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# Non-Linear Trajectory Generation and Lateral Control New Algorithms to Minimize Platoon's Oscillations

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*Abstract*— **The lateral and longitudinal oscillations in a platoon of vehicles depend, fundamentally, on how the reference trajectory of the followers is generated and how the lateral and longitudinal control is designed. In this paper new algorithms are presented to reduce some of these problems. One of them takes charge in generating the reference trajectory of the followers by means of a series of points, where the separation distance between them is based on the kinematics of the vehicle, the trajectory curvature, and on the permissible distance error. The other one, the lateral control algorithm, takes charge of following the desired trajectory (the generated one). This algorithm is based on the "Look-ahead distance" strategy. In addition, this paper presents simulation results for these algorithms and the comparison of the proposed lateral control with other published one.**

## I. INTRODUCTION

IN the recent years, transportation philosophy and technology have received many researches to develop and to enhance the daily transportation system. One of these studies involves cooperative driving, specially a platoon of vehicles in which all members are able to drive in a cooperative way, maybe automatically or semi-automatically in order to reduce the traffic problems produced by human error and congestion [1]. In addition, platoons are classified by the type of connection between the units into: mechanically coupled platoons, in which the leading vehicle is driven manually and the following vehicles are driven automatically, using sensors attached to the mechanical link, which are used for steering control [2,3], and electronically coupled platoons, in which the leader broadcasts its action to all the vehicles in the platoon, and each vehicle is telling its intentions to the following one [4,5].

Therefore, in this kind of driving two important problems (oscillations) appear: Inter-vehicle distance error oscillation of the overall coupled vehicles, which is caused by a speed change of a single vehicle. This oscillation may be amplified upstream leading to what is known as the "slinky-type effect" or "string instability" [6, 7]. The second problem concerns lateral oscillation caused by a position change in the vehicle's lateral axis [8]. Therefore, these oscillations depend on the vehicle's operational control which is classified into longitudinal [9, 10] and lateral control, and on the desired trajectory generation.

Lateral control falls into lane change [14, 15] and path following [16, 17]. In path following field many works have been done to steer the vehicle to follow a desired path in which different strategies, algorithms, and sensors (i.e. GPS [18], camera [19], etc) have been used to get the desired location and to limit the vehicle's lateral oscillations (i.e. the orientation and position errors between the actual and the desired vehicle location). Two strategies used in these works include: 'Current location' and 'Look-ahead distance'. In the former, a lateral controller is designed to limit the errors (orientation and position) between the vehicle's actual location and desired location where the vehicle had to be hit at current time [13,20], and the latter in which lateral control input parameters depend on a look-ahead distance and the output of the controller is the steering angle that the vehicle needs to hit (The future desired location) [16,21].

In the trajectory generation field there are two types of path presentation; continuous [11] and discrete. In the latter, the trajectory is presented as a series of points or markers. These markers can be visible [12] or invisible [13]. The visible markers represented by magnetic, colors, etc; whereas the invisible one represents the vehicle's configuration space -at each point- using three dimensions, two of translation and one of rotation. Finally, the separation distance between any two markers is assumed constantly in many works because of the limitation that is assumed by these researchers. For example, according to Omae [13] the trajectory curvature should be more than 0.2 for which the separation distance is selected at 0.15 m.

However, this paper focuses on the lateral control algorithm for path following using the look-ahead distance strategy and on an adaptive non-linear trajectory generation as a set of points for platoon driving. That is to say, the

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trajectory generator adapts the points-separation distance to the curvature.

Finally, the rest of this paper is organized as follows: section two presents the trajectory generation as a set of points, section three presents the derivation of the lateral control algorithm, section four shows the simulation results by means of the comparison between the lateral control algorithm presented in this paper and one done by Kato et al. [21] and applied to following vehicles in a platoon "Demo 2000 Cooperative Driving". The last section presents the paper conclusions and future work.

## II. Trajectory Generation

As mentioned before, visible and invisible markers are used to represent the desired trajectory. In the former each vehicle measures its position with respect to fixed markers. In the second type, the vehicle control system uses sensors to localize the vehicle's current position and orientation, and compares this information with a stored one (desired location) to steer the vehicle to hit the desired point.

However, this part presents: the invisible markers with a variable separation distance between them, the separation distance (sampling-distance) interval, and the error between the distance covered by the platoon leader and the selected sampling-distance. To make it clear, the following assumptions should be taken into account:

1) The platoon's leader generates the trajectory for all members of the platoon. This strategy reduces the processing time, the computational cost for the other members and unites the reference trajectory for all members of the platoon, which damps the lateral oscillation of these units.

2) Trajectory generation is an on-line operation. That is to say, every so often ($T_{LM}$), the leader generates a sub-trajectory from its current location (at $T_k$) and its location prior to its ultimate maneuver (at $T_{k-1}$). Note here, the overall trajectory is generated by a set of sub-trajectories, each one presenting the leader's maneuver in $T_{LM}$ (see Fig. 1).

3) The leader's maneuver time ($T_{LM}$) represents the variable time needed to move from the position at $T_{k-1}$ to the position at $T_k$, which depends on the leader's velocity, steering angle and GPS frequency.

4) The Spline path generation methods have proven efficiency at generating smooth continuous curvature paths with low computational cost. For this reason, Spline techniques are used to generate the trajectory (by using MATLAB 6).

5) The leader samples the generated sub-trajectory (i.e. between $T_{k-1}$ and $T_k$), by using the sampling distance ($D_s$), into a set of posture that is used by the "trajectory agent" [22] of each members to track the trajectory and to follow the preceding vehicle. The sampling distance used here is related to the leader frame at ($T_{k-1}$). This distance depends on the curvature of the trajectory and on the longitudinal and lateral permissible oscillation errors.

6) The platoon leader sends information about the sub-trajectory generated for each maneuver time ($T_{LM}$), to all the other vehicles.

7) Each member has a lateral controller that steers it towards these points, ensuring that it hits them one after the other; and a longitudinal controller to change its velocity, thus reducing the inter-vehicle spacing error by using additional information about the leader velocity and acceleration; and its preceding vehicle location, velocity, and acceleration.

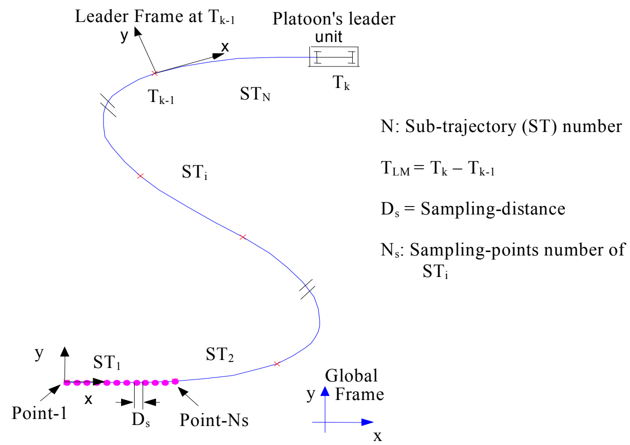


Fig. 1. Platoon trajectory generated by series of sub-trajectories.

Consider the vehicle's kinematics, it is modelled as an object moving in $\Re^2$ (Fig. 2). By experience it is known that in an empty space robot can be drive to any position with any rotation. Hence the vehicle's configuration space has three dimensions, two of translation and one of rotation. Using Fig.2, any configuration of the vehicle is presented by three parameters $(x, y, \theta) \in \Re^2 \times [0, 2\pi)$, with module $2\pi$ arithmetic on $\theta$. At any instant during motion, the project of any path of the vehicle on the $xy$-plane is a curve ($\gamma$) tangent to the main axis of the vehicle, thus implying that the motion is constrained by (1) and the curvature of ($\gamma$) is expressed by (2), where $L$ is the wheelbase length, $R_s$ the curvature radius, $C$ the curvature and $\delta$ is the front wheel angle. For DS electrical Club-Car model (Appendix B):

$|\delta| \leq |\delta max| < 32.62^o$, and $L=1.66$ m.

$$-\sin(\theta) \cdot dx + \cos(\theta) \cdot dy = 0 \quad (1)$$

$$C = 1 / R_s = \tan(\delta) / L \quad (2)$$

Furthermore, due to the non-holonomic constraint that the vehicle has, the space of differential motions (dx,dy,dθ) of the vehicle at any configuration (x, y, θ) is a two dimensional space in which the instantaneous motion is determined by two parameters: the linear velocity along its main axis (v) and the front wheel angle (δ). However, when the wheel angle is non-zero, the vehicle changes orientation (θ), and its linear velocity (v) with it allowing the vehicle's configurations to span a three-dimensional space. Hence, the vehicle's motion between two points in a desired trajectory is presented by (x,y,θ). That is to say, the motion between two points in xy-plane is presented as a curve with length Da (Fig. 2). Where: Da (Arc length) represents the actual distance covered by the platoon leader in one sample time (Ts), e is the error between the actual distance (Da) and the sampling one (Ds) in Ts, and β represents the angle between the perpendicular of rear wheel axis of the vehicle in the current sampling point and the perpendicular one in the next point.

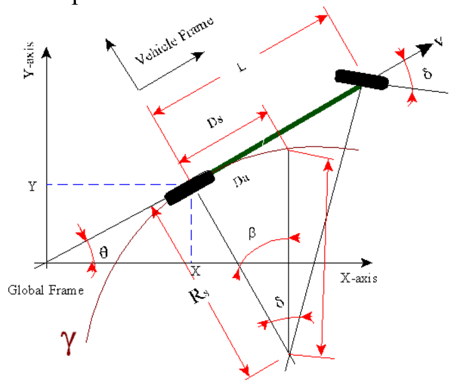

Fig. 2. Vehicle kinematics model

Using Fig. 2 and the curvature estimation based on the steering angle (see (2)), the error produce between the ($D_a$) and ($D_s$) is:

$$e = \frac{\beta}{c} - D_s = \frac{\beta \cdot L}{\tan(\delta)} - D_s \tag{3}$$

But, $D_s = R_s \cdot \sin(\beta)$

So, (3) can be re-written as follows.

$$e = \left(\frac{L}{\tan(\delta)}\right) \arcsin\left(\frac{D_s}{R_s}\right) - D_s \tag{4}$$

Fig. 3 presents the relation between the error ($e$) and the sampling-distance ($D_s$). According to this figure, the error is assumed to be in a range of [$0$ - $0.3\%D_{smin}$] for any curvature radius. Using this assumption, the interval of the sampling-distance is assumed [$0.1$ $0.4$] (see Appendix A).

Finally, the trajectory generation has done using MATLAB Spline functions (MATLAB 6, Fig.4).

## III. Lateral Control Algorithm

As mentioned before, the leader generates the trajectory as a series of points (or as invisible markers) and sends it to the rest of the platoon members to be as reference trajectory for all of them. Therefore, each vehicle is steered to hit these points one after other. Consequently, the essential work here is how to relate the vehicle's actual and desired locations with its front wheel angle.

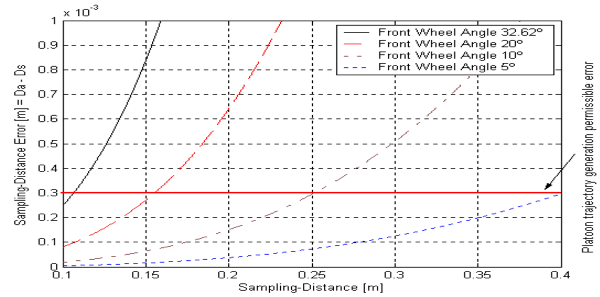

Fig. 3. The relation between the sampling-distance and the error ($e$) between the sampling-distance ($D_s$) and the actual one ($D_a$), with various front wheel angle.

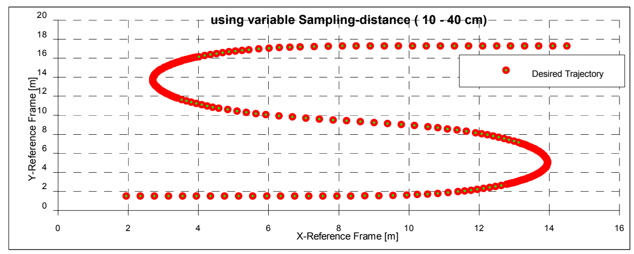

Fig. 4. Generated trajectory as a set of points

Take into account the above considerations, and suppose the front wheel angle in $P_0$ (Fig.5) changed to the angle ($\delta$) which needs to hit $P_1$ and to generate the trajectory with curvature radius $R_s$. The path length between $P_0$ and $P_1$ is expressed by (6) where $\beta$ is the vehicle orientation at $T_{j+1}$ with respect to the vehicle frame at $T_j$:

$$D_a = R_s \cdot \beta = \frac{\beta}{C} \tag{6}$$

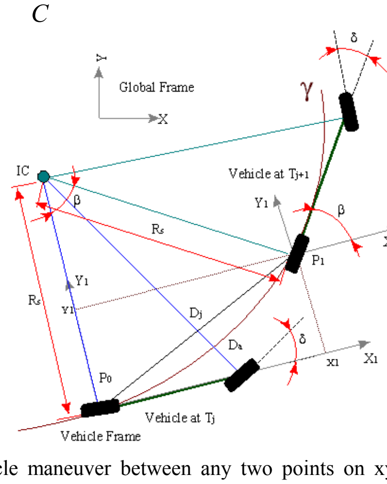

Fig. 5. Vehicle maneuver between any two points on xy-plane, where ($x_1$,$y_1$,$\beta$) are the vehicle coordinates and orientation at $T_{j+1}$ related to the vehicle frame at $T_j$.

According to Fig. 5 the error ($\varepsilon$) between the actual distance ($D_a$) and the string distance $D_j$, is expressed by (see Appendix A):

$$\varepsilon = \left(\frac{1}{C}\right) \cdot \sin^{-1}(C \cdot D_s) - \sqrt{D_s^2 + \left[\left(\frac{1}{C}\right) - \left(\frac{1}{C^2} - D_s^2\right)^{1/2}\right]^2} \tag{5}$$

Using the assumption detailed before ($e \in [0$ - $0.3\%D_s]$) the error (ε) will be less than 0.05% which could be negligible for any curvature radius as indicated in (5) and Fig. 6. Therefore; the string distance ($D_j$) is approximately equal to the actual one ( $D_a \approx D_j$ ), so:

$$D_s = \frac{\beta}{C} = \sqrt{(x_1)^2 + (y_1)^2} \tag{7}$$

From (2) and (7), the relation between the desired front wheel angle and the desired position and orientation of the vehicle at $T_{j+1}$ is expressed by (8).

$$\delta = \arctan\left(\frac{\beta \cdot L}{\sqrt{(x_1)^2 + (y_1)^2}}\right) \tag{8}$$

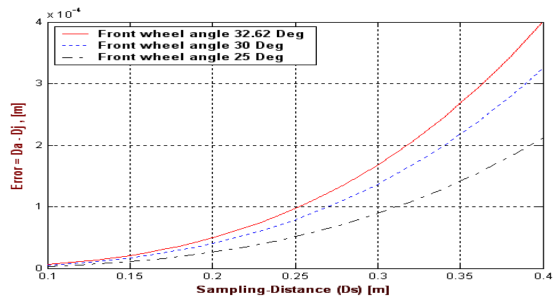


Fig. 6. The relation between the sampling distance and the error (ε) between the string distance (Dj) and the actual on (Da) sampling distance error with various front wheel angle.

## IV. Simulation Results

This section presents the simulation results for the trajectory generation using a series of points and lateral control by means of a comparison between the lateral control algorithm done by Kato et al [21] (see (9)) which is used in "Demo 2000 Cooperative Driving" and the one presented in this work (see (8)).

$$\delta = \arctan\left[2L\left(3y_1 - x_1 \tan(\beta)\right)/x_1^2\right] \tag{9}$$

This comparison has been done for:

1- Non-linear trajectory with constant points-separation (40 cm), see Fig. 7-12.
2- Proposed non-linear trajectory with inter-points adaptive distance (10-40 cm) depending on the curvature of the trajectory, see Fig 13-18.

It is important to note that, in this simulations the vehicle's linear velocity is assumed *24 km/h* and the only limitation which has been used is the steering angle change between any two points in the simulation steps (actually, this limitation depends on the response time of the steering dynamic system and the vehicle velocity). This change has been $\pm 10^o$ as a maximum permissible change ($\Delta\delta = \delta_{T(j+1)} - \delta_{T(j)}$). But this limitation is impossible to apply at Kato's algorithm simulation because of the big lateral oscillation that it produced in this kind of trajectory. In addition, this big oscillation carries the system to an instability state. For that, Kato's algorithm simulation in non-linear trajectory (see Fig. 11 and 17) has been done under the following assumption: The lateral controller is able to change the wheels direction ($\delta$) from the maximum clockwise rotation to the maximum anti-clockwise rotation without delay, that is to say, $\Delta\delta = \pm 2|\delta max| = \pm 2\ (32.62^o)$.

As indicated in Fig. 7-18, using constant sampling-distance strategy ($D_s$ = 0.4 m) the maximum oscillations produced by Kato's and Awawdeh's algorithms presented by Table 1.

TABLE 1.
THE MAXIMUM OSCILLATION PRODUCED BY KATO'S AND AWAWDEH'S ALGORITHMS WITH CONSTANT INTER-POINT SPACING.

| Kato et al. Algorithm | Awawdeh et al. Algorithm |
|---|---|
| ± 22º (Vehicle orientation) | ± 7º (Vehicle orientation) |
| ±1.1m (in Global x-axis) | ±0.23m(in Global y-axis) |

However, the result can be improved using variable sampling-distance strategy as indicated in Table 2.

TABLE 2.
THE MAXIMUM OSCILLATION PRODUCED BY KATO 'S AND AWAWDEH'S ALGORITHMS WITH THE PROPOSED ADAPTIVE STRATEGY FOR TRAJECTORY GENERATION.

| Kato et al. Algorithm | Awawdeh et al. Algorithm |
|---|---|
| ± 13.2º (Vehicle orientation) | 4.5º (Vehicle orientation) |
| ± 0.6m (in Global y-axis) | ±0.012m (in Global y-axis) |

## V. Conclusion And Future Work

As mentioned before, the lateral oscillations produced by using the proposed control algorithm are negligible, and by considering the steering limitation (the steering angle change) the system stays in stability state. Furthermore, this algorithm can be used in free vehicle drive and in platoon of vehicle with different kind of sensors (GPS, camera, etc).

As a future work, the design of the second part of the vehicle operational control in platoon (longitudinal control) and the implementation of this algorithm on a real vehicle (Club-Car) will be carried out.

## Appendix A

Using Fig.2, the error ($\varepsilon$) between the actual distance ($D_s$) and the string distance ($D_j$) can be expressed by (10).

$$\varepsilon = D_a - D_j \tag{10}$$

But

$$D_j = \sqrt{D_s^2 + \left[R_s - (R_s^2 - D_s^2)^{1/2}\right]^2} \tag{11}$$

and

$$D_a = R_s \cdot \beta \tag{12}$$

Substituting, $\beta = \sin^{-1}(D_s/R_s)$ and $C = (1/R_s)$, equation (10) can be re-written as follows:

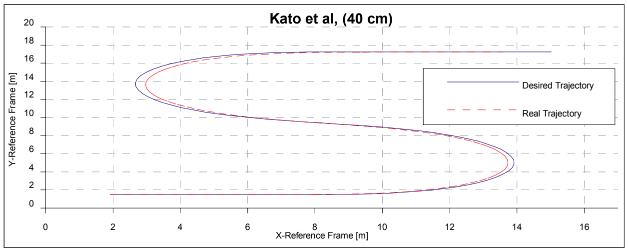


Fig. 7. The difference between the desired and the actual paths in non-linear trajectory (with constant separation distance 40 cm for the Club-Car model) using Kato et al. lateral control algorithm.

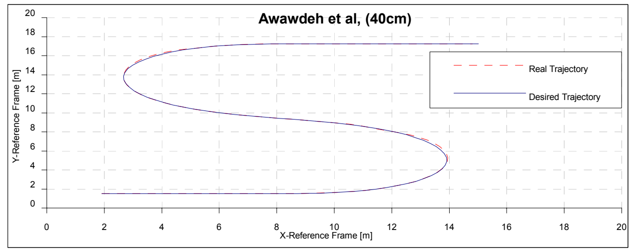


Fig. 8. The difference between the desired and the actual paths in non-linear trajectory (with constant separation distance 40 cm for the Club-Car model) using lateral control algorithm proposed in this work.

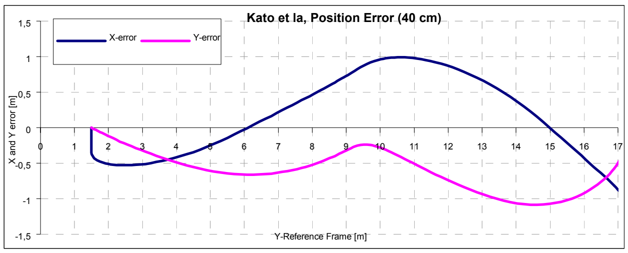


Fig. 9. The error between actual and desired positions in non-linear trajectory (with constant separation distance 40 cm for the Club-Car model) using Kato et al. lateral control algorithm.

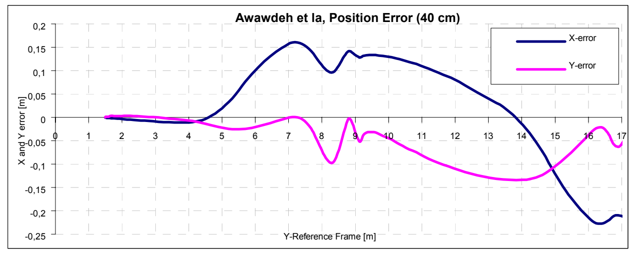


Fig. 10. The error between actual and desired positions in non-linear trajectory (with constant separation distance 40 cm for the Club-Car model) using lateral control algorithm proposed in this work.

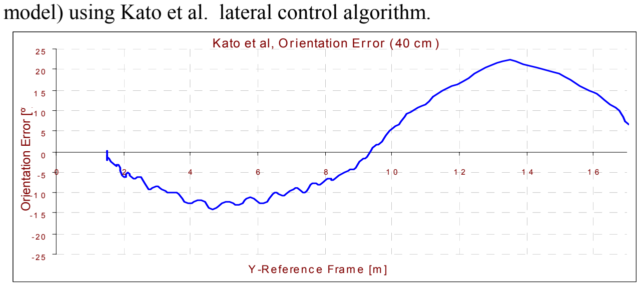


Fig. 11. Vehicle orientation in non-linear trajectory (with constant separation distance 40 cm for the Club-Car model) using Kato et al lateral control algorithm.

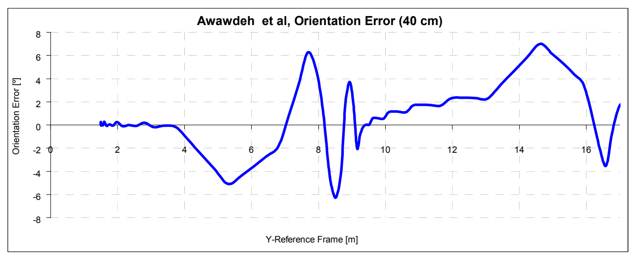


Fig. 12. Vehicle orientation in non-linear trajectory (with constant separation distance 40 cm for the Club-Car model) using the lateral control algorithm proposed in this work.

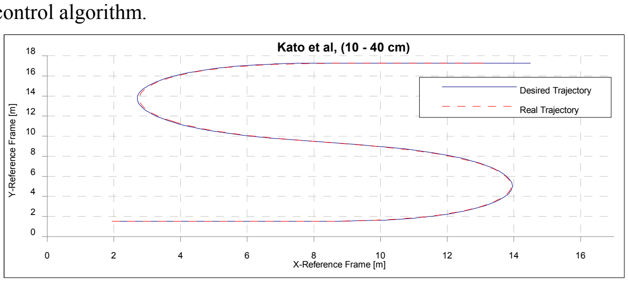


Fig. 13. The difference between the desired and the actual paths in non-linear trajectory (with variable separation distance 10-40 cm for the Club-Car model) using Kato et al. lateral control algorithm.

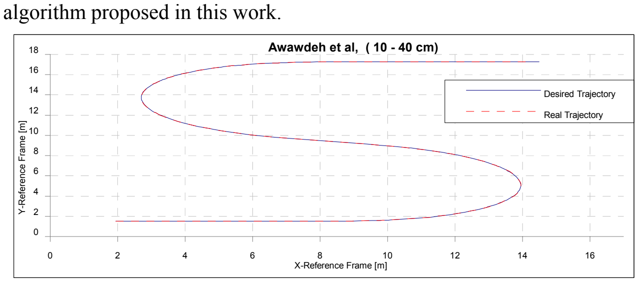


Fig. 14. The difference between the desired and the actual paths in non-linear trajectory (with variable separation distance 10-40 cm for the Club-Car model) using the lateral control algorithm proposed in this work.

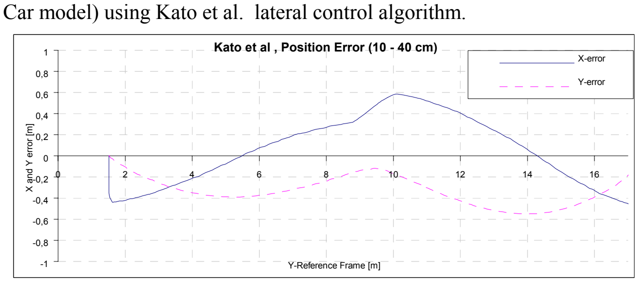


Fig. 15. The error between actual and desired positions in non-linear trajectory (with variable separation distance 10-40 cm for the Club-Car model) using Kato et al. lateral control algorithm.

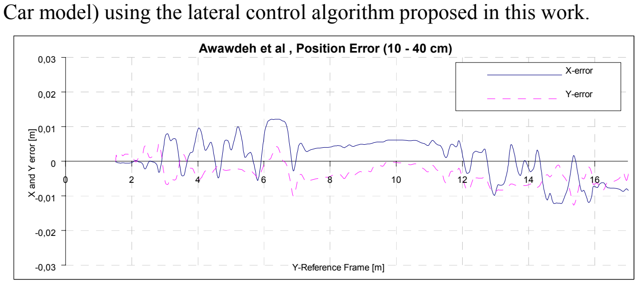


Fig. 16. The error between actual and desired positions in non-linear trajectory (with variable separation distance 10-40 cm for the Club-Car model) using the lateral control algorithm proposed in this work.

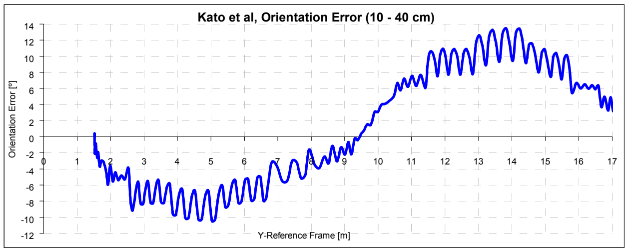


Fig. 17. Vehicle orientation in non-linear trajectory (with variable separation distance 10-40 cm for the Club-Car model) using Kato et al lateral control algorithm.

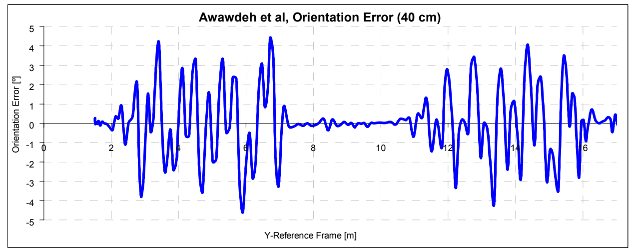


Fig. 18. Vehicle orientation in non-linear trajectory (with variable separation distance 10-40 cm for the Club-Car model) using the lateral control algorithm proposed in this work.

$$\varepsilon = \left(\frac{1}{C}\right)\cdot \sin^{-1}\left(C \cdot D_s\right) - \sqrt{D_s^2 + \left[\left(\frac{1}{C}\right) - \left(\frac{1}{C^2} - D_s^2\right)^{1/2}\right]^2}$$

For platoon applications the sampling-distance is assumed: *0.1* m for the maximum curvature [1] and *0.4* m for zero and small curvature because the maximum permissible error for inter-vehicles spacing should be 0.3%$D_{smax}$.

## Appendix B

The following picture shows the Club-Car of the Electronics Department of Alcalá University. This one is a reference vehicle used by the platoon guidance research group.

Fig. 19. Club-Car photograph of the Electronics Department research group related with platoon guidance.

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