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Decentralized Control of a Large Platoon of Vehicles Using Non-Identical Controllers

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Abstract—**This paper studies the decentralized control of a platoon of identical vehicles when each control agent is assumed to only have knowledge of the distance between itself and its immediate forward neighbor. In particular, it is desired to solve the decentralized robust servomechanism problem (RSP), so that the vehicles' separation distances are regulated to specified set points, independent of the lead vehicle's velocity and such that the system is string stable. It is shown that for a large class of identical decentralized controllers, namely those decentralized controllers which solve the RSP and which have stable stabilizing compensators, e.g. a 3-term controller, that it is impossible to solve the above problem. This gives motivation to consider non-identical decentralized controllers for the platoon vehicle problem, and it is shown in this case that it is possible to solve the above problem. A number of examples are included, including examples which have a large number of vehicles in a platoon, i.e. *N*=2000.**

I. Introduction

RESENTLY IHS (Intelligent Highway Systems) has become an active area of research in the systems control area, where the focus is on developing control methods to allow platoons of identical vehicles to automatically move at a desired velocity with a specified separation distance between vehicles. Earlier works on this problem used optimal centralized control to regulate a string of moving vehicles [1], [2]. Since these works, the emphasis of research is on decentralized control approaches. In [3], the notion of "String Stability" was introduced in platoon control, where it was observed that one does not want the transient error in the separation distance between vehicles to "grow" as one proceeds down a line of vehicles in the platoon; systems which have this property are said to be "string stable". In [4], the so called "Spacing Control Law" where the problem of regulating the separation distance between each vehicle, and the so called "Headway Control Law" where the time duration it takes for a vehicle to travel to the present position of the lead vehicle is of interest, were studied. Reference [5] shows that if the specified separation distance for each agent is proportional to the velocity of the vehicle, then it may be possible to design a decentralized string-stable controller [5], and [6] studies the effect of actuator delays in platoon control problem. The papers [7] and [8] assume that there exists communication between the leader and all other vehicles in the platoon, and under this condition design controllers to satisfy the string stability constraint. In [9], the stability of asynchronous swarms of vehicles was analyzed, assuming that the system has a fixed communication topology. In [10], a complete modeling of a vehicle, including lateral and longitudinal movement, is carried out, and a vehicle control system was developed in which safety is of the highest concern.

In this paper, the so called "Spacing Control Law" problem is considered where one wishes to regulate the separation distance of each vehicle for a platoon independent of the velocity of the lead vehicle where it is assumed that the desired separation distance between each vehicle may vary from vehicle to vehicle. In this problem, it is assumed that there is no communication between the leader and other vehicles, and thus the controller for the platoon will be fully decentralized. It will also be assumed that each local controller for a vehicle only has access to the separation distance between itself and the vehicle in front of it, which makes the local controller very simple to implement; this assumption differs from other studies as in [7], [8], which assumes that the velocity and acceleration of the adjacent vehicle in front is available for measurement. Under these conditions, it is shown that for a large class of decentralized controllers which are identical, namely those decentralized controllers which solve the robust servomechanism problem [11], [12], [13], [14], for constant disturbances/set points and which have a stabilizing controller [12] which are asymptotically stable, i.e. a 3-term controller, it is impossible to design a controller for the platoon, so as to achieve closed loop string stability. This result gives motivation to consider non-identical decentralized controllers for the platoon of vehicles problem, and it is shown that in this case it is possible to design non-identical decentralized controllers so as to regulate the separation distance between vehicles independent of the lead vehicle's velocity, and also to bring about string stability for both the

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vehicle separation distance and vehicle velocity.

A number of examples are included to illustrate the results obtained; in particular it is shown that studies of a platoon with a small number of vehicles, e.g. N= 20, can be misleading, and some examples of platoons with a large number of vehicles e.g. N= 2000 are included.

II. Preliminary Results

Given N+1 identical vehicles traveling in a straight line, let the position of the lead vehicle from a given reference be denoted by y_0, and let the position of the next N vehicles be denoted by y_1, y_2, …, y_N respectively. Let the separation distance of the first vehicle from the lead vehicle be denoted by $d_1=y_0-y_1$, and the separation distance of the i^{th} vehicle to the i-1^{th} vehicle be denoted by $d_i=y_{i-1}-y_i$, i= 2, 3, …, N. Let the velocity of the lead vehicle be denoted by v_0^{ref} and the velocities of the remaining vehicles be denoted by $v_i = dy_i/dt$, i= 1, 2, …, N respectively. Let the force applied to the i^{th} vehicle which has position y_i be denoted by u_i, i= 1, 2, …, N.

Assume that the dynamics of the i^{th} vehicle are given as:

$$\begin{cases} \dot{x}_i = Ax_i + Bu_i + \bar{E}\bar{C}x_{i-1} \\ d_i = Cx_i \end{cases} \quad , i= 2, 3, \ldots, N \qquad (2.1a)$$

and the first vehicle as:

$$\begin{cases} \dot{x}_1 = Ax_1 + Bu_1 + Ev_0^{ref} \\ d_1 = Cx_1 \end{cases} \qquad (2.1b)$$

where $v_i = \bar{C}x_i$ and $x_i \in \Re^n, u_i \in \Re^1, d_i \in \Re^1$, i= 1, 2, …, N, then the model of a platoon of vehicles can be described by:

$$\dot{x} = \begin{pmatrix} A & 0 & 0 & \cdots & 0 & 0 \\ \bar{E}\bar{C} & A & 0 & \cdots & 0 & 0 \\ 0 & \bar{E}\bar{C} & A & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & A & 0 \\ 0 & 0 & 0 & \cdots & \bar{E}\bar{C} & A \end{pmatrix} x + \begin{pmatrix} B & 0 & 0 & \cdots & 0 & 0 \\ 0 & B & 0 & \cdots & 0 & 0 \\ 0 & 0 & B & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & B & 0 \\ 0 & 0 & 0 & \cdots & 0 & B \end{pmatrix} u + \begin{pmatrix} E \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} v_0^{ref}$$

$$d = \begin{pmatrix} C & 0 & 0 & \cdots & 0 & 0 \\ 0 & C & 0 & \cdots & 0 & 0 \\ 0 & 0 & C & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & C & 0 \\ 0 & 0 & 0 & \cdots & 0 & C \end{pmatrix} x \qquad (2.2)$$

where $x=(x_1', x_2', \cdots, x_N')', u_i=(u_1, u_2, \cdots, u_N)', d_i=(d_1, d_2, \cdots, d_N)'$ for some appropriate values of $\bar{E}, \bar{C}$, and E, and this representation should be called a platoon of N vehicles.

As an example of such a representation, assuming a vehicle of mass m at position y_i with force input u_i is described by the simplified representation:

$$m\ddot{y}_i + b\dot{y}_i + 0y_i = u_i \quad , i= 1, 2, \ldots, N \qquad (2.3)$$

where b>0 corresponds to a velocity damping term, then a platoon of N+1 identical vehicles can be described by the model (2.1) with:

$$A=\begin{pmatrix} 0 & -1 \\ 0 & -b/m \end{pmatrix}, B=\begin{pmatrix} 0 \\ 1/m \end{pmatrix}, E=\begin{pmatrix} 1 \\ 0 \end{pmatrix}, C=\begin{pmatrix} 1 & 0 \end{pmatrix}, \bar{E}=\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \bar{C}=\begin{pmatrix} 0 & 1 \end{pmatrix} \qquad (2.4a)$$

and $x = (d_1 \ v_1 \ d_2 \ v_2 \ \cdots \ d_N \ v_N)$ (2.4b)

In this paper we shall often use the following model for numerical experiments:

2.1 *Nominal Model of Platoon of N Vehicles* (*Model I*)

In (2.4), let b=1, m=0.1; then the following model is obtained:

$$\dot{x} = \begin{pmatrix} 0 & -1 & 0 & 0 & \cdots & \cdots & 0 & 0 \\ 0 & -10 & 0 & 0 & \cdots & \cdots & 0 & 0 \\ 0 & 1 & 0 & -1 & \cdots & \cdots & 0 & 0 \\ 0 & 0 & 0 & -10 & \cdots & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \cdots & 0 & 1 & 0 & -1 \\ 0 & 0 & \cdots & \cdots & 0 & 0 & 0 & -10 \end{pmatrix} x + \begin{pmatrix} 0 & 0 & \cdots & 0 \\ 10 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ 0 & 10 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 10 \end{pmatrix} u + \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \\ 0 \end{pmatrix} v_0^{ref} \qquad (2.5a)$$

$$d = \begin{pmatrix} 1 & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \cdots & 1 & 0 \end{pmatrix} \qquad (2.5b)$$

2.2 *Control Problem for Platoon of Vehicles*

The problem of controlling a platoon of vehicles described by (2.2) consists of three parts:

(a) Find a decentralized controller for vehicles to solve the robust servomechanism problem so that the spacing d_i between vehicle i and vehicle i-1 is asymptotically regulated to a constant specified distance d_i^{ref}, independent of the (constant) velocity v_0^{ref} of the lead vehicle, i.e.

$$\lim_{t\to\infty} d_i = d_i^{ref} \ , \forall v_0^{ref} > 0 \ , \ \forall x_i(0) \in \Re^n \ , \ i= 1, 2, \ldots, N$$

and for all controller initial conditions,

(b) The transient error associated with vehicle control should not amplify as i increases in the platoon of vehicles. In particular, assume that the decentralized controller in (a) has been applied to (2.2), and let

$$\frac{d_i(s)}{d_{i-1}(s)} = G_i(s) \quad , i= 2, 3, \ldots, N$$

in the resultant closed loop system, and let $g_i(t)$ be the corresponding impulse response of $G_i(s)$. Then it is desired that $g_i(t)$ should satisfy the property:

$$\|g_i(t)\|_1 \le 1 \quad , i= 2, 3, \ldots, N$$

which implies that:

$$\|d_i(t)\|_\infty \le \|d_{i-1}(t)\|_\infty \quad , i = 2, 3, \ldots, N$$

i.e. the system should be string stable [3].

(c) It is desired that the decentralized controller for vehicle i, i= 1, 2,…, N should require the least amount of information re the knowledge of other vehicles and of the lead vehicle. In particular, it will be assumed that the controller for vehicle i, can only measure the spacing distance d_i between itself and the vehicle i-1 immediately in front of the vehicle i.

This is called the *platoon vehicle control problem* (PVCP).

***Remark* 1:** It is to be noted that previous work on the vehicle control of platoons, has typically assumed that a controller requires additional information to (c) e.g. it is typically assumed that a knowledge of the lead vehicle's velocity is known, in addition to the velocity and acceleration of the preceding vehicle [7], [8]. The assumption that the decentralized controller only requires a knowledge of the spacing distance between the vehicle and the vehicle preceding itself is a much more realistic assumption.

2.3 *Decentralized Robust Servomechanism Problem (DRSP) for a platoon of vehicles*

Given the augmented system (2.2) and given a desired separation distance d_i^{ref} for the i^{th} vehicle, assume that a decentralized controller:

$$\begin{cases} \dot{\eta}_i = \mathcal{A}_i \eta_i + \mathcal{B}_i d_i + \mathcal{E}_i d_i^{ref} \\ u_i = \mathcal{C}_i \eta_i + \mathcal{D}_i d_i + \mathcal{F}_i d_i^{ref} \end{cases} \tag{2.6}$$

is to be found so that the closed loop system has the following property:

i) The closed loop system (2.2),(2.6) is asymptotically stable

ii) Asymptotic error tracking and regulation occurs, i.e. $\forall d_i^{ref}$, i= 1, 2, …, N, for all constant plant v_0^{ref} and for all plant and controller initial conditions:

$$\lim_{t\to\infty}(d_i - d_i^{ref}) = 0 \quad , i = 1, 2, \ldots, N.$$

iii) For any plant perturbation which maintains property *i*), it is desired that property *ii*) still holds.

In this case, the following existence conditions for a solution to the problem as obtained:

***Lemma* 1 [11]-[13]:** There exists a solution to the (DRSP) for (2.2) if and only if the following conditions are all satisfied:

(a) (C, A, B, D) is stabilizable and detectable.

(b) $rank\begin{pmatrix} A & B \\ C & D \end{pmatrix} = n+1$

where (C, A, B, D) is given by (2.1).

***Remark* 2:** The conditions above are just the conditions for a solution to the robust servomechanism problem (RSP) to exist for the isolated centralized system (2.1).

***Remark* 3:** It is to be noted that the conditions of lemma 1, hold for all systems described by the vehicle model (2.4).

***Assumption* 1:** In what follows, we will assume that the conditions of lemma 1 are always satisfied for the model (2.1).

2.4 *Closed loop Model of platoon System*

Given the platoon vehicle model (2.2), assume that the decentralized controller (2.6) has been found to solve the DRSP for (2.2) for the class of constant tracking signals so that the following closed loop system obtained is asymptotically stable:

$$\begin{cases} \dot{\tilde{x}} = \bar{\mathcal{A}}\tilde{x} + \tilde{\mathcal{E}}_v v_0^{ref} + \tilde{\mathcal{E}}_{ref}\, d^{ref} \\ d = \tilde{\mathcal{C}}\tilde{x} \\ v = \tilde{\mathcal{C}}_v \tilde{x} \end{cases} \tag{2.7}$$

where

$$\bar{\mathcal{A}} = \begin{pmatrix} A+B\mathcal{D}_1C & B\mathcal{C}_1 & 0 & 0 & \cdots & \cdots & 0 & 0 \\ \mathcal{B}_1 C & \mathcal{A}_1 & 0 & 0 & \cdots & \cdots & 0 & 0 \\ \bar{E}\bar{C} & 0 & A+B\mathcal{D}_2C & B\mathcal{C}_2 & \cdots & \cdots & 0 & 0 \\ 0 & 0 & \mathcal{B}_2 C & \mathcal{A}_2 & \cdots & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \cdots & \bar{E}\bar{C} & 0 & A+B\mathcal{D}_NC & B\mathcal{C}_N \\ 0 & 0 & \cdots & \cdots & 0 & 0 & \mathcal{B}_N C & \mathcal{A}_N \end{pmatrix} \tag{2.7a}$$

$$\tilde{\mathcal{E}}_v = \begin{pmatrix} E \\ 0 \\ 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \\ 0 \end{pmatrix} \quad , \quad \tilde{\mathcal{E}}_{ref} = \begin{pmatrix} B\mathcal{F}_1 & 0 & \cdots & 0 \\ \mathcal{E}_1 & 0 & \cdots & 0 \\ 0 & B\mathcal{F}_2 & \cdots & 0 \\ 0 & \mathcal{E}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & B\mathcal{F}_N \\ 0 & 0 & \cdots & \mathcal{E}_N \end{pmatrix} \tag{2.7b}$$

$$\tilde{\mathcal{C}} = \begin{pmatrix} C & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 \\ 0 & 0 & C & 0 & \cdots & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \cdots & C & 0 \end{pmatrix} \tag{2.7c}$$

$$\tilde{\mathcal{C}}_v = \begin{pmatrix} \bar{C} & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 \\ 0 & 0 & \bar{C} & 0 & \cdots & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \cdots & \bar{C} & 0 \end{pmatrix} \tag{2.7d}$$

and

$\tilde{x} := (x_1' \ \eta_1' \ x_2' \ \eta_2' \ \cdots \ x_N' \ \eta_N')'$, $d^{ref} := (d_1^{ref} \ d_2^{ref} \ \cdots \ d_N^{ref})'$, $d := (d_1 \ d_2 \ \cdots \ d_N)'$, $v := (v_1 \ v_2 \ \cdots \ v_N)'$.

2.5 *Derivation of the transfer function $d_i(s)/d_{i-1}(s)$ and $v_i(s)/v_{i-1}(s)$*

From (2.7), the state equations of two adjacent vehicles with $v_0^{ref}=0$ and $d_i^{ref}=0$, are given as follows:

$$\dot{x} = \left(\begin{array}{cc:cc} A+B\mathcal{D}_{i-1}C & B\mathcal{C}_{i-1} & 0 & 0 \\ \mathcal{B}_{i-1}C & \mathcal{A}_{i-1} & 0 & 0 \\ \hdashline \bar{E}\bar{C} & 0 & A+B\mathcal{D}_i C & B\mathcal{C}_i \\ 0 & 0 & \mathcal{B}_i C & \mathcal{A}_i \end{array}\right) x + \left(\begin{array}{c:c} B\mathcal{F}_{i-1} & 0 \\ \mathcal{E}_{i-1} & 0 \\ \hdashline 0 & B\mathcal{F}_i \\ 0 & \mathcal{E}_i \end{array}\right) d^{ref} \tag{2.8a}$$

$$d = \left(\begin{array}{cc:cc} C & 0 & 0 & 0 \\ \hdashline 0 & 0 & C & 0 \end{array}\right) x \tag{2.8b}$$

$$v = \left(\begin{array}{cc:cc} \bar{C} & 0 & 0 & 0 \\ \hdashline 0 & 0 & \bar{C} & 0 \end{array}\right) x \tag{2.8c}$$

where $x = (x_{i-1}' \ \ \eta_{i-1}' \ \ x_i' \ \ \eta_i')'$, $d = (d_{i-1} \ \ d_i)'$, and $v = (v_{i-1} \ \ v_i)'$, for i= 2, 3, …, N

and solving for $d_i(s)/d_{i-1}(s) =: G_i(s)$ results in:

$$G_i(s) = (C \quad 0) M_i^{-1} \begin{pmatrix} \bar{E} \\ 0 \end{pmatrix} \bar{C} (sI - A - B\mathcal{D}_{i-1}C)^{-1} B\mathcal{C}_{i-1} (sI - \mathcal{A}_{i-1})^{-1} \mathcal{B}_{i-1}$$

where

$$M_i = \begin{pmatrix} sI - A - B\mathcal{D}_i C & -B\mathcal{C}_i \\ -\mathcal{B}_i C & sI - \mathcal{A}_i \end{pmatrix}, \quad i = 2, 3, \dots, N \tag{2.9}$$

Likewise, solving for $v_i(s)/v_{i-1}(s) =: P_i(s)$ results in:

$$P_i(s) = (\bar{C} \quad 0) M_i^{-1} \begin{pmatrix} \bar{E} \\ 0 \end{pmatrix}, \quad i = 2, 3, \dots, N \tag{2.10}$$

***Definition*:** Let the impulse response of $G_i(s)$ and $P_i(s)$ be denoted as $g_i(t)$ and $p_i(t)$ respectively , i=2 , 3,…,N.

***Definition*:** The numerical model of the platoon system (2.9) is said to be string stable [3] with respect to $d_i(s)$ or $v_i(s)$ if the following condition respectively holds:

There exists constants $\beta_i > 0$ and $\bar{\beta}_i > 0$ such that $\|g_i(t)\|_1 = \beta_i \le 1$, $\|p_i(t)\|_1 = \bar{\beta}_i \le 1$, i= 2 , 3,…, N

III. MAIN RESULTS

3.1 *String Stability for Identical Controllers*

In the following development, it will be assumed that the platoon of vehicles is controlled by a set of identical controllers for each vehicle, and that it is desired to solve the *platoon vehicle control problem* described in section 2.2. In particular, assume that a model of a vehicle (2.1) is described by :

$$v_i(s) = \frac{p(s)}{q(s)} u_i(s), \quad i = 1, 2, \dots, N \tag{3.1a}$$

$$d_i(s) = \frac{1}{s}(v_{i-1}(s) - v_i(s)), \quad i = 1, 2, \dots, N \tag{3.1b}$$

where the transfer function $p(s)/q(s)$ may be either proper or strictly proper and $q(s)$ is assumed to be Hurwitz stable. Assume that a solution to the RSP for the platoon exists for constant set points and constant disturbances, which implies from lemma 1 that the plant (3.1) must necessarily satisfy the property that $p(0) \ne 0$.

Assume now that the following robust feedforward-feedback controllers which have a servo-compensator [12] applied to solve the RSP problem, are used to control each vehicle of the platoon:

$$u_i(s) = \frac{p_c(s)}{sq_c(s)}(d_i(s) - d_i^{ref}(s)) + \frac{\hat{p}_c(s)}{\hat{q}_c(s)} d_i^{ref}(s), \quad i = 1, 2, \dots, N \tag{3.2}$$

where it is assumed that, $q_c(s)$ is Hurwitz stable, no pole-zero cancellation occurs in $p_c(s)/sq_c(s)$ and $\hat{p}_c(s)/\hat{q}_c(s)$, that the resultant closed loop system obtained by equating (3.2) to (3.1) is asymptotically stable, and that $p_c(s)/sq_c(s)$ and $\hat{p}_c(s)/\hat{q}_c(s)$ may be either strictly proper, proper, or improper. The controller (3.2) includes a large class of controllers, i.e. it includes the class of 3-term controllers and observer based controllers.

Then the closed loop system is described as follows:

$$v_i(s) = G(s) v_{i-1}(s) + G^{ref}(s) d_i^{ref}(s) \tag{3.3}$$

where

$$G(s) = \frac{p(s)p_c(s)}{s^2 q(s) q_c(s) + p(s) p_c(s)} \tag{3.4a}$$

$$G^{ref}(s) = \frac{s^2 p(s) q_c(s)}{s^2 q(s) q_c(s) + p(s) p_c(s)} \left(\frac{\hat{p}_c(s)}{\hat{q}_c(s)} - \frac{p_c(s)}{sq_c(s)} \right) \tag{3.4b}$$

(where $p(0)p_c(0) > 0$, since the closed loop system is assumed to be asymptotically stable) which implies that:

$$\frac{v_i(s)}{v_{i-1}(s)} = \frac{d_i(s)}{d_{i-1}(s)} =: G(s)$$

The following result is obtained:

***Theorem* 1:** Consider the vehicle system (3.1) and corresponding identical controllers (3.2) for the platoon; then such a closed loop system will always be string unstable, i.e. $\|G(s)\|_\infty > 1$ where $G(s)$ is given by (3.4).

***Remark* 4:** This result states that it is impossible to find a set of identical controllers described by (3.2) for a platoon of vehicles to solve the platoon vehicle control problem, since such a closed loop system will always be string unstable.

***Proof of Theorem* 1:** In the plant model (3.1) assume that

$$p(s)=p_m s^m + p_{m-1}s^{m-1}+\cdots+p_2 s^2+p_1 s+p_0 \ , \ p_m \neq 0 \quad (3.5a)$$

$$q(s)=s^n + q_{n-1}s^{n-1}+\cdots+q_2 s^2+q_1 s+q_0 \quad (3.5b)$$

where $p_0 \neq 0$ (since it has been assumed a solution to the RSP exists) and $q_0 > 0$ (since it has been assumed that the plant $p(s)/q(s)$ is asymptotically stable). Also in the controller (3.2) assume that:

$$p_c(s)=\bar{p}_{\bar{m}} s^{\bar{m}} + \bar{p}_{\bar{m}-1}s^{\bar{m}-1}+\cdots+\bar{p}_2 s^2+\bar{p}_1 s+\bar{p}_0 \ , \ \bar{p}_{\bar{m}} \neq 0 \quad (3.6a)$$

$$q_c(s)=s^{\bar{n}} + \bar{q}_{\bar{n}-1}s^{\bar{n}-1}+\cdots+\bar{q}_2 s^2+\bar{q}_1 s+\bar{q}_0 \quad (3.6a)$$

where $\bar{q}_0 > 0$ (since it has been assumed that $q_c(s)$ is Hurwitz stable).

It is clear from (3.4) that $G(s)$ has the property that:

$$G(j\omega)\big|_{\omega=0} = 1$$

It will now be shown that $\exists\, \omega^*$ so that $|G(j\omega)| > 1$ for $\omega \in (0, \omega^*]$ which implies that $\|G(j\omega)\|_\infty > 1$, and thus that $\|g(t)\|_1 > 1$ which means that the system is string unstable. Let

$$p(s)p_c(s) = b_{m+\bar{m}}s^{m+\bar{m}} + \cdots + b_3 s^3 + b_2 s^2 + b_1 s + b_0 \quad (3.7a)$$

$$q(s)q_c(s) = s^{n+\bar{n}} + q^*_{n+\bar{n}-1}s^{n+\bar{n}-1} + \cdots + q^*_2 s^2 + q^*_1 s + q^*_0 \quad (3.7b)$$

where

$b_0 = p_0\bar{p}_0 > 0$ (since $p(0)p_c(0) > 0$)

$q^*_0 = q_0\bar{q}_0 > 0$ (since $q(0) > 0$ and $q_c(0) > 0$)

Then in substituting (3.7) with the transfer function $G(s)$ given by (3.4), the following representation is obtained:

$$G(s)=\frac{b_0+b_1 s+b_2 s^2+\cdots+b_{m+\bar{m}}s^{m+\bar{m}}}{b_0+b_1 s+(b_2+q_0\bar{q}_0)s^2+(b_3+q^*_1)s^3+(b_4+q^*_2)s^4+\cdots+s^{n+\bar{n}+2}} \quad (3.8)$$

which implies that:

$$|G(j\omega)|^2=\frac{(b_0-b_2\omega^2+b_4\omega^4\cdots)^2+(b_1\omega-b_3\omega^3+b_5\omega^5+\cdots)^2}{(b_0-(b_2+q_0\bar{q}_0)\omega^2+(b_4+q^*_2)\omega^4+\cdots)^2+(b_1\omega-(b_3+q^*_1)\omega^3+\cdots)^2} \quad (3.9)$$

or

$$|G(j\omega)|^2 = \frac{(b_0-b_2\omega^2)^2}{(b_0-(b_2+q_0\bar{q}_0)\omega^2)^2}\cdot\frac{N(\omega)}{D(\omega)} \quad (3.10)$$

where

$$N(\omega)=\left(1+\omega^2\frac{b_4\omega^2-b_6\omega^4+\cdots}{(b_0-b_2\omega^2)}\right)^2+\omega^2\left(\frac{b_1-b_3\omega^2+\cdots}{(b_0-b_2\omega^2)}\right)^2$$

$$D(\omega)=\left(1+\omega^2\frac{(b_4+q^*_2)\omega^2-(b_6+q^*_4)\omega^4+\cdots}{(b_0-(b_2+q_0\bar{q}_0)\omega^2)}\right)^2+\omega^2\left(\frac{b_1-(b_3+q^*_1)\omega^2+\cdots}{(b_0-(b_2+q_0\bar{q}_0)\omega^2)}\right)^2$$

Now since $q_0\bar{q}_0 > 0$ and $b_0 > 0$ in (3.10), this implies that there exists $\omega^* > 0$ so that $|G(j\omega)|^2 > 1, \forall \omega \in (0, \omega^*]$, which in turn implies that $\|G(j\omega)\|_\infty > 1$ and thus that $\|g(t)\|_1 > 1$, which proves the result.

IV. EXAMPLES

In the examples which follow, a vehicle is assumed to have the following simplified model:

$$\begin{pmatrix}\dot{d}_i\\ \dot{v}_i\end{pmatrix}=\begin{pmatrix}0 & -1\\ 0 & -b/m\end{pmatrix}\begin{pmatrix}d_i\\ v_i\end{pmatrix}+\begin{pmatrix}0\\ 1/m\end{pmatrix}u_i+\begin{pmatrix}1\\ 0\end{pmatrix}v_{i-1}$$
$$d_i = \begin{pmatrix}1 & 0\end{pmatrix}\begin{pmatrix}d_i\\ v_i\end{pmatrix} \quad (4.1)$$

where b=1 and m=0.1 which corresponds to the model (2.4), and a platoon of vehicles is then described by (2.5).

The following simple-to-implement 3-term decentralized controller is assumed to be used in all examples:

$$u_i(s)=(KP_i+\frac{KI_i}{s}+KD_i)e_i(s) \ , \ i= 1 , 2,\ldots,N \quad (4.2)$$

where $e_i(s)=(d_i(s)-d_i^{ref}(s))$, where $d_i^{ref}(s)=0$ is typically assumed

In this case, the following transfer functions are directly obtained from (2.9), (2.10):

$$\frac{d_i(s)}{d_{i-1}(s)}=G_i(s)=\frac{(KD_{i-1}s^2+KP_{i-1}s+KI_{i-1})}{s^2(ms+b)+(KD_i s^2+KP_i s+KI_i)} \quad (4.3)$$

$$\frac{v_i(s)}{v_{i-1}(s)}=P_i(s)=\frac{(KD_i s^2+KP_i s+KI_i)}{s^2(ms+b)+(KD_i s^2+KP_i s+KI_i)} \quad (4.4)$$

for i= 1, 2, …, N . It is to be noted that the controllers (4.2) are not necessarily assumed to be identical.

***Remark* 5:** If $KP_i = KP_{i-1}$, $KD_i = KD_{i-1}$, $KI_i = KI_{i-1}$, i= 2, 3,…, N i.e. identical decentralized controllers are used to control the platoon (2.5), then it follows from theorem 1 that the resultant closed loop system will always be string unstable i.e.:

$\|G_i(s)\|_\infty = \|G_{i-1}(s)\|_\infty = \|P_i(s)\|_\infty = \|P_{i-1}(s)\|_\infty > 1$, i= 2, 3,…, N.

It will be shown in the examples however that if non-identical controllers are used, then it is possible for the resultant closed loop system to be string stable. In the examples to follow, the control input forces are very similar to the velocities, and so to save space, they are not included.

***Example* 1 (Identical controllers, *N*=40)**

In this example, the controller parameters of (4.2) are chosen to be identical for all vehicles and are given by:

KP_i =8, KD_i =18, KI_i =1 (4.5)

In this case, when the velocity of the leader changes, the results of figure 4.1 are obtained and the distance error initially is not amplified when n<20; however when n>20, the peaks of the distance error are amplified, which implies that the resultant system is not string stable. The peaks of the velocity of the vehicles increase for all values of n, also indicating that the resultant system is not string stable. These results obtained are consistent with remark 5.

***Example* 2 (Identical controllers, *N*=2000)**
In this example, the controller parameters of (4.2) are chosen to be identical for all vehicles with KP_i =18, KD_i =4, KI_i =1 and a large number of vehicles are assumed to be contained in the platoon (N=2000). In this case, when the velocity of the leader changes, the results of figure 4.2 are obtained. It is now seen that the peaks of the distance error and the peaks of the velocity are amplified as , $N \to \infty$ with the corresponding effect being that the peaks of the control signal magnitude are now also being amplified as $N \to \infty$. Such a behaviour is clearly undesirable since the peaks of the distance error, the peaks of the velocity, and the peaks of the control signal magnitude are all unbounded, and nonlinear effects such as control signal magnitude constraints will become significant.

***Example* 3 (Non-identical controllers *N*=2000)**
In this example non-identical controllers (4.2) will be designed in order to satisfy the string stability constraint:

$$\left\| \frac{d_i(s)}{d_{i-1}(s)} \right\|_\infty < \quad , i= 2 ,3 ,\ldots, N.$$

The design procedure will be done in a recursive way. Assume that the controller for the i-1th vehicle (4.2) has already been designed with specified parameters KP_{i-1}, KD_{i-1}, KI_{i-1}; then from (4.2) if KP_i , KD_i , KI_i can be chosen so that two stable zeros of the transfer function (4.2) can be cancelled by two stable poles, the remaining transfer function will have only one pole. This implies that if KI_i is chosen so that KI_i> KI_{i-1}, then the DC gain of the resultant first order transfer function (4.3) will be less than or equal to one, and thus the condition

$$\left\| \frac{d_i(s)}{d_{i-1}(s)} \right\|_\infty < 1$$

must hold since (4.3) is a first order transfer function with $DC\ Gain \le 1$.
The updating procedure to design the controller parameters for the controller (4.2) thus becomes:

$$KI_i \ge KI_{i-1} \tag{4.6a}$$

$$KP_i = \frac{KI_i}{KI_{i-1}} KP_{i-1} + \frac{m}{KD_{i-1}} KI_{i-1} \tag{4.6b}$$

$$KD_i = \frac{KI_i}{KI_{i-1}} KD_{i-1} + \frac{m}{KD_{i-1}} KP_{i-1} - b \tag{4.6c}$$

which implies that $\frac{d_i(s)}{d_{i-1}(s)} = G_i(s)$ is now given by:

$$G_i(s) = \frac{KI_{i-1}}{KI_i} \frac{1}{(\frac{KI_{i-1}}{KI_i}\frac{m}{KD_{i-1}})s+1} \tag{4.7}$$

which has the property that $\|G_i(s)\|_\infty < 1$, i.e. distance string stability occurs.
In this case, when the above updating procedure was carried out to design the decentralized controllers with the i =1 controller having controller parameters KP_1 =8, KD_1 =18, KI_1=1, the results of figure 4.3 are obtained for the case when N=2000. It can be seen that the peaks of the distance error show a desirable attenuation as $N \to \infty$, which confirms the fact that the system has distance string stability. However the peaks of the velocity still have an undesirable amplification problem.

***Example* 4 (Non-identical controllers, *N*=2000)**
In this example, it is desired to design a decentralized controller which possesses both distance and velocity string stability, and in this case the same controller parameters as used in example 3 were used in n<500 , but the controller parameters KP, KD are now assumed to increase linearly when n>500, as shown in figure 4.4. In this case the peak distance error has a desirable attenuation as $N \to \infty$, and also the velocity peaks now have an attenuation beginning approximately at n=600, variable the case of example 3.

***Example* 5 (Non-identical controllers, *N*=2000)**
This example shows that if one uses a controller design procedure which is "simple" to implement, such as designing the controller parameters KP, KD so that they increase linearly with respect to vehicle index, (as shown in figure 4.5) that the results obtained may not be entirely desirable. This is seen in figure 4.5 where we now have attenuation in the peaks of the distance and velocity, but undesirable undershooting now occurs in the distance error and velocity.

***Example* 6 (Non-identical controllers, *N*=2000)**
This example shows that if one uses a controller design which is somewhat more complex to implement, such as choosing the controller parameters KP_i, KD_i as given in figure 4.6, that one may obtain attenuation in the peaks of the distance error and velocity, and also obtain avoid the undesirable undershooting which occurs in example 5.

***Example* 7 (Using strictly proper controllers)**
In the previous examples, a 3-term decentralized controller was used to control the platoon of vehicles described by (2.5). The same type of results as obtained in examples 1-6 can also be obtained by applying non-identical strictly proper decentralized controllers, e.g. by replacing the 3-term controller of (4.2) by the controller:

$$u_i(s) = c_i(s)\, e_i(s)$$

where:

$$c_i(s) = 0.21\theta_i \varepsilon^{-5} \frac{(s+1)^2}{s(s+\varepsilon^{-2})^2}, \; i = 1, 2, \ldots, N \qquad (4.8)$$

where $\varepsilon = 10^{-2}; \theta_i > 1$, i=1, 2, ..., N. In this case $c_i(s)$ has been designed using the optimization procedure of [14], [15].

V. Conclusion

The main focus of this paper is to design a decentralized controller for a platoon of identical vehicles, which uses minimal measurement information, i.e. the separation distance between the vehicle and the vehicle immediately in front, in order to regulate the separation distance of the vehicles to desired set points, independent of the lead vehicle's velocity, and such that string stability occurs. This has been done by applying non-identical 3-term controllers to control each vehicle. Example simulations of the resultant controlled system are carried out for the case of a large number of platoon vehicles (N=2000), and illustrate that the proposed decentralized controller can successfully solve this type of problem.

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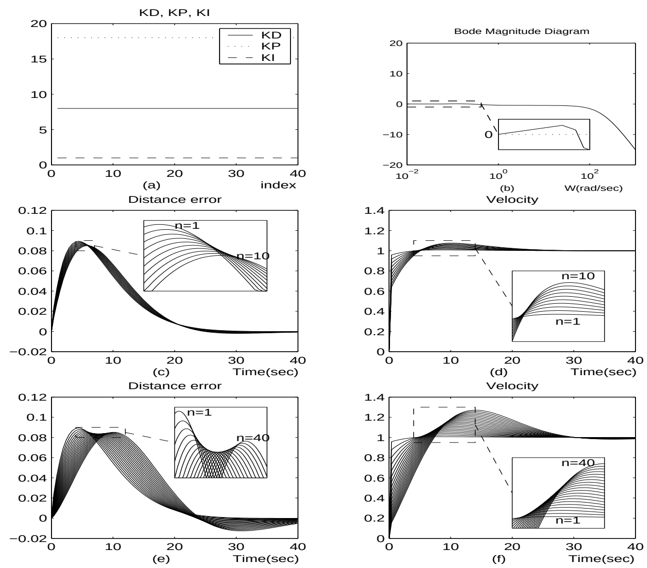

Figure 4.1: (Example 1, Identical controllers, N=40) Results obtained for identical 3-term controller with parameters KP=8, KD=18, KI=1, with a disturbance applied as a unit step change in the leader's velocity. In this case, an undesirable "slinky effect" in the distance error occurs when n>20.

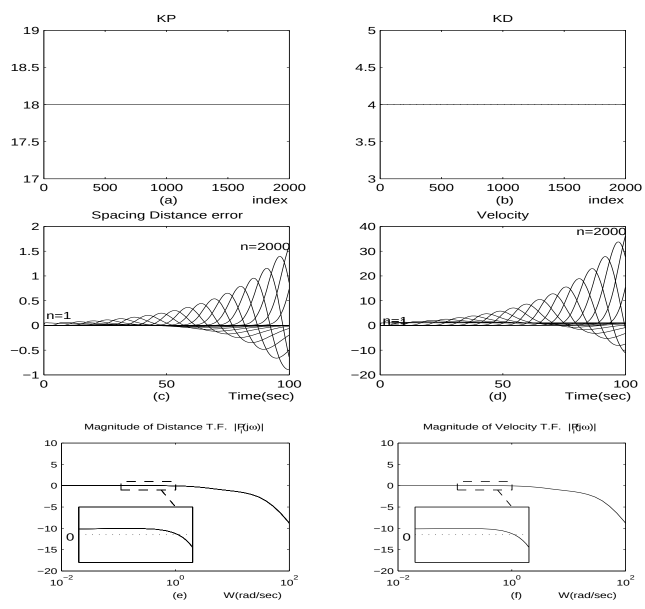

Figure 4.2: (Example 2, Identical controllers, N=2000) Results obtained for identical 3-term controller with parameters KP=18, KD=4, KI=1, with a disturbance applied as a unit step change in the leader's velocity. In this case, an undesirable "slinky effect" in both the distance error and velocity occurs when n>1.

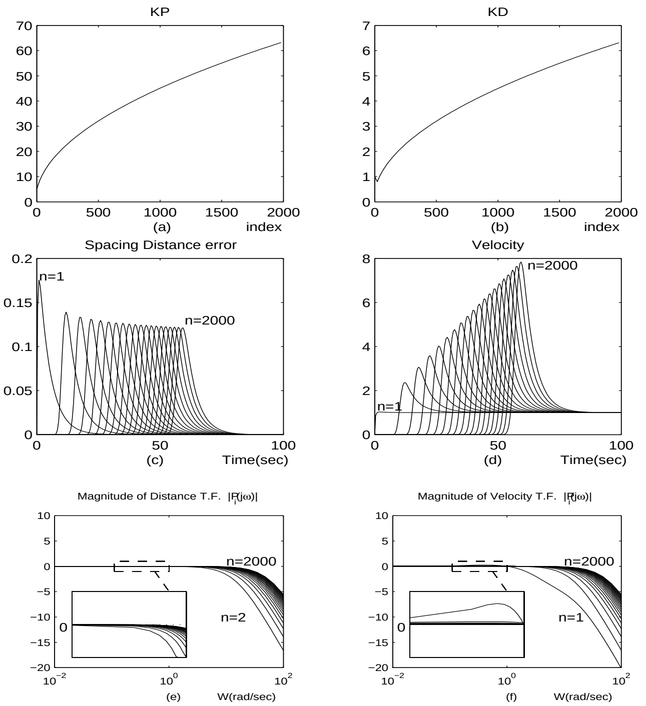

Figure 4.3: (Example 3, Non-identical controllers, *N*=2000) Results obtained for non-identical 3-term controller, with parameters *KP*, *KD* given in (a), (b), and *KI*=1, for case of disturbance applied as a unit step change in the leader's velocity. In this case, the peaking of the distance error is attenuated as $n \to \infty$, but the peak of the velocity is unbounded.

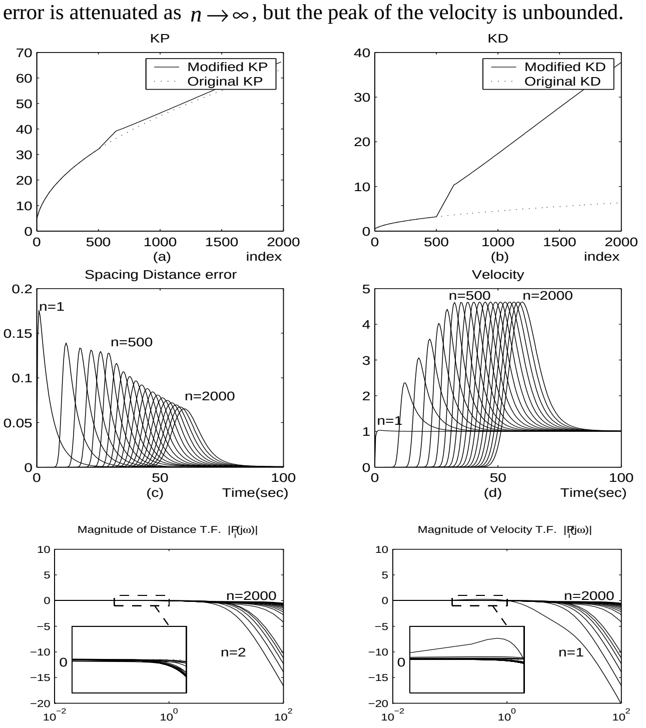

Figure 4.4: (Example 4, Non-identical controllers, *N*=2000) Results obtained for non-identical 3-term controllers, with parameters *KP*, *KD* given in (a), (b), and *KI*=1, for case of disturbance applied as a unit step change in the leader's velocity. In this case, both the peaks of the distance error and velocity are attenuated as $n \to \infty$.

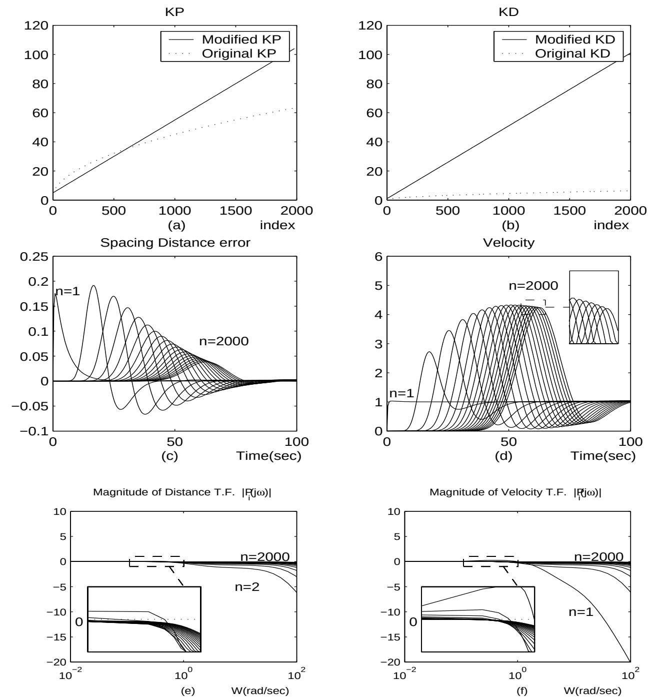

Figure 4.5: (Example 5, Non-identical controllers, *N*=2000) Results obtained for non-identical 3-term controllers, with parameters *KP*, *KD* given in (a), (b), and *KI*=1, for case of disturbance applied as a unit step in the leader's velocity. In this case, both the peaks of the distance error and velocity are attenuated as $n \to \infty$, but undesirable undershooting occurs in the distance error and velocity.

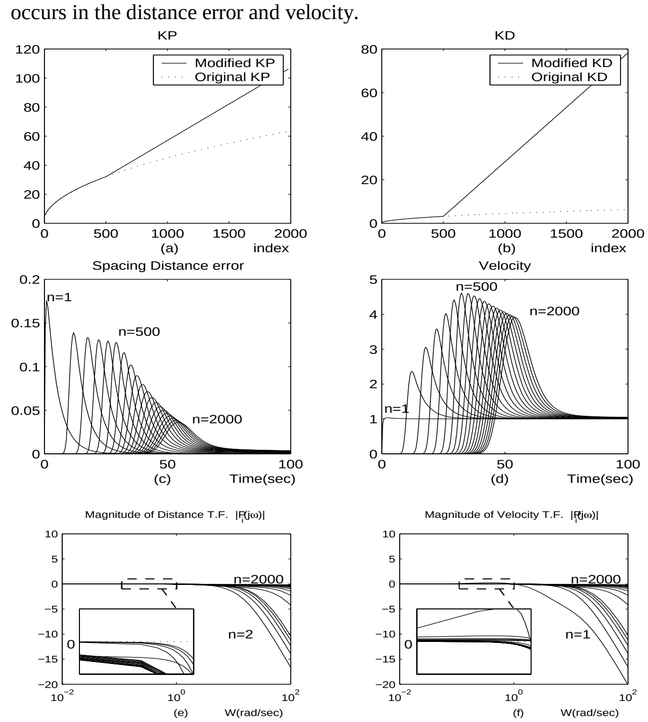

Figure 4.6: (Example 6, Non-identical controllers, *N*=2000) Results obtained for non-identical 3-term controllers, with parameters *KP*, *KD* given in (a), (b), and *KI*=1, for case of disturbance applied as a unit step in the leader's velocity. In this case, both the peaks of the distance error and velocity are attenuated as $n \to \infty$, and no undesirable undershooting occurs in the distance error and velocity.