A Riccati-Genetic Algorithms Approach To Fixed-Structure Controller Synthesis

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Abstract A practical approach to the design of controllers with fixed structure (low order, decentralized etc.) that can be tuned via H_2 or H_∞ performance measures is proposed. The design problem is split into a convex subproblem that can involve a large number of decision variables, and a nonconvex subproblem with a small number of decision variables. The former problem can be solved with efficient Riccati solvers, while the latter one is solved using genetic algorithms. The proposed method is flexible and can be used for different H_2 or H_∞ performance or robustness measures. In this paper low-order robust H_2 design and low-order decentralized mixed sensitivity design are presented. Application of these methods to a benchmark problem and a large scale industrial problem demonstrates that the approach is numerically efficient and leads to performance comparable or superior to that of previously published methods.

Key words: Fixed structure controller, robust control, decentralized control, algebraic Riccati equation, genetic algorithms.

I. Introduction

In many practical control problems, the structure of admissible controllers is restricted, e.g. by an upper bound on the controller order or by constraints on the information that is available for feedback in each loop. In this paper we present a new approach to solving such problems and illustrate the method by applying it to the problem of designing robust low order and decentralized controllers. The key idea is to split the problem in two parts: a convex part that can involve a large number of decision variables and is solved efficiently by solving an algebraic Riccati equation, and a non-convex part that involves a small number of decision variables and is solved by Genetic Algorithms.

The problem of designing decentralized controllers was extensively studied in the 1970's, see e.g. [11] or [12]. For full state feedback a complete solution was given in [6]. In [5] a sufficient nonlinear matrix inequality condition for the existence of a solution is provided.

The low order controller design problem has no complete solution up to date either. It is well known however that a constraint on the controller order can be formulated as a rank constraint on a Lyaponuv matrix (see e.g. [13],[9]). In [10] an alternative projection method was proposed as an efficient computational tool for handling an LMI plus the rank constraint problem. In [8] a new heuristic algorithm called *the dual iteration algorithm* was proposed using an LMI formulation. Due to the non-convexity of this problem, most of the published methods consider simple design techniques like pole placement.

In this paper as one application we consider the design of fixed structure controllers that are robust against parameter uncertainty. An approach that has proven to be useful in practical applications is the design of robust H_2 optimal controllers, i.e. controllers that minimize the worst-case H_2 norm. In [7] a design procedure was presented for controllers referred to as "H_2/Popov" controllers. The main problem with this approach is the computational effort needed for estimating the gradients and for the gradient search algorithm. In [2] an iterative method for designing full order H_2/Popov controllers that minimize an upper bound on the worst H_2 norm is proposed. The problem is formulated as Bilinear Matrix Inequality (BMI) and solved using a similar approach to that proposed in [4], however this approach is not useable if constraints on the controller structure are imposed. In this paper we present a numerically efficient procedure for designing robust H_2 controllers with fixed order and compare the performance with that achievable with the method proposed in [2]. To illustrate the flexibility of our approach, we also present a mixed-sensitivity H_∞ optimal design of a low-order decentralized controller for a large scale industrial problem.

The paper is organized as follows: Section 2 introduces the idea of splitting a problem into a convex and a non-convex subproblem and solving it iteratively by using Riccati solvers and genetic algorithms. In Section 3 this approach is used to construct an algorithm for solving the low-order robust H_2 problem. The proposed method is applied to a standard benchmark problem and compared with the method given in [2]. In Section 4, we present an algorithm for designing low-order decentralized H_∞ optimal controllers and illustrate it with an application to a large scale industrial problem. Finally conclusions are drawn in section 5.

II. A Combined Riccati Equation - Genetic Algorithms Approach

Let A, Q and R be real $n \times n$ matrices with Q and R symmetric, and B and V matrices of compatible size. Consider the algebraic Riccati equation

$$A^T P + PA - (PB + V)R(B^T P + V^T) + Q = 0 \quad (1)$$

Efficient solvers are available for finding a solution $P > 0$ (if it exists); this problem appears often in a wide range of control applications.

Consider now the modified problem where the above matrices are all functions of a common parameter vector $\theta \in \mathbb{R}^m$, i.e. $A(\theta)$, $B(\theta)$, $V(\theta)$, $Q(\theta)$, $R(\theta)$: Find $P = P^T > 0$ and θ that satisfy

$$A(\theta)^T P + PA(\theta) - (PB(\theta) + V(\theta))R(\theta)(B(\theta)^T P + V(\theta)^T) + Q(\theta) = 0 \quad (2)$$

This problem is non-convex and cannot be solved using standard solvers. Note however that for any fixed value $\theta = \theta_o$ the problem can again be solved via Riccati solvers. The approach proposed in this paper for designing fixed structure robust H_2 or H_∞ optimal controllers is based on transforming these problems into the form of (2).

Note that the solution P is symmetric and contains $N = \frac{1}{2}(n+1)n$ decision variables. It will be seen below that in applications we typically have $N \gg m$. This observation motivates the idea to split the original problem into a small non-convex part solved by a GA, and a large convex part solved with a Riccati solver. The rationale for doing this is

- to let a fast and efficient Riccati solver take care of the large convex part of the problem: for a given θ find the unique solution P (if it exists), and
- to let GA - which may be unreliable for a large number of decision variables - deal with the smaller non-convex part and search over θ (which usually contains the controller parameters and stability multipliers).

Thus GA is used to construct the vector θ , and then a Riccati solver is applied to calculate P (if exists). The full chromosome is constructed by adjoining the decision variables in θ and P. On the other hand, if a standard GA is used alone to solve the original problem, the GA chromosomes must code both θ and P, and if P is large, the chromosome consequently will be too long for an efficient and reliable solution. Reducing the dimension of the solution space for the GA not only accelerates the evolution process, but also increases the chances of converging to the global solution of the problem.

The overall algorithm is shown in Figure 1. Once the full chromosomes are constructed, the fitness value is evaluated. Note that all the standard GA operations (reproduction, crossover, mutation) are performed on θ only (effective population). The full chromosome length is used only for the fitness evaluation process as shown in Figure 1.

All numerical results presented in this paper were implemented using Matlab 6.5. The GA preferences are as follows:

- Floating point representation of chromosomes

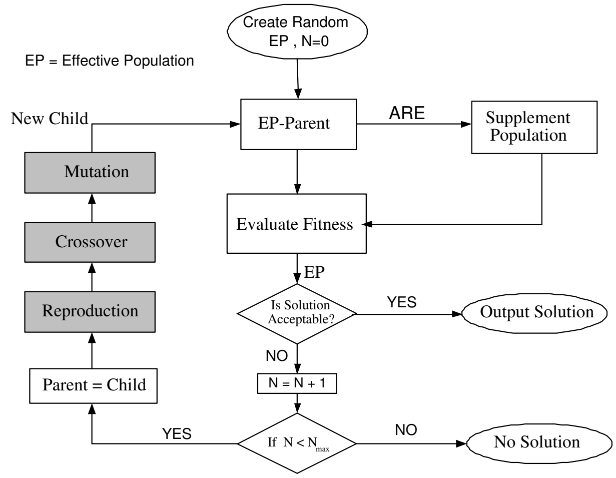

Fig. 1. The structure of the ARE-GA algorithm

- Stochastic universal sampling with linear ranking fitness assignment is used for selection.
- Non-uniform mutation was implemented for better fine-tuning characteristic
- Simple and arithmetic cross-over were applied during each evolution process
- Elitism mechanism is applied.

III. Parametric Robust H_2 Synthesis: ARE-GA Approach

In this section a combined ARE-GA approach is used to address the problem of designing fixed structural controllers that minimizes an upper bound on the worst-case H_2 norm. The analysis results given in this section are taken from [3] (chapter 8) and [2].

A. Problem Formulation

Consider the following LTI system subject to sector bounded nonlinear uncertainty, i.e. a Lur'e system ([3] page 119), described by:

$$\begin{aligned} \dot{x} &= A_0 x + B_1 w_1 + B_2 w_2 + Bu \\ z_1 &= C_1 x + D_1 w_1 \\ z_2 &= C_2 x + D_{2u} u \\ y &= Cx + D_{2w} w_2 \\ w_1 &= \phi(z_1) \end{aligned} \quad (3)$$

where $x \in R^n$ is the state vector, $u \in R^{n_u}$ is the control input, $w_2 \in R^{n_{w_2}}$ is a unit intensity white noise process, $y \in R^{n_y}$ is the measured output, $z_2 \in R^{n_{z_2}}$ is the performance output, $w_1 \in R^{n_{z_1}}$ and $z_1 \in R^{n_{z_1}}$ are the input and output of the nonlinear uncertainty ϕ. The nonlinear perturbation ϕ is assumed to satisfy the sector bound $[0, 1]$, $\phi \in \Phi$ see [3] (page 129) for definition of Φ. If we consider the special case where the function ϕ is linear, i.e, $\phi(z_1) = \Delta z_1$ where Δ is a diagonal matrix that satisfies

$\Delta = diag(\delta_1, \delta_2, ..., \delta_{n_{z_1}}),\ \delta_i \in [0,1], i = 1, ..., n_{z_1}\}$ the above description simplifies to an important class of uncertain system considered in many references [7]. These systems are described by $\dot{x} = (A_0 + \tilde{A})x + B_2 w_2 + Bu, \tilde{A} \in \tilde{\mathcal{A}}$, where $\tilde{\mathcal{A}} = \{\tilde{A} \in R^{n \times n} : \tilde{A} = B_1 \Delta C_1\}$. Such systems are known systems subject to parametric uncertainty.

Using the definition of worst-case performance of non-linear systems in [15], [2] we define the worst-case H_2 performance J for (3) as

$$J = \sup \sum_{i=1}^{n_{w_2}} \int_0^\infty z_i(t)^T z_i(t) dt, \tag{4}$$

where the supremum is taken over all nonzero output trajectories $\{z_1(t), ..., z_{n_{w_2}}(t)\}$ of the nonlinear system (3) starting from $x(0) = 0$.

Theorem III.1 *If there exists a Lyapunov function*

$$V(x) = x^T P x + 2 \sum_{i=1}^{n_{z_1}} \lambda_i \int_0^{C_{1,i} x} \phi_i(\sigma)\, d\sigma \tag{5}$$

*where $C_{1,i}$ denotes the ith row of C_1, and $\Lambda =$ **diag**$(\lambda_1, ..., \lambda_n) \geq 0$,and $T =$ **diag**$(\tau_1, ..., \tau_n) \geq 0$, satisfying*

$$\begin{bmatrix} A^T P + PA + C_z^T C_z & PB_1 + A^T C_1^T \Lambda + C_1^T T \\ B_1^T P + \Lambda C_1 A + TC_1 & \Lambda C_1 B_1 + B_1^T C_1 T \Lambda - 2T \end{bmatrix} \leq 0 \tag{6}$$

*then the upper bound of $\mathcal{J}$ is finite and can be computed by minimizing **Tr** $B_w^T (P + C_1^T \Lambda C_1) B_w$, over the variables $P, \Lambda, and\, T$, i.e. by solving the problem*

$$minimize\ \textbf{Tr}\, B_2^T (P + C_1^T \Lambda C_1) B_2 \tag{7}$$

$$subject\, to: \ (6), \ \ P > 0\ , \Lambda \geq 0\ \ , T \geq 0$$

Proof*: See [3], (pages 121-122).*

B. Controller Synthesis

The problem considered in this section is to find a strictly proper controller

$$K(s) \begin{cases} \dot{\zeta}(t) = A_K \zeta(t) + B_K y(t) \\ u(t) = C_K \zeta(t)) \end{cases} \tag{8}$$

where $\zeta \in \mathbb{R}^{n_c}$ with n_c fixed, such that the worst-case performance upper bound **Tr** $B_w^T (P + C_1^T \Lambda C_1) B_w$ of the closed-loop system is minimized; such a controller is referred to as Popov controller [10].

The closed-loop representation of the above system is given by

$$\begin{bmatrix} \dot{x} \\ \dot{\zeta} \end{bmatrix} = \bar{A} \begin{bmatrix} x \\ \zeta \end{bmatrix} + [\bar{B}_1\ \ \bar{B}_2] \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} \bar{C}_1 \\ \bar{C}_2 \end{bmatrix} \begin{bmatrix} x \\ \zeta \end{bmatrix}$$
$$w_1 = \phi(z_1)$$

where

$$\bar{A} = \begin{bmatrix} A_0 & BC_K \\ B_K C & A_K \end{bmatrix}, \quad \bar{B}_1 = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \quad \bar{B}_2 = \begin{bmatrix} B_2 \\ B_K D_{2w} \end{bmatrix}$$

$$\bar{C}_1 = [C_1\ \ D_1 C_K], \quad \bar{C}_2 = [C_2\ \ D_{2u} C_K]$$

Using the Schur complement, the inequality constraint (6) of Theorem III.1 can be written as

$$\bar{A}^T P + P\bar{A} + \bar{C}_z^T \bar{C}_z - (P\bar{B}_1 + \bar{A}^T \bar{C}_1^T \Lambda + \bar{C}_1^T) \cdot$$
$$R^{-1} (\bar{B}_1^T P + \Lambda \bar{C}_1 A + T\bar{C}_1) \leq 0 \tag{9}$$

$$R = \Lambda C_1 B_1 + B_1^T C_1 T \Lambda - 2T \leq 0 \tag{10}$$

Since the optimal solution P subject to the above inequality constraint occurs always on the boundary, the inequality can be replaced by an equation. Note that the left hand side of inequality (9) has the same form as the Riccati equation (2), where $\theta = \text{col}(A_k, B_k, C_k, \Lambda, T)$ and $\text{col}(M_1, \ldots, M_n)$ denotes the columns of the matrices M_i stacked together in one column vector.

Note that the size of the matrix $P = P^T$ is $(n + n_c) \times (n + n_c)$. Even if the order of the controller is low, the size of the Lyaponuv matrix will be large when dealing with large scale systems. For example, consider the task of designing a first order controller for a 10th order SISO uncertain system with scalar uncertainty. Solving (9) with GA alone means that each chromosome should consist of 72 decision variables (floating number). Splitting the problem means that GA is searching only for the controller and the multiplier (the chromosomes consist of just 6 variables), and a Riccati solver is used to search for the unique P which allows the evaluation of the objective function (7).

The population structure for this example is shown in Table I below. Once the solution P is available, the upper bound on J is calculated as **Tr** $B_2^T (P + C_1^T \Lambda C_1) B_2$. Then the fitness of each chromosome ($K(s), P, T, \Lambda$) is evaluated as a linear function of $J(P_i, \Lambda_i)$.

GA		ARE	Worst-Case	Fit-
Controller variables	Multipliers	P	H_2 Cost	ness
$A_{k_1}, B_{k_1}, C_{k_1}, D_{k_1}$	Λ_1, T_1	P_1	$J(P_1, \Lambda_1)$	f_1
$A_{k_2}, B_{k_2}, C_{k_2}, D_{k_2}$	Λ_2, T_2	P_2	$J(P_2, \Lambda_2)$	f_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$A_{k_{n_p}}, B_{k_{n_p}}, C_{k_{n_p}}, D_{k_{n_p}}$	Λ_{n_p}, T_{n_p}	P_{n_p}	$J(P_{n_p}, \Lambda_{n_p})$	f_{n_p}

TABLE I

GA POPULATION STRUCTURE

IV. NUMERICAL EXAMPLE: THREE-MASS-SPRING SYSTEM

The efficiency of the approach presented in the previous section is illustrated by comparing it with the method proposed in [2], one of the most efficient synthesis techniques reported so far. To make the comparison fair, the same example used in [2] was used here again. The system consists of three masses connected by two springs, in which the spring uncertainty between the second and the third mass is expressed in the form $K_2 = k_{2,nom}(1+\delta)$ where $k_{2,nom}$ is the nominal value, and the uncertainty is captured by $\delta \in R$. All the system parameters are set to a nominal value ($m_1 = m_2 = m_3 = 1$, $k_1 = k_{2,nom} = 1$). The uncertainty in the spring stiffness is approximated as $K_2(y) = k_{2,nom}[y + \rho\phi(y)]$, where $\rho > 0$ is a measure of the relative guaranteed uncertainty bound, and $\phi(y)$ is a $[-1, 1]$ sector-bounded memoryless nonlinear function of the spring displacement y.

The population size is 60, the number of iterations is 300, with an average computation time of less than 5 minutes for one complete run (computed on a Pentium 5 2.0G cpu speed, 256 DDR ram).

The plant model is of 6th order, and it turns out that with the ARE-GA approach it can be stabilized with controllers of 3rd order or higher. Table IV shows that while a 3rd order controller leads to higher bounds on the cost J, a 4th order controller leads to cost values almost identical with the full order controllers reported in [2]. Figure IV below shows that ARE-GA controllers achieve robustness against a larger range of variation in the spring constant K_2 compared with the controller derived using the approach proposed in [2] and a standard LQG controller.

Controller order	Upper bound on J, 5% Uncertainty	Upper bound on J, 10% Uncertainty	Upper bound on J, 20% Uncertainty
3rd order ARE-GA	4.6	4.84	5.75
4th order ARE-GA	3.22	3.36	3.69
Full order [2]	3.19	3.34	3.69

TABLE II

COMPARISON OF THE WORST-CASE H_2 BOUND OF ARE-GA AND LMI-BASED POPOV CONTROLLERS

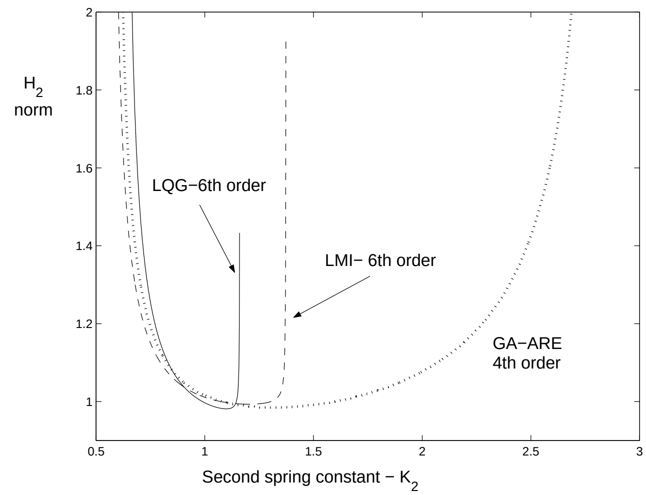

Fig. 2. H_2 cost vs. parameter variation for Popov and ARE-GA controller, both designed assuming 10% uncertainty in K_2. A standard LQG controller is included for comparison.

V. MIXED SENSITIVITY DECENTRALIZED LOW-ORDER CONTROLLER DESIGN FOR A HVDC SYSTEM

In this section we present the application of the ARE-GA approach to a large-scale industrial problem: control of a high voltage direct current (HVDC) system. HVDC systems are used in electrical power grids as a supplement to AC transmission. Power transfer by means of HVDC is used in case of (i) inter-connecting asynchronous AC systems with different power frequencies, (ii) high voltage cables longer than about 80km, and (iii) long overhead lines with lengths in excess of about 600km. A two-input two-output state space model for such a plant was presented in [1]; the controlled outputs are the direct current I_{DC1} on the rectifier side and the direct voltage V_{DC2} on the inverter side and control inputs are excursion in firing angles u_1 and u_2. The high dynamic order (35 state variables) reflects the large number of passive elements. This model has been linearized for nominal AC voltages (1pu) and nominal firing angles (rectifier firing angle $\alpha = 21^o$, inverter extinction angle $\gamma = 25.6^o$) and extensively validated against nonlinear EMTDC simulation for small changes. A controller is to be designed that achieves a fast response to command step changes with little overshoot and little cross-coupling between channels. Moreover, the controller should have low order and must use only the information available on one side of the DC link for feedback, i.e. only measurement of I_{DC1} is used to generate u_1 and measurement of V_{DC2} for u_2.

To solve this problem, we use the generalized plant model

$$\begin{aligned} \dot{x} &= Ax + B_w w + Bu \\ z &= C_z x + D_{zw} w + D_z u \\ y &= Cx + D_w w \end{aligned}$$

The task is to find a biproper ($D_K \neq 0$) controller $K(s)$ such that the H_∞ norm of the closed loop system $T(s)$ is less than γ, where T(s) is given by:

$$T(s) \left\{ \begin{array}{l} \dot{x}_{cl} = \bar{A}x_{cl} + \bar{B}w \\ z = \bar{C}x_{cl} + \bar{D}w \end{array} \right.$$

In addition, the controller must satisfy the constraints on the information available for feedback. We will design a mixed sensitivity H_∞ optimal controller, and make use of the following result.

Theorem V.1 *The matrix $\bar{A}$ is stable and $\|T\|_\infty < \gamma$ if and only if there exists a symmetric solution $P = P^T > 0$ to the system of LMIs*

$$\left[\begin{array}{cc} \bar{A}^T P + P\bar{A} + \gamma^{-1}\bar{C}^T\bar{C} & P\bar{B} + \gamma^{-1}\bar{C}^T\bar{D} \\ \bar{B}^T P + \gamma^{-1}\bar{D}^T\bar{C} & -\gamma I + \gamma^{-1}\bar{D}^T\bar{D} \end{array} \right] \leq 0 \quad (11)$$

Proof*: See [3].*

Again using the Schur complement, the inequality (11) of Theorem V.1 holds if and only if

$$\bar{A}^T P + P\bar{A} + \gamma^{-1}\bar{C}^T\bar{C} - (P\bar{B} + \gamma^{-1}\bar{C}^T\bar{D})R^{-1}\cdot$$

$$(\bar{B}^T P + \gamma^{-1}\bar{D}^T\bar{C}) \leq 0 \qquad (12)$$

$$R = -\gamma I + \gamma^{-1}\bar{D}^T\bar{D} \leq 0 \qquad (13)$$

For a mixed sensitivity design we solve the problem

$$\min_{K(s)\in\mathcal{K}} \gamma \quad \text{subject to} \quad \left\| \begin{array}{c} W_S(s)S(s) \\ W_T(s)T(s) \end{array} \right\|_\infty < \gamma$$

where $\mathcal{K}$ denotes the set of admissible controllers. We consider two possible choices of controller structures: $\mathcal{K}$ can either contain all controllers of the form

$$K_f(s) = \left[\begin{array}{cc} g_1\frac{T_1 s+1}{T_2 s+1} & g_3 \\ g_4 & g_2\frac{T_3 s+1}{T_4 s+1} \end{array} \right]$$

with variables g_i and T_i, or all controllers

$$K_d(s) = \left[\begin{array}{cc} g_1\frac{T_1 s+1}{T_2 s+1} & 0 \\ 0 & g_2\frac{T_3 s+1}{T_4 s+1} \end{array} \right]$$

where g_3 and g_4 have been fixed to zero. In this example the parameter vector θ in (2) becomes $\theta = [g_1\ g_2\ g_3\ g_4\ T_1\ T_2\ T_3\ T_4]^T$. Note that the size of the of Lyapunov matrix P in (12) is n + n_c + order of weighting filters $= 35 + 2 + 4 = 41$. If GA is used alone to solve the above problem, there will be 861 variables in P in addition to the controller variables. This huge number of variables requires a length of chromosomes which would render the use of GA impractical.

In contrast, if the ARE-GA approach is applied the chromosomes will consist of only 8 variables for the controller $K_f(s)$ and 6 variables for the decentralized controller $K_d(s)$. For a given controller, equation (12) is solved easily using a Riccati solver. The advantage is that the size of the plant does not contribute to the size of the chromosomes, leading to a much faster evolution process and better chances of convergence to the global minimum.

To limit the size of this paper, all minor details (filter selection and tuning) will be omitted. Applying the ARE-GA algorithm yields the controllers

$$K_f(s) = \left[\begin{array}{cc} 0.171\frac{1.555s+1}{1.145s+1} & 0.122 \\ 0.394 & -0.254\frac{1.120s+1}{1.266s+1} \end{array} \right]$$

and

$$K_d(s) = \left[\begin{array}{cc} 3.155\frac{0.552s+1}{6.537s+1} & 0 \\ 0 & -1.6306\frac{0.6214s+1}{2.263s+1} \end{array} \right]$$

The singular values of the sensitivity functions $S(s)$ and $T(s)$ (see e.g. [14]) and the corresponding weighting filters for the two controllers $K_f(s)$ and $K_d(s)$ are shown in Figures 3 and 4 respectively. Note that a second order filter W_s is used to shape the low frequency response of the system (for good tracking and fast response). On the other hand, the first order filter W_T was just used to limit the peak

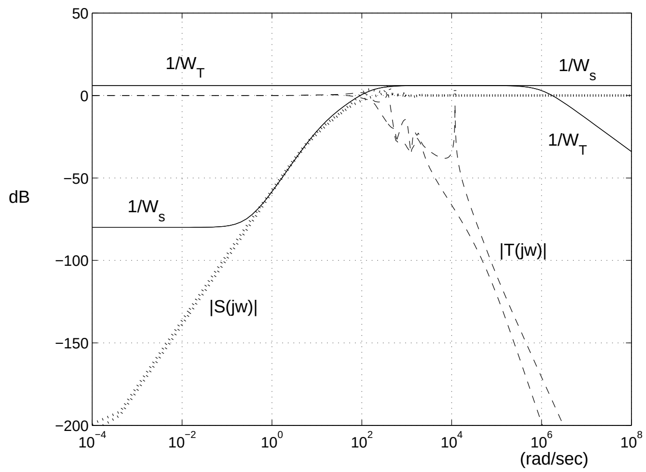

Fig. 3. Singular values of sensitivity and complementary sensitivity function with $K_f(s)$

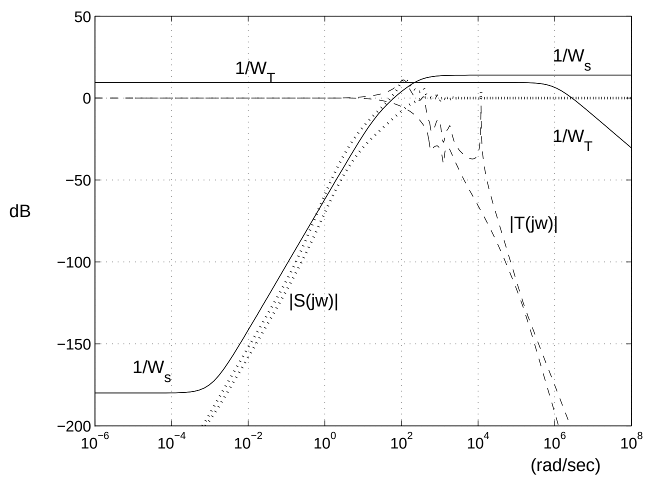

Fig. 4. Singular values of sensitivity and complementary sensitivity function with $K_d(s)$

of $T(s)$ within the frequency range of interest to control the oscillatory behavior of the closed loop system.

The tracking behavior in terms of settling time, steady state error and peak overshoot can be further improved by shaping the response with a pre-filter

$$K_{pre}(s) = \left[\begin{array}{cc} \frac{1}{0.015s+1} & 0 \\ 0 & \frac{1}{0.01s+1} \end{array} \right]$$

Figures 5 and 6 show the closed loop step response of the HVDC system with $K_f(s)$ and $K_d(s)$ respectively. Figure 5 also shows the responses obtained with a manually designed lead-lag compensator that uses unconstrained information for feedback (not decentralized). When comparing the performance of both controllers it should be kept in mind that the manual design of a lead-lag compensator for each channel is a time-consuming, tedious trial and error procedure, whereas the GA-ARE design is carried out in a semi-automatic manner and can be easily repeated for controllers of different order or structure. It can also be seen that the cross-coupling between I_{DC1} and V_{DC2} with controller $K_d(s)$ is worse than with controller $K_f(s)$; this is due to the constraint on the information available for feedback.

The total computation time for 200 iterations with 50 chromosomes is less than 12 minutes (all calculations were performed on the full order model with 35 states).

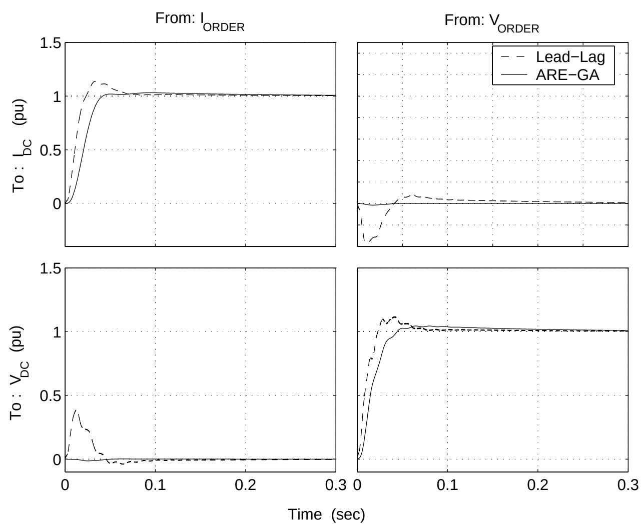

Fig. 5. Closed-loop step response with $K_f(s)$

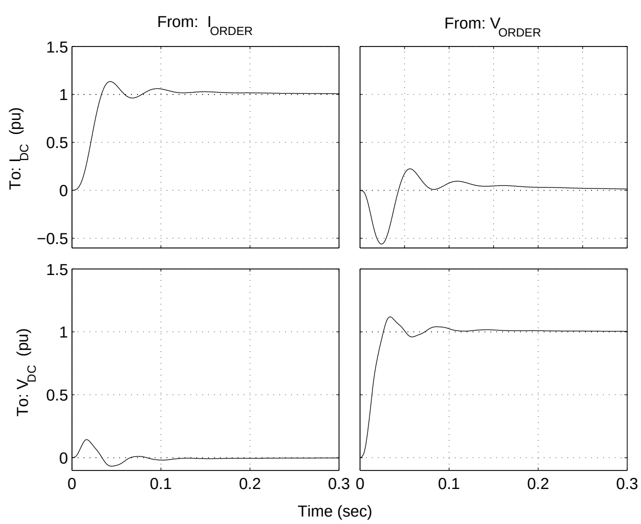

Fig. 6. Closed-loop step response with $K_d(s)$

VI. Conclusion

This paper introduces a novel approach that can be used for a variety of control applications involving non-convex constraints. Many powerful analysis results involving a Lyapunov matrix P have been developed over recent years, and the possibility of turning them into synthesis algorithms for controllers with a fixed structure is of considerable practical value. The flexibility of genetic algorithms allows its use for a wide range of applications. Here we are proposing a way of integrating this non-convex optimization tool with a convex one to form a much more efficient combined tool. The idea is to split the problem into two parts: a convex part solved via efficient Riccati solvers, and a non-convex part solved by GA. This simple idea can be extended to other design techniques such as μ-synthesis, and can help in bridging the gap between the rich theory of robust control and practical applications that require fixed structure controllers.

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