilize (12) (17), one must choose the parameters $\tilde{\lambda}_i$, $i \in \bar{m}$, such that these polynomials are stable. If these polynomials are stable, then the fast-motion subsystem achieves a quasi-steady state response at which:

$$v_i[k+1] = v_i[k] \quad \forall k > k_f, \quad i \in \bar{m},$$

for some $k_f > 0$. The values of these steady-state variables are given by:

$$v_i^{\text{ss}} = \frac{1-e^{-\theta T_s}}{T_s} (r_i[k-1] - y_i[k-1]) - \frac{\mathbf{c}_i M\mathbf{x}[k-1]}{T_s},$$

which, once substituted into (12) (17), show that the desired output (15) is attained. A full proof of the stability of the closed-loop system is given in [11].

Remark 1: The stability of (12) (17) depends strongly on the choices of T_s and B_d because the location of the approximate DFM's, defined in [14], and transmission zeros in the open-loop sampled system is a function of these two parameters.

D. Application to decentralized control

For the TTSM controller to be decentralized, the matrix G must be diagonal. This requirement is satisfied only if the matrices B and C present certain properties, which are studied next.

Let the matrix C have rank m; then, the matrix $C^\dagger \in \mathbb{R}^{n \times m}$, the right pseudoinverse of C, is such that $CC^\dagger = I_m$. Let also $\mathbf{d}_i$ be the i-th column of $C^\dagger$ and let $\mathbf{b}_i$ be the i-th column of B. Assume that for $i \in \bar{m}$ the controllable subspace of $(A, \mathbf{b}_i)$, denoted by $\mathcal{C}_i$, is not empty. Then the next theorem provides necessary and sufficient conditions for the existence of a decentralized TTSM controller for (1).

Theorem 1: There exists a TTSM decentralized stabilizing controller for the linear time-invariant system (1) if and only if:

i) $\mathbf{d}_i \in \mathcal{C}_i$, $i \in \bar{m}$, and
ii) there are $\bar{m}$ constants a_i, $i \in \bar{m}$, defining $B_d = [a_1\mathbf{d}_1, \ldots a_m\mathbf{d}_m]$, such that zero dynamics of the discrete-time system (C, A_d, B_d) are stable,

where $\mathbf{d}_i$ and $\mathcal{C}_i$ are given above.

Proof: We separately prove sufficiency and necessity. Sufficiency:

i) If $\mathbf{d}_i \in \mathcal{C}_i$ then, from the results in [13], there exist GSHF's $\phi_i(t), i \in \bar{m}$, such that:

$$a_i \mathbf{d}_i = \int_0^{T_s} e^{A(T_s - \tau)} \mathbf{b}_i \phi_i(\tau) d\tau$$

for any constant a_i. Consequently, the matrix B_d can be designed as:

$$B_d = [a_1 \mathbf{d}_1, \ldots, a_m \mathbf{d}_m],$$

implying that:

$$\begin{aligned} G &= CB_d \\ &= C[\mathbf{d}_1, \ldots, \mathbf{d}_m] \mathrm{diag}[a_1, \ldots, a_m] \\ &= CC^\dagger \mathrm{diag}[a_1, \ldots, a_m] \\ &= \mathrm{diag}[a_1, \ldots, a_m]. \end{aligned}$$

the feedback transformation (10) becomes:

$$\tilde{u}_i[k] = \frac{1}{a_i} v_i[k], \quad i \in \bar{m}.$$

Since the signal $v_i[k]$ depends only on y_i and r_i, $i \in \bar{m}$, the controller is decoupled as:

$$u_i(t) = \frac{\phi(t)}{a_i} v_i[k], \quad kT_s \le t < (k+1)T_s, \qquad (18)$$

for $i \in \bar{m}$, and the resulting TTSM controller becomes decentralized.

ii) If there are constants a_i such that zero dynamics of the discrete-time system (C, A_d, B_d) are stable and if $\mathbf{y}[k]$ in (11) is kept bounded through proper design of $\mathbf{v}[k]$, then the state of (1) is bounded as well; in fact, the state of the zero dynamics tends to 0 and so do $e_i^F[k]$, $i \in \bar{m}$. Therefore, the TTSM controller achieves output tracking.

Necessity:

i) Evidently, if $\mathbf{d}_i \notin \mathcal{C}_i$, $i \in \bar{m}$, then there is no matrix B_d such that G is diagonal and the controls cannot be decoupled as in (18).

ii) If there are no constants a_i such that zero dynamics of the discrete-time system (C, A_d, B_d) are stable then the internal dynamic state would evolve uncontrolled and eventually would result in an unstable closed-loop system.

■

IV. Example

Consider the plant described by the matrices:

$$A = \frac{1}{100} \begin{bmatrix} 1 & 5 & 0 \\ 1 & 10 & -1 \\ 0 & -5 & 1 \end{bmatrix},$$

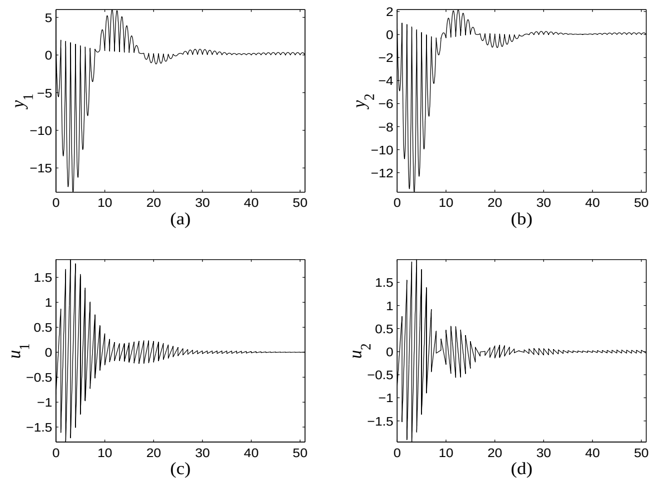

Fig. 3. Response of the system to the nonzero initial condition and zero input. (a) The output signal $y_1(t)$; (b) the output signal $y_2(t)$; (c) the input signal $u_1(t)$; (d) the input signal $u_2(t)$.

$$B = \begin{bmatrix} 0 & 20 \\ 10 & 20 \\ 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

This system has relative degree 1 with an unstable UDFM at $s = 0.01$; thus, continuous-time output control cannot stabilize it. If a zero-order hold with, for example, $T_s =$ 1sec samples the system, this UDFM becomes an unstable open-loop transmission zero at $z = 1.01$ and the matrix G is invertible but not diagonal. Notice that the sampling period can be large because the poles of the plant are small, i.e., the eigenvalues of A are $\{0,\ 0.01,\ 0.11\}$.

If instead of the zero-order hold, the GSHF's:

$$\phi_1(t) = -23.89t + 11.74 \quad \phi_2(t) = -11.65t + 5.76$$

are used, the discrete-time equivalent system will be represented by the following matrices:

$$A_d = \begin{bmatrix} 1.0103 & 0.0529 & -0.0003 \\ 0.0106 & 1.1057 & -0.0106 \\ -0.0003 & -0.0529 & 1.0103 \end{bmatrix}$$

$$B_d = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}^T \quad C_d = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

The associated TTSM controller is decentralized because, as desired, the matrix $G = \mathrm{diag}[1, 2]$ is diagonal. Moreover, for the above choice of B_d, the sampled system has stable zero dynamics because the only transmission zero is located at $z = 0.9995$ and, as expected, the system does not have any DFM.

Fig. 3 gives the input and output signals for the closed-loop system response to nonzero initial conditions $\mathbf{y}(0) = [1, 1, -2]^T$ and zero reference signal $r[k] = 0, \forall k$, when the controller parameters are $\theta_1 = 0.1$, $\theta_2 = 0.3$, $\tilde{\lambda}_1 = 0.75$, and $\tilde{\lambda}_1 = 0.5$. Figs. 3(a) and 3(b) show the output signals

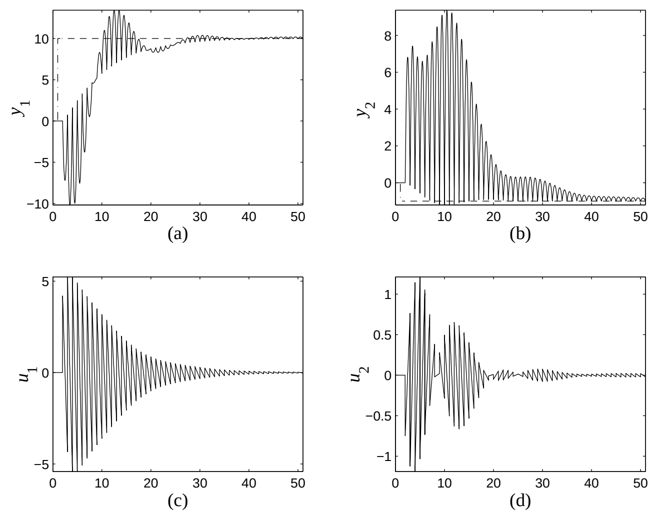

Fig. 4. Response of the closed-loop system due to step input and zero initial conditions. (a) The output signal $y_1(t)$; (b) the output signal $y_2(t)$; (c) the input signal $u_1(t)$; (d) the input signal $u_2(t)$.

$y_1(t)$ and $y_2(t)$, respectively. Figs. 3(c) and 3(d) give the input signals $u_1(t)$ and $u_2(t)$, respectively.

Fig. 4 gives the input and output signals for the closed-loop system subject to zero initial conditions and a step reference $r[k] = [10, -1]^T$ at $t = 1$sec with the controller parameters given above. Figs. 4(a) and 4(b) give the output signals $y_1(t)$ and $y_2(t)$, respectively. Figs. 4(c) and 4(d) give the input signals $u_1(t)$ and $u_2(t)$, respectively. The dashed lines in Figs. 4(a) and 4(b) represent the reference signal.

It can be seen from the simulations that the magnitude of the intersampling ripple in the output depends on the magnitude of the control input. This implies that in transient state the intersampling ripple will be present and in steady-state it will remain in the output signals only if the control inputs do not approach zero as t increases [15]. A good choice of sampled-data hold function can significantly improve the performance of the controller [16], [17] by reducing the amplitude of the intersampling swing. Measurement and system noise are additional matters of concern originated by such large state ripple. Given the flexibility in the design of the GSHF's, one can improve the closed-loop robustness by choosing functions that provide more tolerance with respect to those disturbances [12].

V. Conclusions

The combination of the TTSM controller and GSHF's conveys advantages as well as disadvantages. On one hand, the lack of robustness of GSHF's sacrifices the insensitivity to parameter variations of the TTSM controller. On the other hand, the class of linear time-invariant systems to which this type of controller is applicable is significantly enlarged.

One important point that requires consideration is the selection of the sampling rate. T_s must be selected small enough to satisfy the requirements of the TTSM control but also as big as possible in order to reduce the magnitude of the GSHF's. Large GSHF's would lead to large control inputs and large state swings that in many applications may not be tolerable. Since the GSHF's are not uniquely determined for a desired discrete-time system, the designer must look for the functions ϕ_i with minimum magnitude that satisfy (4)–like those proposed in [16] [17]. The designer must also take additional criteria into account [12] when noise-rejection is required. Since these improvements have proven feasible, the results presented here can be extended to provide better performance and robustness. Therefore, the proposed controller can be applied to a broad class of decentralized control problems.

References

[1] E.J. Davison, W. Gesing, and S.H. Wang, "An algorithm for obtaining the minimal realization of a linear time-invariant system and determining if a system is stabilizable-detectable," in *IEEE Transactions on Automatic Control*, vol. AC-23, 1978, pp. 1048-1054

[2] M. E. Sezer and D. D. Šiljak, "Structurally Fixed Modes", in *Systems and Control Letters*, vol.1, no. 1, July 1981, pp. 60-64

[3] Ü. Özgüner and E. J. Davison, "Sampling And Decentralized Fixed Modes," in *Proceedings of the 1985 American Control Conference*, 1985, pp. 257-262

[4] S. Lee and M. Meerkov and T. Runolfsson, "Vibrational Feedback Control: Zero Placement Capabilities," in *IEEE Transactions on Automatic Control*, vol. AC-32, no. 7, July 1987, pp. 604-611

[5] L. Trave and A. Tarras and A. Titli, "An Application of Vibrational Control to Cancel Unstable Decentralized Fixed Modes," in *IEEE Transactions on Automatic Control*, vol. AC-30, no. 3, March 1985, pp. 283-286

[6] B. O. D. Anderson and J. B. Moore, "Time-Varying Feedback Laws for Decentralized Control," in *IEEE Transactions on Automatic Control*, vol. AC-26, no. 5, May 1981, pp. 1133-1138

[7] S.H. Wang, "Stabilization of decentralized control systems via time-varying controllers," in *IEEE Transactions on Automatic Control* no. 27, 1982, pp. 741-744

[8] J.L. Willems, "Elimination of fixed modes in decentralized systems by means of sampling," in *Systems and Control Letters* no. 10, 1988, pp. 1-8

[9] P. P. Khargonekar and A. B. Özgüler, "Decentralized Control and Periodic Feedback," in *IEEE Transactions on Automatic Control*, vol. 39, no. 4, April 1994, pp. 877-882

[10] A.G. Aghdam and E.J. Davison, "Application of Generalized Sampled-data Hold Functions to Decentralized Control Structure Modification," in *Proceedings of the 15th IFAC World Congres* Barcelona Spain, July 2002, Area Code 2d, Session Slot T-Th-M21

[11] V.D. Yurkevich, "Design of digital MIMO control system with two-time-scale motions," in *Proceedings of the 15 IFAC World Congress*, Barcelona, Spain, July 2002, Area Code 2d, Session slot T-Tu-M21

[12] P.T. Kabamba, "Control of linear Systems Using Generalized Sampled-Data Hold functions," in *IEEE Transactions on Automatic Control*, vol. AC-32, no. 9, September 1987

[13] A.G. Aghdam and E.J. Davison, "Decentralized control of systems, using Generalized Sampled-data Hold Functions," in *Proceedings of the 38th Conference on Decision and Control* Arizona USA, December 1999

[14] A. F. Vaz and E. J. Davison, "On the Quantitative Characterization of Approximate Decentralized Fixed Modes using Transmission Zeros," in *Mathematics of Control Signals, and Systems* vol. 2, 1989, pp. 287-302

[15] A. Feuer and G.C. Goodwin, "Generalized Sample Hold Functions–Frequency Domain Analysis of Robustness, Sensitivity, and Inter-sample Difficulties," in IEEE Transactions on Automatic Control. vol. 39, no. 5, 1994, pp. 1042-1047

[16] Y.C. Juan and P.T. Kabamba, "Optimal hold functions for sampled data regulation," in *Automatica*, vol. 27, no.1, 1991, pp. 177-181

[17] A.G. Aghdam and E.J. Davison, "An Optimization Algorithm for Decentralized Digital Control of Continuous-Time Systems which Accounts for Inter-Sample Ripple," to appear in *Proceedings of 2004 American Control Conference* Boston USA, July 2004