Proceeding of the 2004 American Control Conference
Boston, Massachusetts June 30 - July 2, 2004

Dynamic precompensation and output feedback control of integrated process networks

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***Abstract*— In integrated process networks, the presence of large flowrates induces a time-scale separation of the dynamics where the individual units evolve in a fast time scale while the overall process evolves in a slow time scale. The slow dynamics of such networks are modeled by a high index differential algebraic equation system which, in the case of cascaded control configurations, has a control dependent state-space. We propose a minimal-order dynamic extension to obtain a modified DAE system of index two with a control invariant state-space that can be subsequently used as the basis for controller design. We illustrate this method for a distillation column with large recycle where the top and the bottom compositions are the key outputs to control.**

Keywords DAEs; process networks; non-linear control; distillation column

I. Introduction

Integrated process networks, i.e. process networks interconnected with large recycle of material and/or energy, are the rule rather than the exception in chemical plants. The behavior of such networks is typically highly non-linear due to the feedback interactions induced by recycle. Effective control of the network behavior is critical in the current industrial environment which dictates frequent changes in operating conditions and targets.

This work focuses on process networks with large material recycle compared to throughput. Owing to the coexistence of large and small flowrates, such networks typically exhibit dynamics in two distinct time scales. The dynamics of the individual units evolve in a fast time scale while the dynamics of the overall network or process evolve in a slow time scale. The natural approach for the control of such two time scale systems consists of deriving separate controllers that address the control objectives for the fast and slow time scales [4] and, very often, involves cascaded structures where set points are used as manipulated inputs in the slow time scale. In such cases, the underlying models of the slow dynamics are high index differential algebraic equation (DAE) systems for which the constrained state-space depends explicitly on these manipulated inputs. Due to this dependency, such DAE systems do not possess a control-invariant state space, which precludes a direct derivation of the underlying ODE representation. In [5], a dynamic state feedback precompensator was proposed to modify a general class of such DAE systems such that the state-space of the resulting system is independent of the new manipulated inputs. Such an approach relies, of course, on the availability of state measurements, which clearly limits its applicability.

In this paper, we show that structural rank properties of these DAE systems allow the derivation of a minimal order dynamic extension leading to a DAE with control invariant state-space. The approach is applied to a high-purity distillation column with large recycle flowrate. Control of high-purity distillation column is a challenging problem owing to highly non-linear behavior, ill-conditioning and a strong coupling between the top and the bottom of the column [3]. We focus in particular on a two-point control problem in such a column, i.e. control of both the bottom and the top compositions (known to be especially challenging), and illustrate the appropriateness of the proposed method.

II. Integrated process networks

We consider the generic network shown in Fig.1, consisting of N processes (e.g. reactors, separation systems) and a material recycle stream for which the flowrate F_R is much larger than the feed flowrate F_0.

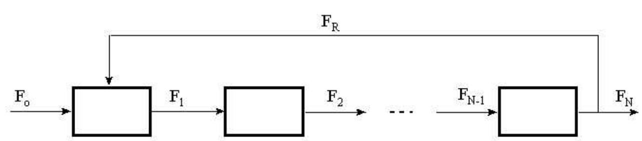

Fig. 1. Process network

Defining the singular perturbation parameter $\epsilon = F_{0s}/F_{Rs}$, where the subscript s denotes steady state values, the mathematical model describing the overall and component material balances has the form [2]:

$$\dot{x} = f(x) + g^s(x)u^s + \frac{1}{\epsilon}\, b(x)g^l(x)u^l \tag{1}$$

where x is the vector of state variables ($x \in \mathcal{X} \subset \mathbb{R}^n$), $u^s \in \mathbb{R}^{m_s}$ is a vector of scaled input variables corresponding to the small flowrates (F_0 and F_N), $u^l \in \mathbb{R}^{m_l}$ is a vector of scaled input variables corresponding to the large flowrates (F_R and F_j for $j = 1, ..., N-1$), $b(x)$ is a $n \times p$ full column rank matrix and $g^l(x)$ is a $p \times m_l$ matrix.

The system in Eq.1 exhibits dynamics in two time-scales, albeit it is not in a standard singularly perturbed form.

In the fast time scale ($\tau = t/\epsilon$), in the limit $\epsilon \to 0$, the dynamics of Eq. 1 take the form:

$$\frac{dx}{d\tau} = b(x)g^l(x)u^l \tag{2}$$

Note that only the variables u^l associated with the large flowrates are available for control in this fast time scale.

In the slow time scale t, multiplying Eq. 1 by ϵ and considering the limit $\epsilon \to 0$, since the matrix $b(x)$ has full column rank, the quasi steady state constraints $g^l(x)u^l = 0$ are obtained. Defining $z = lim_{\epsilon \to 0} \frac{g^l(x)u^l}{\epsilon}$, the slow dynamics of the network take the form:

$$\begin{array}{rcl} \dot{x} & = & f(x) + b(x)z + g^s(x)u^s \\ 0 & = & g^l(x)u^l \end{array} \tag{3}$$

The slow dynamics are thus modeled by a high index DAE, since the solution for the algebraic variables z cannot be obtained directly from the algebraic equations.

For control purposes, let y^l denote the output variables that are associated with control objectives in the fast time scale (e.g. holdups that need to be stabilized) and y^s associated to control objectives in the slow time scale (e.g. product quality). For simplicity, we consider a static state feedback law, $u^l = \alpha^l(x) + \beta^l y^l_{sp}$, where y^l_{sp} denotes the set point for the outputs y^l, to stabilize the fast dynamics and induce a desired output response in the fast time scale. Then, the DAE model of the slow dynamics of the network takes the form:

$$\begin{array}{rcl} \dot{x} & = & f(x) + b(x)z + g^s(x)u^s \\ 0 & = & g^l(x)\left[\alpha^l(x) + \beta^l y^l_{sp}\right] \\ y^s & = & H^s x \end{array} \tag{4}$$

where it is assumed that the outputs y^s are linear combinations of the state variables. Note that, in the slow-time scale, the small flowrates u^s are available as manipulated inputs. Typically, some of the set points of the fast output variables, y^l_{sp}, are also used as additional manipulated inputs. Such a cascaded control configuration becomes necessary when the number of controlled outputs y^s in the slow time scale exceeds the number of available variables in u^s. In the case of such configurations, the algebraic constraints in the DAE description of the slow dynamics explicitly involve the manipulated input variables, which leads to a DAE model with control-dependent state-space.

Without loss of generality, it can be assumed that the n_s first components of the vector y^l_{sp} correspond to the set points used as additional manipulated inputs. After splitting the matrix $g^l(x)\beta^l$ in a similar way, the constraints of the DAE model of Eq. 4 yield:

$$0 = g^l(x)\alpha^l(x) + \left(g^l(x)\beta^l\right)_2 y^l_{sp_2} + \left[\begin{array}{cc} \left(g^l(x)\beta^l\right)_1 & 0 \end{array}\right] u$$

where $u = [y^l_{sp_1}{}^T \;\; u^{sT}]$ is the $(n_s + m_s)$ vector of manipulated inputs, so that the DAE model of the slow dynamics takes the form:

$$\begin{array}{rcl} \dot{x} & = & f(x) + b(x)z + \left[\begin{array}{cc} 0 & g^s(x) \end{array}\right] u \\ 0 & = & g^l(x)\alpha^l(x) + \left(g^l(x)\beta^l\right)_2 y^l_{sp_2} + \left[\begin{array}{cc} \left(g^l(x)\beta^l\right)_1 & 0 \end{array}\right] u \\ y^s & = & H^s x \end{array} \tag{5}$$

In typical examples of such networks (e.g. [2]) modeled by Eq. 5, the following key rank properties can be verified:

- a) the $n \times p$ matrix $b(x)$ is full column rank,
- b) the $(p \times n_s)$ matrix $\left[\begin{array}{c} \left(g^l(x)\beta^l\right)_1 \end{array}\right]$ is full column rank,
- c) the jacobian of the vector $g^l(x)\alpha^l(x) + \left(g^l(x)\beta^l\right)_2 y^l_{sp_2}$ has full row rank.

Given the rank property $b)$, the constraints can be multiplied by a constant invertible matrix such that the DAE model of Eq. 5 is written

$$\begin{array}{rcl} \dot{x} & = & f(x) + b(x)z + g(x)u \\ 0 & = & \left[\begin{array}{c} \bar{k}(x) \\ \underline{k}(x) \end{array}\right] + \left[\begin{array}{cc} \hat{C}(x) & 0 \\ 0 & 0 \end{array}\right] u \\ y & = & Hx \end{array} \tag{6}$$

where the following rank properties are fulfilled:

- i) the $n \times p$ matrix $b(x)$ is full column rank,
- ii) $\hat{C}(x)$ is an invertible $n_s \times n_s$ (with $n_s < p$) matrix that accounts for the use of n_s of the set points y^l_{sp} as additional manipulated inputs,
- iii) the jacobian of the vector $\left[\bar{k}(x)^T \;\; \underline{k}(x)^T\right]^T$ is full row rank.

We consider DAE systems in the form of Eq. 6. The dependence of the state-space on the manipulated inputs precludes a direct derivation of an expression for the algebraic variables by differentiation of the constraints. However, the rank conditions mentioned earlier allow us to modify the DAE system into a new DAE system of index two with a control-invariant state-space.

III. PRECOMPENSATOR DESIGN

For DAE systems of the form in Eq. 6, a natural approach consists of designing a precompensator with the goal of modifying the constraints that involve the n_s set points used as additional manipulated inputs in order to obtain a modified DAE system with a control-independent state-space. The following proposition gives the general form of such a precompensator:

Proposition: *Consider a DAE system of the form in Eq. 6 where the rank conditions $i), ii), iii)$ are satisfied. Then the following dynamic extension:*

$$\begin{array}{rcl} \dot{w} & = & v_1 \\ u & = & \left[\begin{array}{c} I_{n_s} \\ 0 \end{array}\right] w + \left[\begin{array}{cc} 0 & 0 \\ 0 & I_{m-n_s} \end{array}\right] v \end{array} \tag{7}$$

where w is the n_s vector of the precompensator states yields the desired index-two DAE system.

Proof:

The direct substitution of the dynamic extension of Eq. 7 into the DAE system of Eq. 6 yields the following DAE system:

$$\begin{aligned}
\begin{bmatrix} \dot{x} \\ \dot{w} \end{bmatrix} &= \begin{bmatrix} f(x) \\ 0 \end{bmatrix} + \begin{bmatrix} b(x) \\ 0 \end{bmatrix} z + \begin{bmatrix} g_1(x) \\ 0 \end{bmatrix} w \\
&\quad + \begin{bmatrix} 0 & g_2(x) \\ I_{n_s} & 0 \end{bmatrix} v \\
0 &= \begin{bmatrix} \bar{k}(x) \\ \underline{k}(x) \end{bmatrix} + \begin{bmatrix} \hat{C}(x) \\ 0 \end{bmatrix} w
\end{aligned} \tag{8}$$

where the matrix $g(x)$ is partitioned in the $n \times n_s$ and $n \times (m - n_s)$ matrices $g_1(x)$ and $g_2(x)$, respectively, as $g(x) = [\ g_1(x)\ g_2(x)\ ]$. The state-space of this modified DAE system is clearly independent of the new manipulated inputs v.

Upon one differentiation of the new constraints, the matrix coefficient for the algebraic variables takes the following form:

$$K = \left[\begin{bmatrix} \dfrac{\partial \bar{k}}{\partial x} \\ \dfrac{\partial \underline{k}}{\partial x} \end{bmatrix} b(x) \right] \tag{9}$$

Given the properties $i)$ and $iii)$, it is clear that the matrix K as the product of a full row rank matrix and a full column rank matrix is invertible, which completes the proof.

The constraints obtained after one differentiation are now solvable in z:

$$\begin{aligned}
z &= -\left[\underline{\underline{L}}_{b(x)} k(x)\right]^{-1} \left[\underline{\underline{L}}_{f(x)} k(x) + \underline{\underline{L}}_{g_1(x)} k(x)\ w \right. \\
&\quad \left. + \underline{\underline{L}}_{g_2(x)} k(x)\ v_2 + \bar{C}(x) v_1 \right] \\
&= R(x,w) + S_1(x)\ v_1 + S_2(x)\ v_2
\end{aligned}$$

where,

$$k(x) = \begin{bmatrix} \bar{k}(x) \\ \underline{k}(x) \end{bmatrix}, \qquad \bar{C}(x) = \begin{bmatrix} \hat{C}(x) \\ 0 \end{bmatrix}$$

$$\underline{\underline{L}}_{b(x)} k(x) = \left[\begin{bmatrix} \dfrac{\partial \bar{k}}{\partial x} \\ \dfrac{\partial \underline{k}}{\partial x} \end{bmatrix} b(x) \right] \tag{10}$$

A direct substitution of the solution for z in the differential equation for x yields a state-space realization of the original system. This is given in the following proposition:

Proposition: *Consider a DAE system of the form in Eq. 6 for which the rank conditions $i), ii), iii)$ are satisfied, subject to the dynamic precompensator of Eq. 7. Then the dynamic system:*

$$\begin{aligned}
\dot{\bar{x}} &= \bar{f}(\bar{x}) + \bar{g}(\bar{x}) v \\
y &= \bar{H} \bar{x}
\end{aligned} \tag{11}$$

is a state-space realization of the modified DAE system, where $\bar{x} = [x^T\ w^T]^T$, the extended state vector, is constrained to evolve on the manifold defined by the constraints in the DAE system in Eq. 8, v is the new manipulated input vector, and

$$\begin{aligned}
\bar{f}(\bar{x}) &= \begin{bmatrix} f(x) + b(x) R(x,w) + g_1(x) w \\ 0 \end{bmatrix} \\
\bar{g}(\bar{x}) &= \begin{bmatrix} b(x) S_1(x) & b(x) S_2(x) + g_2(x) \\ I_{n_s} & 0 \end{bmatrix}
\end{aligned} \tag{12}$$

where $g(x) = [g_1(x)\ g_2(x)]$.

Note that the integrators were added only to the manipulated input channels associated with the matrix $\hat{C}$ that accounts for the use of n_s set points as manipulated inputs. Moreover, the matrix coefficient for z takes the form in Eq. 9 which does not involve u or any of its time derivatives, so that we require only that the DAE system has a finite index for some smooth $u(t)$.

On the basis of this state-space realization, an output feedback controller can be designed using existing techniques for non-linear ODE systems.

IV. CASE STUDY

We consider a distillation column with N trays (numbered from top to bottom), to which a saturated liquid containing a mixture of three components with mole fractions x_{1f}, x_{2f} of components 1 and 2 respectively, is fed at (molar) flowrate F_0 on tray N_f. The heavy component 3 which is the desired product is removed at the bottom from the reboiler at a flowrate B, while the lighter components 1 and 2 are removed at the top from the condenser at a flowrate D. In this column, a large vapor boilup V_B and liquid recycle R are used compared to the feed, distillate and bottom product flowrates, to attain a high purity of the desired component 3 in the bottom product. The key outputs to be controlled are the bottom purity $x_{3,B}$, the top composition $x_{1,D}$, and the two liquid holdups M_C and M_R that behave like integrators. Under the above assumptions, a standard dynamic model of the column is obtained, which is given by the following

ODE system [6]:

Condenser

$$\begin{cases} \dot{M}_C = V_B - R - D \\ \dot{x}_{1,D} = \dfrac{V_B}{M_C}(y_{1,1} - x_{1,D}) \\ \dot{x}_{3,D} = \dfrac{V_B}{M_C}(y_{3,1} - x_{3,D}) \end{cases}$$

Tray $i < N_f$

$$\begin{cases} \dot{x}_{1,i} = \dfrac{1}{M_i}[V_B(y_{1,i+1} - y_{1,i}) + R(x_{1,i-1} - x_{1,i})] \\ \dot{x}_{3,i} = \dfrac{1}{M_i}[V_B(y_{3,i+1} - y_{3,i}) + R(x_{3,i-1} - x_{3,i})] \end{cases}$$

Feed tray $i = N_f$

$$\begin{cases} \dot{x}_{1,i} = \dfrac{1}{M_i}[V_B(y_{1,i+1} - y_{1,i}) + R(x_{1,i-1} - x_{1,i}) \\ \qquad +F(x_{1f} - x_{1,i})] \\ \dot{x}_{3,i} = \dfrac{1}{M_i}[V_B(y_{3,i+1} - y_{3,i}) + R(x_{3,i-1} - x_{3,i}) \\ \qquad +F(x_{3f} - x_{3,i})] \end{cases}$$

Tray $i > N_f$

$$\begin{cases} \dot{x}_{1,i} = \dfrac{1}{M_i}[V_B(y_{1,i+1} - y_{1,i}) + R(x_{1,i-1} - x_{1,i}) \\ \qquad +F(x_{1,i-1} - x_{1,i})] \\ \dot{x}_{3,i} = \dfrac{1}{M_i}[V_B(y_{3,i+1} - y_{3,i}) + R(x_{3,i-1} - x_{3,i}) \\ \qquad +F(x_{3,i-1} - x_{3,i})] \end{cases}$$

Reboiler

$$\begin{cases} \dot{M}_R = R - V_B + F - B \\ \dot{x}_{1,B} = \dfrac{1}{M_R}[R(x_{1,N} - x_{1,B}) - V_B(y_{1,B} - x_{1,B}) \\ \qquad +F(x_{1,N} - x_{1,B})] \\ \dot{x}_{3,B} = \dfrac{1}{M_R}[R(x_{3,N} - x_{3,B}) - V_B(y_{3,B} - x_{3,B}) \\ \qquad +F(x_{3,N} - x_{3,B})] \end{cases}$$

where $M_C, x_{1,D}$ and $x_{3,D}$ are the molar liquid holdup and mole fractions of components 1 and 3 in the condenser, $M_i, x_{1,i}$ and $x_{3,i}$ are the molar liquid holdup and mole fractions of components 1 and 3 in tray i, and $M_R, x_{1,B}$ and $x_{3,B}$ are the corresponding holdup and mole fractions in the reboiler.

The presence of large vapor boilup V_B and liquid recycle R, and hence, large internal liquid and vapor flowrates in the column, compared to the inlet and outlet flowrates from the column, induces a time-scale separation in the column dynamics with the dynamics of individual stages evolving in a fast time scale, and the dynamics of the overall column in a slow time scale [6]. Defining the singular perturbation parameter $\epsilon = D_{nom}/R_{nom}$, and $\kappa_1 = V_{Bnom}/R_{nom} = O(1)$, where the subscript nom refers to nominal steady state values and $O(.)$ is the standard order of magnitude notation, the process model, under standard modeling assumptions, takes the general form of Eq. 1 [6], where x is the vector of state variables (compositions and holdups in each stage), $u^s = [D \;\; B]^T \in \mathbb{R}^2$ is the vector of manipulated inputs corresponding to small flowrates and $u^l = [\bar{R} \; \bar{V}_B]^T \in \mathbb{R}^2$ is the vector of manipulated inputs corresponding to large flowrates where $\bar{R} = R/R_{nom}$ and $\bar{V}_B = V_B/V_{Bnom}$.

In the fast time scale ($\tau = t/\epsilon$), in the limit $\epsilon \to 0$, only the outputs u^l are available for control purposes. In particular, the control of the liquid holdups in the condenser and the reboiler (M_C and M_R) is easily achieved by using simple proportional controllers:

$$\begin{aligned} \bar{R} &= 1 - \bar{K}_{c1}(M_{Cnom} - M_C) \\ \bar{V}_B &= 1 - \bar{K}_{c2}(M_{Rnom} - M_R) \end{aligned}$$

In the slow time scale t, the dynamics take the general form of Eq. 4. Only the small flowrates D and B affect the slow dynamics. At this time scale, the outputs to be controlled consist of the total liquid holdup that needs to be stabilized as it is not affected by the large flowrates [6], the top and the bottom compositions. Hence, we need an additional manipulated input. A natural approach to this end is a cascaded control configuration where one of the set points for the condenser/reboiler holdups used in the fast proportional control is treated as an additional manipulated input variable. Considering M_{Cnom} as an additional manipulated input, and $M_C + M_R, x_{3,B}, x_{3,D}$ as the controlled outputs, the DAE system for the slow dynamics can be expressed in the form of Eq. 6:

$$\begin{aligned} \dot{x} &= f(x) + b(x)z + g(x)u \\ 0 &= \begin{bmatrix} \bar{k}(x) \\ \underline{k}(x) \end{bmatrix} + \begin{bmatrix} \hat{C}(x) & 0 \\ 0 & 0 \end{bmatrix} u \\ y_1 &= M_C + M_R = x_1 + x_{2N+4} \\ y_2 &= x_{3,B} = x_{2N+6} \\ y_3 &= x_{1,D} = x_2 \end{aligned}$$

where $u = \begin{bmatrix} M_{Cnom} & D & B \end{bmatrix}^T$ is the vector of manipulated inputs,

$$\hat{C}(x) = \begin{bmatrix} \bar{K}_{c1} \end{bmatrix}$$
$$\bar{k}(x) = \begin{bmatrix} -1 - \bar{K}_{c1}M_C - \kappa_1\bar{K}_{c2}M_{Rnom} + \kappa_1(1 + \bar{K}_{c2}M_R) \end{bmatrix}$$

and the $2N + 2$ linearly independent constraints that do not involve the inputs are:

$$\underline{k}(x) = \begin{bmatrix} \kappa_1 \bar{V}_B (y_{3,1} - x_{3,D}) \\ \kappa_1 \bar{V}_B (y_{1,1} - x_{1,D}) \\ \kappa_1 \bar{V}_B (y_{1,i+1} - y_{1,i} + x_{1,i-1} - x_{1,i}) \\ \kappa_1 \bar{V}_B (y_{3,i+1} - y_{3,i} + x_{3,i-1} - x_{3,i}) \\ \vdots \end{bmatrix}$$

where $1 \leq i \leq N$, $x_{1,i}, x_{3,i}$ are the liquid mole fractions of 1 and 3 in tray i, and $y_{1,i}, y_{3,i}$ are the vapor mole fractions in tray i. It can easily be verified that the rank conditions necessary for the application of the proposition are fulfilled, so that the following precompensator of the form in Eq. 7 is obtained:

$$\begin{aligned} \dot{w} &= v_1 \\ u &= \begin{bmatrix} w \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} v \end{aligned}$$

The dynamic extension corresponds to adding an integrator to the channel of the manipulated input M_{Cnom}.

The resulting DAE system is used to derive a state-space realization on the basis of which an output feedback controller with integral action is designed. The relative orders of the three outputs are $r_1 = r_2 = r_3 = 1$ with respect to the new manipulated inputs v, so that the controller is designed to enforce the response:

$$y_i + \gamma_i \frac{dy_i}{dt} = y_{i,sp} \ , \quad i = 1,\ 2,\ 3 \qquad (13)$$

where $y_{1,sp},\ y_{2,sp}$ and $y_{3,sp}$ denote the set points for the respective outputs. The controller consists of an input/output linearizing state feedback controller coupled with an 'open-loop' observer and an external linear controller [1]. The controller was tuned with the parameters $\gamma_1 = \gamma_2 = \gamma_3 = 20\ min$. The following table gives the nominal values of the process variables at steady state.

Variable	Description	Value
B	bottom product flowrate (mol/min)	50.0
D	distillate flowrate (mol/min)	50.0
F	feed flowrate (mol/min)	100.0
K_{c1}	proportional controller gain (min^{-1})	20.0
K_{c2}	proportional controller gain (min^{-1})	20.0
M_C	condenser liquid holdup (mol)	180.0
M_i	liquid holdup on tray i (mol)	175.0
M_R	reboiler liquid holdup (mol)	200.0
N	total number of trays	15
N_f	feed tray	8
R	liquid recycle flowrate (mol/min)	1000.0
V_B	vapor boilup flowrate (mol/min)	1050.0
α_1	relative volatility of component 1	1.5
α_2	relative volatility of component 2	1.3

Figures $2, 3, 4, 5, 6$ show the performance of this nonlinear output feedback controller for the slow dynamics of the column. We consider a 2.7% increase in the bottom purity $x_{3,B}$, and a 27% decrease in the top composition $x_{1,D}$ in the presence of 2% error in α_1, -3% and 7% unmeasured step disturbances in the feed composition x_{1f} and x_{2f}, at $t = 15$ min. Note that the increase in the bottom purity and the decrease in the top composition have to lead to a consistent steady-state. Clearly, the controller eliminates the effect of disturbances and induces the desired input/output behavior.

V. ACKNOWLEDGEMENT

Financial support by ACS-PRF and by NSF/CTS grant No 0234440 is gratefully acknowledged.

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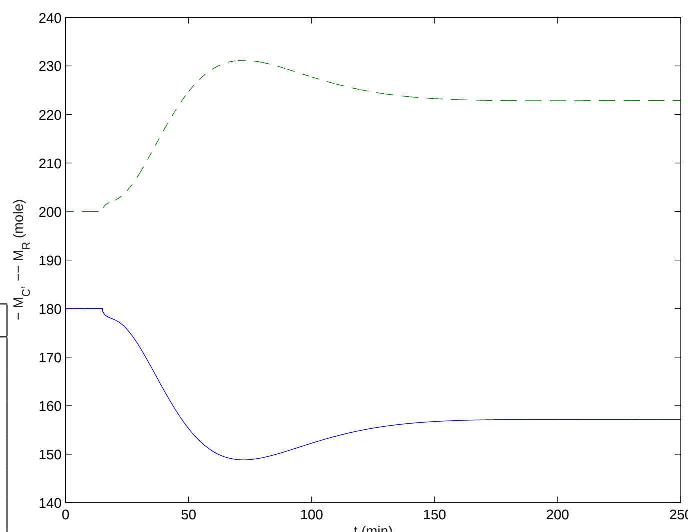

Fig. 2. Liquid holdups

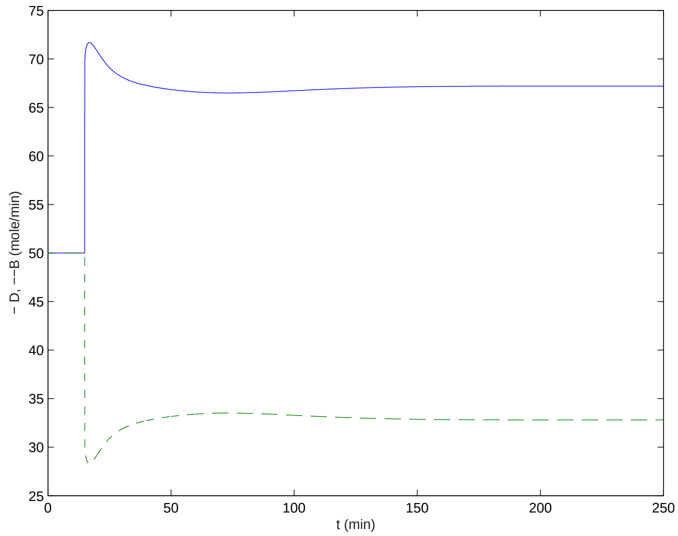

Fig. 3. Small flowrates

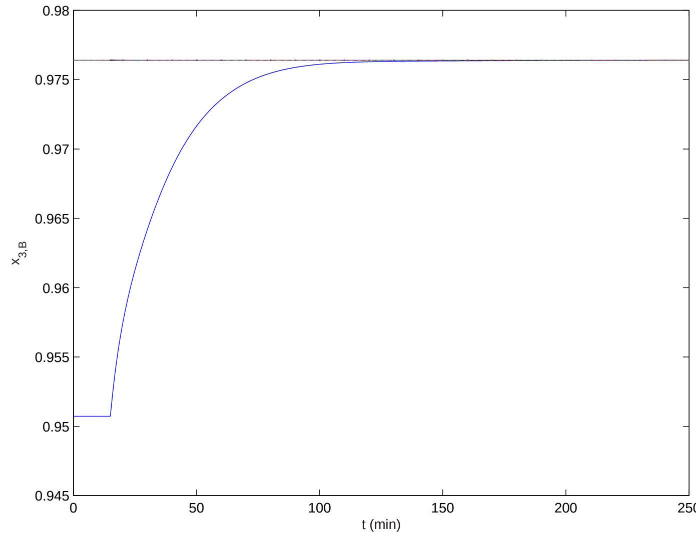

Fig. 5. Bottom purity

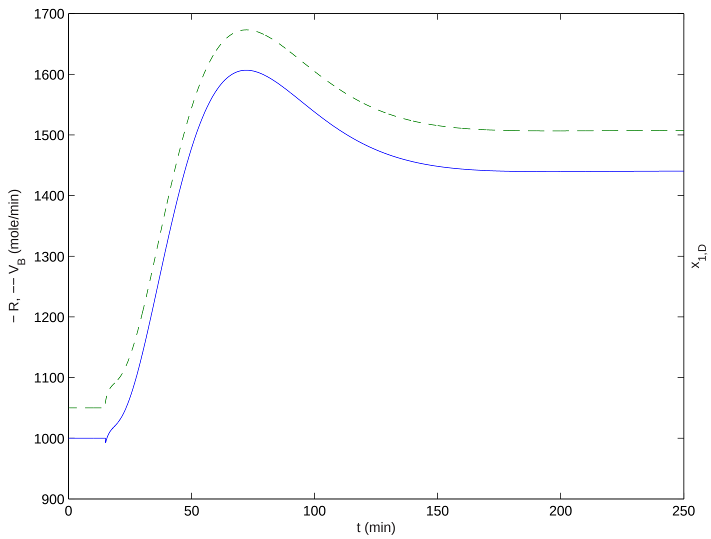

Fig. 4. Large flowrates

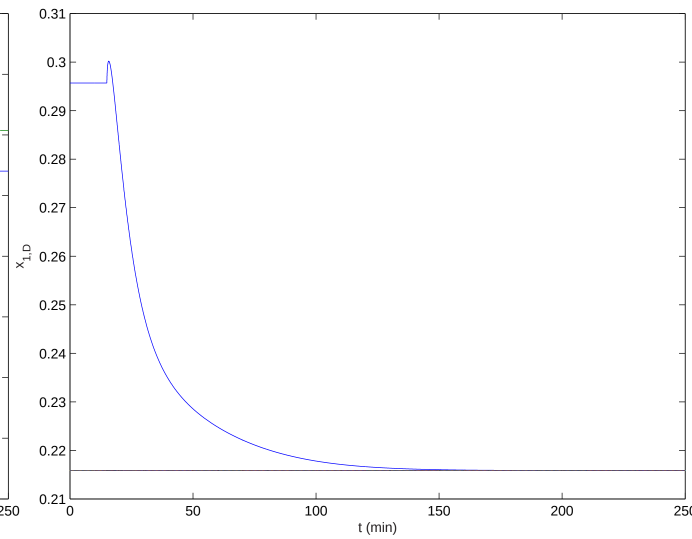

Fig. 6. Top composition