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CONTROL OF MULTIVARIABLE SYSTEMS WITH INTERVAL PARAMETERS

Yan Shao
Department of Electrical Engineering, School of Engineering, Design and Technology, Bradford University, Bradford, West Yorkshire, BD7 1DP, United Kingdom

YuMing Zhang
University of Kentucky Center for Manufacturing and Department of Electrical and Computer Engineering, Lexington, KY 40506, USA

ABSTRACT. This paper considers a class of *m*-input *m*-output interval systems. The systems are described by impulse response functions. The parameters in the system model are unknown, but bounded by given intervals. An algorithm that determines the control actions based on the intervals of the model parameters is proposed. It is proved that the robust steady-state performance in tracking a given set-point is guaranteed despite possible open-loop overshooting, delay, and nonminimum phase of the interval plants. A simulation example is given.

Key words: robust control, MIMO system, uncertainty.

1. INTRODUCTION

Interval models are useful descriptions for many uncertain dynamic processes. Much of the present success in interval plant control is related to system analyses. Effective design methods [1] for the interval plant control are desired. Recently, Abdallah et al. [2], and Olbrot & Nikodem [3] addressed a class of interval plants with one interval parameter. Datta and Patel synthesized a robust control for a single-input single-output (SISO) interval plant by converting the parametric uncertainty to an uncertainty band and then applying the standard H_∞ method [4]. Datta and Bhattacharyya used an interval approach for adaptive control [5]. In [6], Baras and Patel established methods that yield a conceptual approach for constructing controlled-invariant sets and stabilizing controllers for set-valued discrete-time dynamic uncertain nonlinear systems.

This paper uses a predictive algorithm to control multivariable interval plants. The author observed that predictive controllers were traditionally designed primarily based on the nominal model without explicitly using the uncertainty of the controlled process [7, 8]. Campo and Morari [9] and Allwright and Papavashiliou [10] have developed predictive control algorithms for models with interval parameters. However, their efforts were towards the computational aspects and no performance results have been either given or proven. Recently, Nicolao et al. developed a robust predictive control for uncertain impulse response functions with one uncertain parameter [11]. In [12], a predictive control algorithm was proposed for SISO interval plants. The steady-state stability of the resultant closed-loop system was proved under uncertain conditions. This algorithm has been applied to control a gas metal arc welding process where the process uncertainty is mandatory [13]. In this paper, multivariable systems will be addressed.

2. SYSTEM

2.1 Problem description:

Consider the following *m*-input *m*-output discrete system:

$$Y_k = H(z^{-1})U_k \tag{1}$$

where k is the current instant,

$$\begin{cases} U_k = [u_k^{(1)}, u_k^{(2)}, \ldots, u_k^{(m)}]^T \\ Y_k = [y_k^{(1)}, y_k^{(2)}, \ldots, y_k^{(m)}]^T \end{cases} \tag{2}$$

are the input and output vectors at k,

$$H(z^{-1}) = \sum_{j=1}^{n} H(j) z^{-j} \tag{3}$$

is the transfer function matrix,

$$H(j) = \begin{bmatrix} h_{11}(j) & h_{12}(j) & \ldots & h_{1m}(j) \\ h_{21}(j) & h_{22}(j) & & h_{2m}(j) \\ \ldots & \ldots & \ldots & \ldots \\ h_{m1}(j) & h_{m2}(j) & \ldots & h_{mm}(j) \end{bmatrix} \tag{4}$$

while n and $h_{i_1 i_2}(j)'s\ (i_1 = 1, 2, \ldots, m;\ i_2 = 1, 2, \ldots, m; j = 1, 2, \ldots n)$ are the order and the real number parameters, respectively.

Assume $h_{i_1 i_2}(j)'s\ (i_1 = 1, 2, \ldots, m;\ i_2 = 1, 2, \ldots, m; j = 1, 2, \ldots n)$ are time-invariant. They are unknown but bounded by the following intervals:

$$h_{i_1 i_2 \min}(j) \le h_{i_1 i_2}(j) \le h_{i_1 i_2 \max}(j) \tag{5}$$

where $h_{i_1 i_2 \min}(j) \le h_{i_1 i_2 \max}(j)\ (i_1 = 1, 2, \ldots, m;\ i_2 = 1, 2, \ldots, m; j = 1, 2, \ldots n)$ are known. Denote the set-point as

$$Y_0 = [y_0^{(1)}, y_0^{(2)}, \ldots, y_0^{(m)}]^T \tag{6}$$

The objective is to design a controller for determining the feedback control actions $\{U_k\}'s$ so that the closed-loop system achieves the following robust steady-state performance:

$$\lim_{k\to+\infty} Y_k = Y_0 \tag{7}$$

where Y_k is the output of the closed-loop system.

The unit step response function

$$S(i)=\sum_{j=1}^{i} H(j)=\begin{bmatrix} s_{11}(i) & s_{12}(i) & \dots & s_{1m}(i) \\ s_{21}(i) & s_{22}(i) & & s_{2m}(i) \\ \dots & \dots & \dots & \dots \\ s_{m1}(i) & s_{m2}(i) & \dots & s_{mm}(i) \end{bmatrix} \tag{8}$$

where $s_{i_1i_2}(i)$ and their upper and lower limits $s_{i_1i_2\max}(i)$ and $s_{i_1i_2\min}(i)$ are:

$$\begin{aligned} s_{i_1i_2\max}(i) &= \sum_{j=1}^{i} h_{i_1i_2\max}(j) \ge s_{i_1i_2}(i) = \sum_{j=1}^{i} h_{i_1i_2}(j) \\ &\ge s_{i_1i_2\min}(i) = \sum_{j=1}^{i} h_{i_1i_2\min}(j) \quad (1\le i\le n) \end{aligned} \tag{9A}$$

$$\begin{aligned} s_{i_1i_2\max}(i) &= s_{i_1i_2\max}(n) \ge s_{i_1i_2}(i) = s_{i_1i_2}(n) \\ &\ge s_{i_1i_2\min}(i) = s_{i_1i_2\min}(n) \quad (i > n+1) \end{aligned} \tag{9B}$$

It is apparent that

$$S(i) = S(n) \quad (i > n) \tag{10}$$

2.2 System assumption:

In order to ensure $|S(n)| \ne 0$, $\max(S(n)\Delta U_k)\min(S(n)\Delta U_k) > 0 \quad (\forall \Delta U_k \ne 0)$ is required. Thus, the following conditions are assumed

$$\begin{cases} \max(S(n)\Delta U_k)\min(S(n)\Delta U_k) > 0 \\ \qquad\qquad (\forall \Delta U_k \ne 0) \\ |S(n)| \ne 0 \end{cases} \tag{11}$$

In the previous paper for SISO system [12], it was assumed that the following can be guaranteed through proper mathematical transformations:

$$\begin{cases} s_{\max}(n) \ge s_{\min}(n) > 0 \\ y_0 \ge 0 \end{cases}$$

where $s_{\max}(n) \ge s_{\min}(n)$ are the upper limit and lower limit of the static gain of the SISO system and y_0 is the set-point for the single output y. In this paper, such unnecessary condition is not assumed.

3. Derivation

Consider instant k $(k = 1,\ 2,\ 3,\ \dots)$. Assume the feedback Y_k is available and U_k needs to be determined. Model (1) yields

$$\Delta Y_k = \sum_{j=1}^{n} H(j)\Delta U_{k-j} \tag{12}$$

where

$$\left.\begin{aligned} \Delta Y_k &= [\Delta y_k^{(1)} - \Delta y_{k-1}^{(1)}, \Delta y_k^{(2)} - \Delta y_{k-1}^{(2)}, \dots, \Delta y_k^{(m)} - \Delta y_{k-1}^{(m)}]^T \\ \Delta U_{k-j} &= [\Delta u_{k-j}^{(1)} - \Delta u_{k-j-1}^{(1)}, \Delta u_{k-j}^{(2)} \\ &\quad - \Delta u_{k-j-1}^{(2)}, \dots, \Delta u_{k-j}^{(m)} - \Delta u_{k-j-1}^{(m)}]^T \end{aligned}\right\} \tag{13}$$

Denote

$$Y_{k+i}(\Delta U_k) = Y_{k+i}\big|_{\Delta U_{k+j}=0\ (\forall j>0)} \tag{14}$$

where $Y_{k+i}\big|_{\Delta U_{k+j}=0\ (\forall j>0)}$ is the output vector at instant $k+i\ (i>0)$ when the control actions are kept unchanged after instant k, i.e., $U_{k+j} = U_k\ (\forall j > 0)$. Based on (12), the following equations can be obtained:

$$Y_{k+1}(\Delta U_k) = Y_k + \sum_{j=1}^{n} H(j)\Delta U_{k+1-j} \tag{15}$$

$$\begin{aligned} Y_{k+2}(\Delta U_k) &= Y_{k+1}(\Delta U_k) + \sum_{j=1}^{n} H(j)\Delta U_{k+2-j} \\ &= Y_k + \sum_{j=1}^{n} H(j)(\Delta U_{k+2-j} + \Delta U_{k+1-j}) \end{aligned} \tag{16}$$

$$Y_{k+i}(\Delta U_k) = Y_k + \sum_{j=1}^{n} H(j) \sum_{l=1}^{\min(i,j)} \Delta U_{k+l-j} \quad (i = 1, 2, 3, \dots) \tag{17}$$

When $i = n$, Eq. (17) results in

$$\begin{aligned} Y_{k+n}(\Delta U_k) &= Y_k + \sum_{j=1}^{n} H(j) \sum_{l=1}^{j} \Delta U_{k+l-j} \\ &= Y_k + \sum_{j=1}^{n} H(j) \sum_{l=1}^{j-1} \Delta U_{k+l-j} + \sum_{j=1}^{n} H(j)\Delta U_k \\ &= Y_k + \sum_{j=2}^{n} H(j) \sum_{l=1}^{j-1} \Delta U_{k+l-j} + \sum_{j=1}^{n} H(j)\Delta U_k \end{aligned} \tag{18}$$

Thus,

$$Y_{k+n}(\Delta U_k) = Y_k + \sum_{j=2}^{n} H(j)(U_{k-1} - U_{k-j}) + S(n)\Delta U_k \tag{19}$$

It is evident that

$$\begin{aligned} Y_{k+n}(\Delta U_{k-1}) &= Y_{k+n}(\Delta U_k)\big|_{\Delta U_k=0} \\ &= Y_k + \sum_{j=2}^{n} H(j)(U_{k-1} - U_{k-j}) \end{aligned} \tag{20}$$

Thus,

$$Y_{k+n}(\Delta U_k) = Y_{k+n}(\Delta U_{k-1}) + S(n)\Delta U_k \tag{21}$$

4. CONTROL

4.1 Control Criterion:

In the proposed control algorithm, ΔU_k is so determined that:

$$\begin{aligned} &\max Y_{k+n}(\Delta U_k) \\ &\quad = \max Y_{k+n}(\Delta U_{k-1}) + \max(S(n)\Delta U_k) = Y_0 \end{aligned} \tag{22}$$

4.2 Properties:

Theorem 1: For the given interval plant control problem (1), (5), and (11),

$$\lim_{k\to+\infty} Y_k = Y_0 \tag{23}$$

when algorithm (22) is used.

Proof: For the given nth order model (1),

$$Y_{k+n-1+i}(\Delta U_{k-1}) = Y_{k+n-1}(\Delta U_{k-1}) \quad (\forall i > 0) \tag{24}$$

Thus,

$$Y_{k+n}(\Delta U_{k-1}) = Y_{k+n-1}(\Delta U_{k-1}) \tag{25}$$

From (25) and (20),

$$\begin{aligned} Y_{k+n-1}(\Delta U_{k-1}) &= Y_{k+n}(\Delta U_{k-1}) \\ &= Y_k + \sum_{j=2}^{n} H(j)(U_{k-1} - U_{k-j}) \end{aligned} \tag{26}$$

Hence,

$$Y_{k+n}(\Delta U_k) = Y_{k+n-1}(\Delta U_{k-1}) + S(n)\Delta U_k \tag{27}$$

The control criterion (22) states that

$$\max Y_{k+n}(\Delta U_k) = Y_0 \tag{28}$$

Thus,

$$Y_{k+n}(\Delta U_k) \le Y_0 \tag{29}$$

Here vector inequality (29) implies that any element in Y_0 is larger than or equal to the corresponding element in $Y_{k+n}(\Delta U_k)$. Eqs. (29) and (25) generate

$$Y_{k+n+1}(\Delta U_k) = Y_{k+n}(\Delta U_k) \le Y_0 \tag{30}$$

Therefore, (29) and (22) produce

$$\max(S(n)\Delta U_{k+1}) = Y_0 - \max(Y_{k+n+1}(\Delta U_k)) \ge 0 \tag{31}$$

Since

$$\max(S(n)\Delta U_k)\min(S(n)\Delta U_k) > 0 \quad (\forall \Delta U_k \ne 0) \tag{32}$$

then

$$\begin{cases} \max(S(n)\Delta U_{k+1}) > 0 \Leftrightarrow S(n)\Delta U_{k+1} > 0 \\ \max(S(n)\Delta U_{k+1}) = 0 \\ \qquad \Leftrightarrow S(n)\Delta U_{k+1} = 0 \Leftrightarrow \Delta U_{k+1} = 0 \end{cases} \tag{33}$$

Thus,

$$\begin{aligned} Y_0 \ge Y_{k+n+1}(\Delta U_{k+1}) &= Y_{k+n}(\Delta U_k) \\ &+ S(n)\Delta U_{k+1} \quad (k = 1, 2, 3, ...) \end{aligned} \tag{34}$$

Denote

$$\begin{aligned} E(\Delta U_{k+1}) &= S(n)\Delta U_{k+1} \\ &= (e^{(1)}(\Delta U_{k+1}), e^{(2)}(\Delta U_{k+1}), ..., e^{(m)}(\Delta U_{k+1}))^T \end{aligned} \tag{35}$$

Assume $\lim_{k\to\infty} \Delta U_{k+1} \ne 0$. Because $S(n)\Delta U_{k+1} > 0 \quad (\forall \Delta U_{k+1} \ne 0)$, $\lim_{k\to\infty} E(\Delta U_{k+1}) \ne 0$. Thus, for any given positive integer M, a $\delta > 0$ and j $(1 \le j \le m)$ exist so that the number of ΔU_{k+1}'s that satisfy

$$e^{(j)}(\Delta U_{k+1}) > \delta \quad (k > M) \tag{36}$$

is infinite. Hence, $\lim_{k\to\infty} y^{(j)}_{k+n+1}(\Delta U_{k+1}) = \infty$. This contradicts inequality (30) or control criterion (22). Therefore, $\lim_{k\to\infty} \Delta U_k = 0$. From (17), $\lim_{k\to\infty} \Delta U_k = 0$ yields

$$\begin{aligned} \lim_{k\to\infty} \max Y_{k+n+1}(\Delta U_{k+1}) &= \lim_{k\to\infty} Y_{k+n+1}(\Delta U_{k+1}) \\ &= \lim_{k\to\infty} \min Y_{k+n+1}(\Delta U_{k+1}) \end{aligned} \tag{37}$$

Hence,

$$\lim_{k\to\infty} Y_{k+n+1}(\Delta U_{k+1}) = \lim_{k\to\infty} \max Y_{k+n+1}(\Delta U_{k+1}) = Y_0 \tag{38}$$

That is,

$$\lim_{k\to+\infty} Y_k = Y_0 \tag{39}$$

Remark 1: If the control criterion were

$$\max Y_{k+1}(\Delta U_k) = Y_0 \tag{40}$$

the resultant control would be similar to a one-step-ahead prediction based control. In this case, the robustness of the resultant closed-loop performance is not guaranteed. In fact, the poor robustness of one-step-ahead prediction based controls is the major reason for developing the more robust long-range predictive controls.

Remark 2: In general, for many interval plants, criterion

$$\max Y_{k+p}(\Delta U_k) = Y_0 \tag{41}$$

may obtain the performance (39) with $1 \le p < n$. However, theoretical work which can be used to judge whether a p $(1 \le p < n)$ exists for guaranteeing the performance (39) for a given interval plant has not been established. When a p $(1 \le p < n)$ is used, the regulation speed would improve when p decreases, whereas the robustness of the performance would tend to be poorer.

Remark 3: It can be shown that if a constant disturbance vector is applied, $\lim_{k\to+\infty} Y_k = Y_0$ is not affected. Also, $\lim_{k\to+\infty} Y_k = Y_0$ is proved without assuming conditions on the system other than (11). Non-minimum phase, overshooting, delay, and large parameter intervals thus do not influence the steady-state performance $\lim_{k\to+\infty} Y_k = Y_0$.

5. IMPLEMENTATION

The control algorithm can be implemented based on the following equation:

$$\max(S(n)\Delta U_k) = Y_0 - \max Y_{k+n}(\Delta U_{k-1}) \tag{42}$$

where

$$\max Y_{k+n}(\Delta U_{k-1})$$
$$= \max\{Y_k + [\sum_{j=2}^{n}\sum_{i_2=1}^{m} h_{1i_2}(j)(u_{k-1}^{(i_2)} - u_{k-j}^{(i_2)}),$$
$$\sum_{j=2}^{n}\sum_{i_2=1}^{m} h_{2i_2}(j)(u_{k-1}^{(i_2)} - u_{k-j}^{(i_2)}),\dots,\sum_{j=2}^{n}\sum_{i_2=1}^{m} h_{mi_2}(j)(u_{k-1}^{(i_2)} - u_{k-j}^{(i_2)})]^T\}$$
$$= Y_k + [\sum_{j=2}^{n}\sum_{i_2=1}^{m} h_{1i_2}^*(j)(u_{k-1}^{(i_2)} - u_{k-j}^{(i_2)}),$$
$$\sum_{j=2}^{n}\sum_{i_2=1}^{m} h_{2i_2}^*(j)(u_{k-1}^{(i_2)} - u_{k-j}^{(i_2)}),\dots,\sum_{j=2}^{n}\sum_{i_2=1}^{m} h_{mi_2}^*(j)(u_{k-1}^{(i_2)} - u_{k-j}^{(i_2)})]^T \tag{43}$$

and

$$h_{i_1 i_2}^*(j) = \begin{cases} h_{i_1 i_2 \min}(j) \ \ (u_{k-1}^{(i_2)} - u_{k-j}^{(i_2)} < 0) \\ h_{i_1 i_2 \max}(j) \ \ (u_{k-1}^{(i_2)} - u_{k-j}^{(i_2)} \geq 0) \end{cases} \tag{44}$$
$$(\forall i_1 = 1,\dots,m; i_2 = 1,\dots,m;\ j = 2,\dots,n)$$

It can be shown that

$$\max(S(n)\Delta U_k) = \max[\sum_{i_2=1}^{m} s_{1i_2}(n)\Delta u_k^{(i_2)},$$
$$\sum_{i_2=1}^{m} s_{2i_2}(n)\Delta u_k^{(i_2)},\dots,\sum_{i_2=1}^{m} s_{mi_2}(n)\Delta u_k^{(i_2)}]^T \tag{45}$$
$$= [\sum_{i_2=1}^{m} \tilde{s}_{1i_2}(n)\Delta u_k^{(i_2)}, \sum_{i_2=1}^{m} \tilde{s}_{2i_2}(n)\Delta u_k^{(i_2)},\dots,$$
$$\sum_{i_2=1}^{m} \tilde{s}_{mi_2}(n)\Delta u_k^{(i_2)}]^T$$

where

$$\tilde{s}_{i_1 i_2}(n) = \begin{cases} s_{i_1 i_2 \min}(n) \ \ (\Delta u_k^{(i_2)} < 0) \\ s_{i_1 i_2 \max}(n) \ \ (\Delta u_k^{(i_2)} \geq 0) \end{cases} \tag{46}$$
$$(\forall i_1 = 1,\dots,m; i_2 = 1,\dots,m)$$

Thus,

$$\max(S(n)\Delta U_k) = \tilde{S}(n)\Delta U_k \tag{47}$$

where the i_1th row, i_2th column element of $\tilde{S}(n)$ is $\tilde{s}_{i_1 i_2}(n)$ as defined in (46). Condition (11) ensures that $|\tilde{S}(n)| \neq 0$. Hence, the control law is

$$\Delta U_k = (\tilde{S}(n))^{-1}(Y_0 - \max Y_{k+n}(\Delta U_{k-1})) \tag{48}$$

However, matrix $\tilde{S}(n)$ is unknown. It is known that each element in $\tilde{S}(n)$ has only two possible values given in (46). If the signs of $\Delta u_k^{(1)}, \Delta u_k^{(2)}, \dots,$ and $\Delta u_k^{(m)}$ are given, $\tilde{S}(n)$ will be certain so that ΔU_k can be calculated using (48). However, $\Delta u_k^{(1)}, \Delta u_k^{(2)}, \dots,$ and $\Delta u_k^{(m)}$ are the elements of ΔU_k being determined, and their signs are unknown.

One method to implement (48) is to assume the signs for the elements in ΔU_k so that a corresponding $\tilde{S}(n)$ can be determined to perform (48). There are 2^m possible combinations of the signs for the elements in ΔU_k. Hence, 2^m possible matrices that include the actual $\tilde{S}(n)$ can be used to obtain 2^m possible solutions for ΔU_k. If the signs of the elements in a resultant ΔU_k are the same as the assumed, (42) will be satisfied. This ΔU_k can therefore be applied to the control system. Because the actual $\tilde{S}(n)$ is included in the 2^m possible matrices, its corresponding ΔU_k which satisfies (42) must be included in the 2^m possible solutions. That is, the solution of control law (42) is guaranteed.

The implementation procedure for the control algorithm can summarized as following:

1. Calculate $Y_0 - \max Y_{k+n}(\Delta U_{k-1})$ based on (43).
2. For $j_1 = 1,\ 2; j_2 = 1,\ 2; \dots, j_m = 1,\ 2;$
   Assume $\Delta u_k^{(1)} \begin{cases} < 0 \ (j_1 = 1) \\ \geq 0\ (j_1 = 2) \end{cases}, \Delta u_k^{(2)} \begin{cases} < 0 \ (j_2 = 1) \\ \geq 0\ (j_2 = 2) \end{cases},$
   $,\dots\dots, \Delta u_k^{(m)} \begin{cases} < 0 \ (j_m = 1) \\ \geq 0\ (j_m = 2) \end{cases}$. Then calculate ΔU_k based on (48) and (46).
   If the signs of the elements in a calculated ΔU_k are the same as the assumed, go to Step 3.
3. Calculate $U_k = U_{k-1} + \Delta U_k$.

6. SIMULATION

Consider the following second order 2-input 2-output interval plant:

$h_{11\min}(j)'s$: 0.5, -0.4, 0.3, 0.2, 0.1;
$h_{11\max}(j)'s$: 0.7, 0, 0.5, 0.3, 0.2.
$h_{12\min}(j)'s$: 0.1667, -0.1333, 0.1, 0.0667, 0.0333;
$h_{12\max}(j)'s$: 0.2333, 0, 0.1667, 0.1, 0.0667.
$h_{21\min}(j)'s$: 0.125, 0, 0.075, 0.05, 0.025;
$h_{21\max}(j)'s$: 0.1625, 0, 0.0975, 0.065, 0.0325.
$h_{22\min}(j)'s$: 0.5, 0, 0.3, 0.2, 0.1;
$h_{22\max}(j)'s$: 0.65, 0, 0.39, 0.26, 0.13.

Here each type of $h(j)$ is given in order of $j = 1,\ 2,\ 3,\ 4,\ 5.$ The actual model parameters are: $h_{11}(j)'s$: 0.58, -0.24, 0.38, 0.24, 0.14; $h_{12}(j)'s$: 0.2333, 0, 0.1667, 0.1, 0.0667; $h_{21}(j)'s$: 0.125, 0, 0.075, 0.05, 0.025; $h_{22}(j)'s$: 0.605, 0, 0.363, 0.242, 0.121.

The set-point is $[1,\ 2]^T$. A constant disturbance vector $[1,\ 1]^T$ is applied at $k = 50$ so that the actual system becomes:

$$\begin{cases} Y_k = H(z^{-1})U_k & (k < 50) \\ Y_k = H(z^{-1})U_k + [1,\ 1]^T & (k \geq 50) \end{cases}$$

The response of the closed-loop system is illustrated in Fig. 1. It can be seen that despite the applied disturbance and the uncertainty, the desired outputs are maintained.

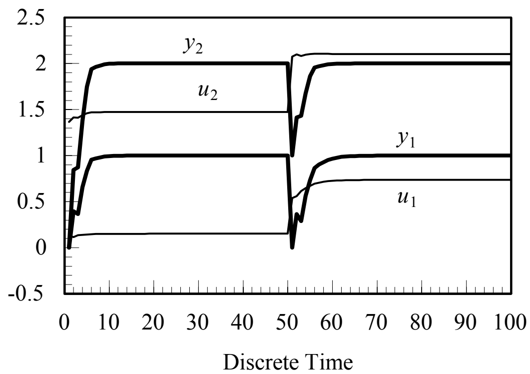

Fig. 1 Closed-loop response under disturbance.

7. CONCLUSIONS

An algorithm is proposed for the control of multivariable interval plants. The steady-state stability is guaranteed under assumed conditions (11). The implementation procedure is outlined. A simulation example is given to illustrate the effectiveness of the proposed algorithm.

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