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Design of a Decoupling Controller for Electrostatically Coupled Microcantilevers Based on Current Measurement

Mariateresa Napoli and Bassam Bamieh

Abstract— **In this paper we tackle the problem of decoupling the dynamics of closely spaced microcantilevers by using feedback control. In particular, the device we consider consists of two electrostatically actuated cantilevers, that are coupled both mechanically and electrostatically. For this pair, which is described by a time-varying periodic system of equations, we design an optimal $\mathcal{H}_\infty$ observer, based on the measurement of the current induced on the cantilevers by the voltage applied. The estimate of the cantilevers displacement is then used to generate the control signal that effectively decouples their dynamics. Simulation results are provided that demonstrate the efficacy of the control scheme proposed.**

I. INTRODUCTION

Over the past years, research in the field of scanning probe technology and electro-mechanical devices in general, has been characterized by two main trends, namely miniaturization and parallelizing. Indeed, the use of array architectures of micro probes not only significantly increases the throughput of the device, it enhances its functionality as well, allowing for more complex, multipurpose instruments. Examples of such devices can be found in data storage and retrieval applications [1], biosensors [2], and multi-probe scanning devices [3] to cite but a few.

Currently, these multi-probe devices are designed with large spacing between the individual elements. This essentially decouples the dynamics of the individual probes, that can be considered to behave as isolated units. The drawback of this configuration is, of course, a decrease in the potential throughput of the system.

The device that we consider in this paper consists of a pair of closely spaced microcantilevers, that can be independently actuated and sensed. The extension to the case of an array of tightly packed cantilevers is not conceptually difficult and is obtained as a generalization of the analysis we present here. In our design each microcantilever constitutes the movable plate of a capacitor and its displacement is controlled by the voltage applied across the plates. We have preferred capacitive actuation over other integrated schemes (e.g. piezoelectric [4], [5], [6], piezoresistive [7], [8], thermal [9]) because, as we show in this paper, it offers both electrostatic actuation as well as integrated detection, without the need for an additional position sensing device.

The dynamical behavior of the cantilever pair has been characterized in detail elsewhere [10]. Here we just recall the main results for the readers' benefit. In [10] it was demonstrated that the close spacing and the fact that the cantilevers are connected to a common base introduces a coupling in their dynamics, which is both electrostatic and mechanical. Moreover, it was shown that the system is described by a pair of second order periodic differential equations.

In this paper we focus on the problem of designing a controller able to ”electronically” decouple the cantilevers. In fact, the approach that we propose, an observer based feedback controller, involves at the same time the synthesis of a novel sensing scheme to reconstruct the displacement of the cantilevers : a dynamical state observer, whose input is the current through the cantilevers. By using an optimal observer, or by tuning the observer gains, it is conceivable that a high fidelity position measurement can be obtained, thus improving resolution in atomic force microscopy applications. The results shown in Section 3 prove our claim.

We formulate and solve the optimal observer problem as an $\mathcal{H}_\infty$ filtering problem for periodic systems. Simulation results demonstrate its excellent performance in estimating the cantilevers displacement. As a second step, this estimate is used by a feedback controller to decouple the dynamics of the cantilevers, so that they can be effectively operated as independent units, in spite of the physical coupling. The performance of the decoupling controller is tested in simulations, which indeed demonstrate a very satisfactory

Both authors are with the Department of Mechanical & Environmental Engineering, University of California, Santa Barbara, CA 93106, U.S.A. {napoli,bamieh}@engineering.ucsb.edu

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performance, even in the presence of noise.

The paper is organized as follows: In Section 2 we describe the mathematical model of the electrostatically actuated cantilever pair. In Section 3 we formulate and solve the optimal observer problem. In Section 4 we design the decoupling controller based on the observer estimate of the cantilever displacement. Simulation results are provided to prove the efficacy of the control scheme proposed. Finally, we present our conclusions in Section 5.

II. MODEL DESCRIPTION

Fig.(1) shows the geometry of our device. It consists of two microbeams connected to the same base, each forming a micro-capacitor, with the second (rigid) plate placed underneath the (movable) cantilever visible in the picture. The vertical displacement of each cantilever can be controlled by applying a voltage across the plates. Though each cantilever is independently actuated, its dynamics are influenced by the presence of the other cantilever. More precisely, the coupling is both mechanical, because the microbeams are connected to the same base, and electrical, due to the fringing fields generated by the capacitor nearby.

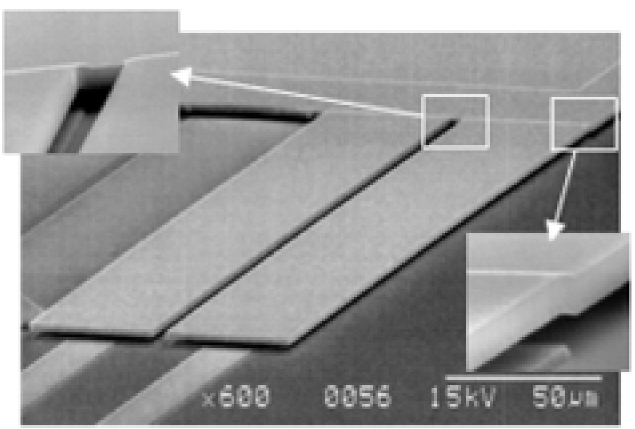

Fig. 1. SEM micrograph of the device. The insets show details of the mechanical connection to the base and between the cantilevers.

The force acting on each microbeam can be split into several components, so that the overall linearized equation of motion for the vertical displacement z_i, $i = 1, 2$, of each cantilever can be written as

$$\ddot{z}_i + \nu_i \dot{z}_i + \omega_{ri}^2 z_i = F_{a,i} + F_{mech,i} + F_{elec,i} \qquad (1)$$

where ν_i and ω_{ri} are respectively the normalized damping coefficient and the natural resonant frequency of the i-th cantilever. Here

$$F_{a,i} = \frac{\epsilon_o A}{2md^2}(1 + 2\frac{z_i}{d})V_i^2$$

expresses the attractive force between the capacitor plates of the i-th cantilever, with d gap between the electrodes, A area of the capacitor plates, m mass, and V_i voltage applied.

$$F_{mech,i} = \Gamma(z_i - z_j)$$

represents the mechanical coupling force, modeled as a spring like force. Due to the symmetry of the device, the coefficient of mechanical coupling Γ is the same for both cantilevers.

$$F_{elec,i} = [K_{ii}V_i^2 + K_{i,j}V_iV_j + K_{j,j}V_j^2](z_i - z_j)$$

is the linearized vertical component of the electrostatic interaction force, derived from a point charge model. The interested reader can find in [10] the details regarding the derivation of the model and its experimental validation.

For the special case of $V_i = V_{oi} \cos \omega_i t$, and for small values of the amplitude V_{oi}, we can neglect the time varying terms in Eq. (1) and rewrite it as

$$\ddot{z}_i + \nu_i \dot{z}_i + \omega_i^2 z_i + \gamma_1 z_j = b_1 V_i^2, \qquad (2)$$

where $\omega_i^2 = \omega_{ri}^2 - \Gamma - (K_e V_{oi}^2 + K_T)$, $\gamma_1 = \Gamma + K_T$, $K_T = (K_{11}V_{o1}^2 + K_{12}V_{o1}V_{o2} + K_{22}V_{o2}^2)/2$, $b_1 = K_e d/2$ and $K_e = \epsilon_o A/(md^3)$.

We consider the current through the cantilevers as the output y_i of the system

$$y_i = i_i(t) = \frac{d}{dt}(C_i V_i) \approx c_{1i}(t) z_i + c_{2i}(t)\dot{z}_i + v_f(t),$$

where the last term represents the first order approximation of y_i, with $c_{1i}(t) = -\frac{\epsilon_o A V_{oi} w_i}{d^2} \sin \omega_i t$, $c_{2i}(t) = \frac{\epsilon_o A V_{oi}}{d^2} \cos \omega_i t$, and $v_f(t) = \frac{\epsilon_o A V_{oi} w_i}{d} \sin \omega_i t$. Note that the peculiar dependence of the output on the input signal forces us to treat this system as time varying, even though in the case of small input signals the state equations are time invariant.

III. OPTIMAL OBSERVER DESIGN

The dependence of the output on the input signal, as illustrated in the previous paragraph, makes the system time varying. Moreover, if the inputs have different, non commensurable frequencies, the coupled equations are not even periodic. This makes the design of the observer for the coupled system much harder, since very few theoretical results are available for general time varying systems. We claim that this

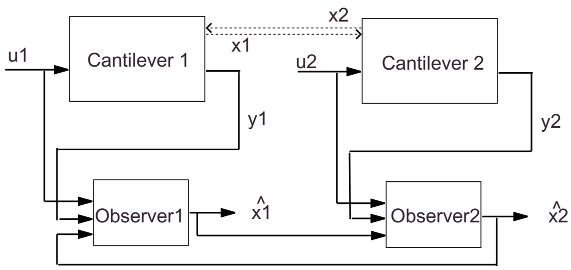

Fig. 2. A schematic of the observer. The dashed lines represent the coupling interaction.

complication is unnecessary and this section is devoted at proving it.

Fig.(2) is a schematic of the observer we propose for the cantilever pair. The subsystems corresponding to the observers are designed for each cantilever as if they were decoupled, i.e. treating the coupling variables z_j in (2) as if they were exogenous inputs. Notice that in this fashion not only we can recast the problem in the framework of periodic systems theory. At the same time we gain flexibility, in the sense that the design does not depend on the number of units (cantilevers) considered and can be easily extended to consider the case of an array of microcantilevers.

For a single cantilever the observer problem can be formulated, in the LFT framework, as an $\mathcal{H}_\infty$ filtering problem [11], by defining the regulated variable $\tilde{z} = z - \hat{z}$ (estimation error), and considering the generalized plant

$$G_{gen} := \left[\begin{array}{c|cc} \text{A} & [\text{M} \;\; 0] & 0 \\ \hline \text{I} & 0 & \text{-I} \\ \text{C(t)} & [0 \;\; \text{N}] & 0 \end{array}\right] = \left[\begin{array}{c|cc} \text{A} & B_1 & 0 \\ \hline C_1 & 0 & D_{12} \\ C_2(t) & D_{21} & 0 \end{array}\right], \tag{3}$$

where the exogenous input $w = [d \quad n]^T$ represents process and measurement noise, the matrices A, $C(t)$ are derived from (2) and the input $u = \hat{z}$ is the output of the observer system.

In this framework the problem amounts at finding a dynamical system G_{obs} such that the $\mathcal{H}_\infty$ norm of the transfer function $T_{\tilde{z}w}$ from w to $\tilde{z}$ is minimized. In particular it has been proved [12] that if the time-varying system has the structure of (3), then G_{obs} is a standard observer, whose gain $L(t)$ comes from the solution of a differential Riccati equation,

$$\begin{aligned} \dot{P}(t) = {} & A(t)P(t) + P(t)A(t)' - \\ & -P(t)[C(t)'R^{-1}C(t) - \frac{1}{\gamma^2}I]P(t) + B(t)B(t)'. \end{aligned} \tag{4}$$

More precisely, if the periodic non-negative definite solution of this equation, $P(t)$, is stabilizing, the

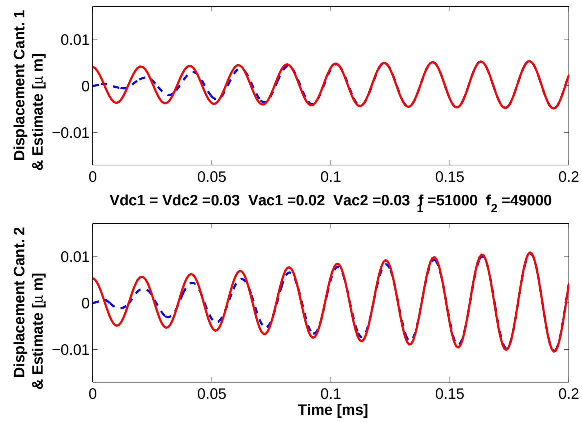

Fig. 3. Optimal observer performance. The parameters for the cantilevers are as in [10]. The dashed lines represent the observer estimate.

optimal filter is given by

$$\dot{\hat{z}} = A(t)\hat{z} + \mathbf{P(t)C(t)}'[y(t) - C(t)\hat{z}].$$

This procedure requires to compute the periodic

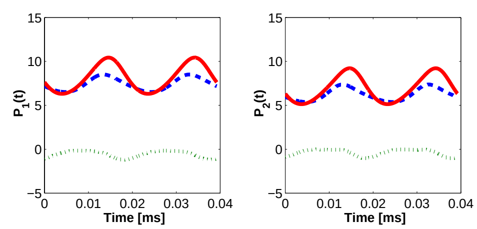

Fig. 4. Components of $P_1(t)$ and $P_2(t)$. The solid line represents the first component, the dashed line the second (and third) and the dotted line the fourth.

stabilizing solution of (4). In order to do so, we have defined the mapping $\mathcal{P} : R^{n \times n} \to R^{n \times n}$,

$$\mathcal{P}(M) = M - P(T),$$

where $P(T)$ is the solution, computed at time T of equation (4), with initial condition $P(0) = M$. If M is a matrix corresponding to any of the steady state periodic solutions, then $\mathcal{P}(M) = 0$. Thus the problem is converted to finding the fixed points of this equation. This is done numerically by using the secant method and defining the iterative scheme :

$$\begin{aligned} M_{k+1} \; = \; & M_k - [M_k - M_{k-1}] \cdot \\ & [\mathcal{P}(M_k) - \mathcal{P}(M_{k-1})]^{-1}\mathcal{P}(M_k). \end{aligned}$$

At this point it should be noted that the fact that the single optimal observers have stable error dynamics is not sufficient to guarantee that when combined as in Fig.(2) they remain stable. If we denote by e_1, e_2 the estimation errors on the state variables of the first and second cantilever respectively, their dynamics are described by the following equations

$$\dot{e}_1 = [A - P(t)C'(t)C(t)]e_1 + Ge_2,$$
$$\dot{e}_2 = Ge_1 + [A - P(t)C'(t)C(t)]e_2$$

i.e. they are coupled. G depends on the coupling coefficient γ_1. By means of the small gain theorem we can claim the stability of this system, as long as the norm of the two error subsystems is small enough, condition that can be included in the optimal problem.

Fig.(3) is a simulation of the performance of the observer obtained with the procedure outlined above. Fig.(4) shows the components of $P_1(t)$ and $P_2(t)$, obtained as a solution to (4) for $\gamma = 10$ and the parameters corresponding respectively to cantilever 1 and 2 as identified in [10]. In Fig.(3) the cantilevers are excited at their resonant frequency, which is assumed to be different. The other parameters of the simulation are reported in the figure. Observe the fast convergence of the estimate within few tens of nanometers from the exact value.

IV. THE DECOUPLING CONTROLLER

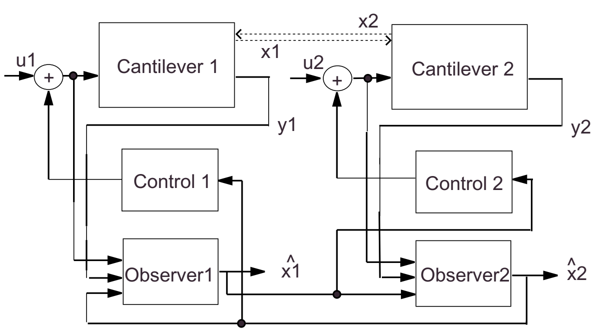

Fig. 5. Schematic of the observer based decoupling controller. The dashed lines represent internal coupling.

It is evident that in order to increase the throughput of a multi probe device, it is desirable to have the largest number of probes in the smallest possible space. On the other side, the proximity of the probes induces coupling in their dynamics, which increases the complexity of the overall device and can deteriorate its performance. For these reasons multi probe devices are currently designed with large spacings between the individual elements.

Our approach is radically different. We believe that one should not worry about coupling at the design stage of the device. As a matter of fact, we demonstrate that coupling can be removed by using an appropriate control action. Indeed if we consider our system, from equation (2) it is clear that we would be able to cancel out the effects of coupling if we could generate an input signal of the form

$$V_i(t) = \sqrt{V_{DC} + V_{AC}\cos(\omega_i t) + \frac{\gamma_1}{b_1} z_j},$$

where the DC offset needs to be large enough so that V_i is always well defined. Note that this does not represent an unfeasible constraint since, in the linear regime of operation, both z_j and V_{AC} are small. This strategy, however, requires the direct measurement of z_j which is something we want to avoid, since it requires a cumbersome apparatus (see optical lever methods commonly used in AFM, or scanning vibrometry), defeating the efforts of reducing the scale of the device.

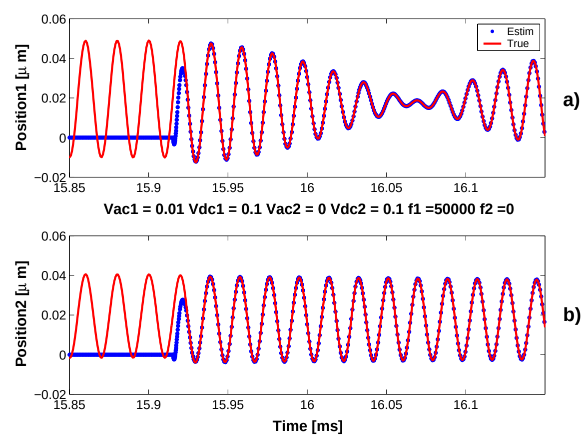

Fig. 6. Steady state oscillation of Cantilever 1 (**a**) and Cantilever 2 (**b**) and detail of the transient when the controller is switched on. The solid line is the cantilever oscillation, the dotted line its estimate. Notice the fast transient of the observer.

The method that we propose is represented schematically in Fig.(5). Here the i-th controller uses an estimate of the displacement of the j-th cantilever, produced by the corresponding observer, so that the input signal is

$$V_i(t) = \sqrt{V_{DC} + V_{AC}\cos(\omega_i t) + \frac{\gamma_1}{b_1} \hat{z}_j}.$$

Notice that, given the dependence of the current from the input, this choice of control signal represents a problem for the synthesis of the optimal oberver gain. In fact, it requires to know a priori $\hat{z}_j$. On the other side, in the linear regime of operation of the device, this signal is much smaller than V_{DC} and V_{AC}, so that we can neglect it in the computation of $P(t)$, without compromising the performance of the observer.

Fig.(6) shows the result of a simulation where one of the cantilevers (Cantilever 1) is excited close to its resonance frequency ($V_{DC} = .1V$, $V_{AC} = 10mV$, $f_1 = 50kHz$), while the other has a constant input ($V_{DC} = .1V$). In particular, we show the instant when the observer/controller is switched on. Notice how prior to this time, as a consequence of coupling, both cantilevers are oscillating (at the same frequency).

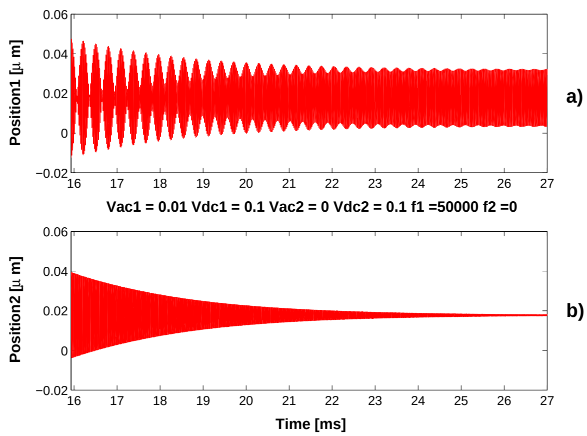

Fig. 7. Oscillation of Cantilever 1 (**a**) and Cantilever 2 (**b**) after the controller is switched on, showing the controller transient. This longer transient is dominated by the system's time constant.

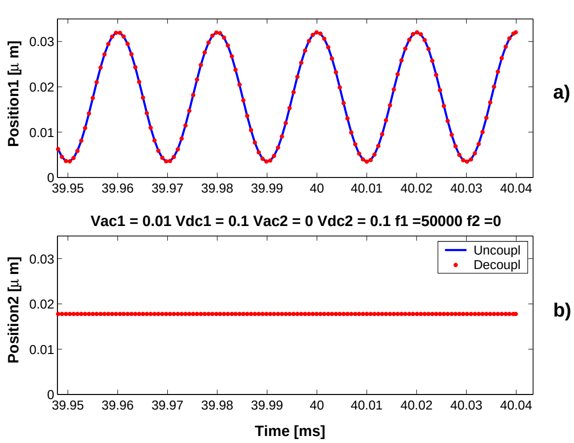

Fig. 8. Comparison between the decoupled cantilevers oscillations and equal single (uncoupled) cantilevers subject to the same external inputs : **a)** Cantilever 1, **b)** Cantilever 2.

Fig.(7) shows the transient of the controller. In spite of the very fast response of the observer, whose estimation error goes to zero almost instantaneously, the time constant of the controller is much longer. This is due to the fact that, after removing the coupling, the evolution of each cantilever is dictated by its own time constant, and because of its very lightly damped modes, it takes some time to reach steady state.

Fig.(8) is a comparison of the steady state oscillation of the now decoupled cantilevers, with the oscillation that isolated (uncoupled) identical cantilevers would exhibit if excited by the same input. Notice the excellent performance of the decoupling controller, confirmed also in Fig.(9), which shows the decoupling error, defined as the difference between the outputs of the corresponding coupled and uncoupled cantilevers : at steady state this error amounts only to few pm.

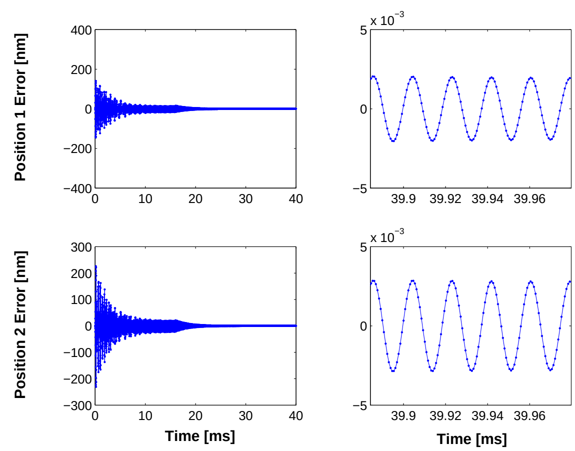

Fig. 9. Decoupling error. Note that the scale is nm.

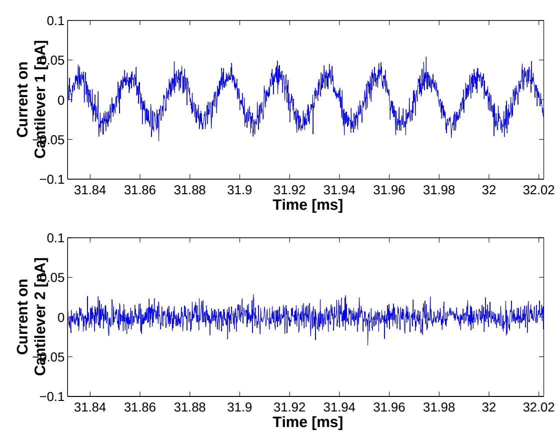

Fig. 10. Simulation of sensor noise : current measured on both cantilevers.

Simulations were performed also to check the effect of measurement noise on the performance of the decoupling controller. Keeping all other parameters unchanged, noise has been added to the output signal, as shown in Fig.(10). Fig.(11) compares the oscillations of the coupled cantilever pair, decoupled by the controller action, and the oscillation of single uncoupled cantilevers subject to the same input. Fig.(12) represents the decoupling error. Compared to the noiseless case, the performance of the decoupling controller is certainly degraded, but remains satisfactory, with a decoupling error in the order of few nm.

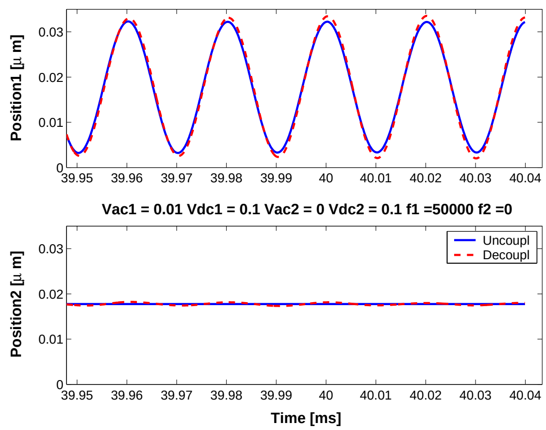

Fig. 11. Comparison between decoupled and uncoupled oscillation of same cantilevers in the presence of measurement noise.

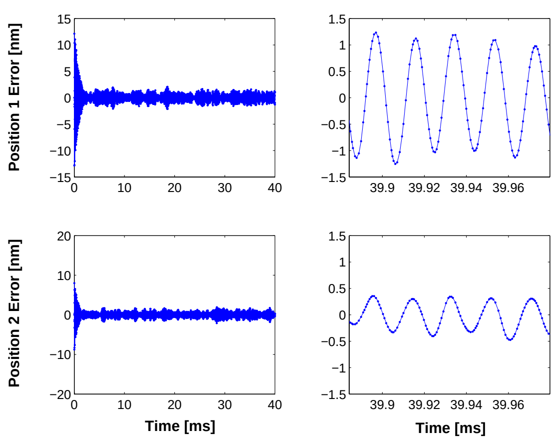

Fig. 12. Decoupling error in the presence of measurement noise.

V. Conclusions

In this paper we have considered the problem of designing a controller able to decouple the dynamics of closely spaced cantilevers. The device we have considered consists of two independently actuated microcantilevers, that are coupled both mechanically and electrostatically. Using a current measurement, we have shown the design of an optimal observer, which can reconstruct the position of the cantilevers with very high accuracy. Based on this observer a decoupling controller has been designed and its performance has been verified in simulations. Even though the performance of the control scheme has been tested only in the case of two cantilevers, the methodology proposed here lends itself to an easy extension to the case of arrays of probes.

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