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Model Reference Control Approach for Safe Longitudinal Control

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***Abstract*— In this paper we report a work done in the context of the ARCOS[1] French program. We introduce a new inter-distance reference model that can be used in cruise control and stop-and-go scenarios. The proposed model is non-linear and provides dynamic solutions which verify comfort and safety criteria simultaneously. The proposed reference model is based on a compliant model for contact, and has the particularity that its solutions can be described by explicit integral curves.**

I. INTRODUCTION

Adaptive cruise control (ACC), and stop-and-go scenarios are examples of problems related with longitudinal control. The former concerns the inter-distance control in highways where the vehicle velocity mainly remains constant, whereas the latter deals with the vehicle circulating in towns with frequent stops and accelerations. In both situations, goals of safety and comfort oppose each other. Safety imposes a minimum inter-vehicle distance while ensuring that the acceleration and deceleration are compatible with the vehicle braking and engine capabilities. Good comfort implies low jerks values.

In most of the reported works, these two categories of problems are treated separately with little regard to the comfort specifications. Indeed, the behavior of the inter-distance dynamics often results from a particular feedback loop, which makes difficult to ensure *a priori* computable bounds on the inter-distance and the vehicle acceleration and jerks. It is also suited that external factors such as road characteristics, weather conditions, and traffic load (among others), must be considered while defying the safety and the comfort metrics. This last point is naturally reinforced by the new safety programs including vehicles/infrastructure communication.

The purpose of this work is to design a reference model for the vehicle inter-distance that allows to provide safety specifications in a simple manner (few parameters). This model is intended to be used independently of the control design, and it should be able to account for external road information.

A. Safe inter-distance policies.

The notion of the safety distance in longitudinal control has been often studied in feedback configurations. [1] proposes that the inter-distance must be proportional to the vehicle velocity; here, the proportional constant has time units and is commonly named Time Headway T_h; the Time Headway is usually defined as the difference between the passage times of two successive vehicles.

[2] proposes a control strategy where the safety inter-distance is computed as a non-linear function of the speed, i.e. the Time Headway T_h is seen as a function of the speed. [3] presents an ACC system for low speed motion, where the desired acceleration was obtained from an estimated model using data of a real driver's behavior.

Typically, safety is treated in a quite conservative way by increasing the inter-vehicle distance at higher velocity [1],[4],[2]. For motions at low velocities, variable time headway is sometimes used, its effect is to reduce the distance at relative velocity close to zero. Nevertheless, there are no clear conditions ensuring collision avoidance for all possible vehicle operation conditions.

Safety criteria in longitudinal control can be treated as the problem to ensure a minimal distance, e.g. collision avoidance [5], when the leader vehicle does full stop, or for more conservative cases, when the following vehicle finds a fixed obstacle at a few meters. The natural concept of "virtual bumpers" is introduced in [6], where the vehicle behaves as if there were imaginary springs and dampers interacting with the leader vehicle. However, the authors did not elaborate this idea further, it was only presented at the conceptual level. Along the same vein, [4] demonstrates how a potential force-based controller can generate conservative forces obtained from an artificial damping. Nevertheless, a priori bounds on the forces are not guaranteed. A more elaborated result can be found in [5], where the authors consider some bounds on the vehicle acceleration and find a region in state space within which the initial conditions can be taken, resulting in a safe operation.

B. Comfort criteria.

Studies on comfort criteria are scarce. Passenger comfort in public ground transportation is determined by the changes in motion felt in all directions, as well as by the other environmental effects. Typically, acceleration magnitude is taken as a comfort metric, but in [7] comfort due to the motion changes in a vehicle's longitudinal direction (the "jerk") has been treated, i.e. the acceleration's derivative is the best to reflect a human comfort criteria.

As its name suggests, jerk is important when evaluating the destructive effect of motion on a mechanism or the discomfort caused to the passengers in a vehicle. The movement of delicate instruments needs to be kept within

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[1] ARCOS is a French program on safety vehicle and secure roads. For details, see http://www.arcos2004.com

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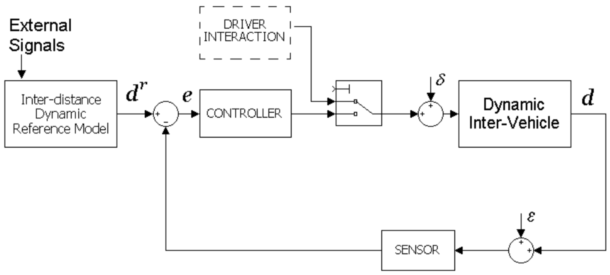

Fig. 1. Inter-distance Control

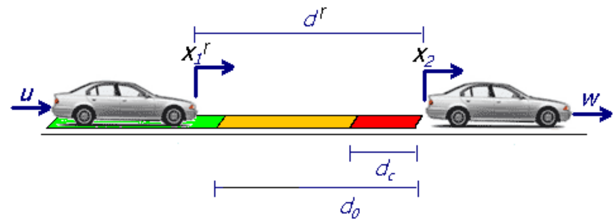

Fig. 2. Inter-distance system

SAFETY		
Collision avoidance	:	$d^r > d_c$
Bounded velocity	:	$\dot{x}_1^r \leq V_{max}$
Bounded braking	:	$\ddot{x}_1^r \geq -B_{max}$
COMFORT		
Bounded acceleration	:	$\ddot{x}_1^r \leq A_{max}$
Bounded Jerks	:	$\lvert\dddot{x}_1^r\rvert \leq J_{max}$

TABLE I

REFERENCE VEHICLE SPECIFICATIONS

specified limits of jerk as well as acceleration to avoid damage. When designing a train and elevators, engineers will typically be required to keep the jerk less than 2 m/s^3 for passenger comfort. Moreover, in the aerospace industry a jerkmeter is frequently used. Then, an accepted criteria is that bounded longitudinal accelerations and jerks can guarantee a certain degree of comfort in longitudinal control, especially in Stop-and-go scenarios.

In this paper, we propose for the first time a new inter-distance reference model that can be used in cruise control and in stop-and-go scenarios as well. The proposed model is nonlinear and provides dynamic solutions which *a priori* verify safety criteria. The model is based on physical laws of compliant contact and has the particularity that its solutions can be described by explicit integral curves. This allows to explicitly characterize the set of initial condition for which the safety specifications can be met. The model has few parameters that can be also set to account for other external factors, such as the road conditions and the traffic load. The model is described by a nonlinear set of equations that are driven by the vehicle leader acceleration.

II. PROBLEM STATEMENT

The figure 1 shows the control scheme which the inter-distance reference model is designed for. It can be understood as a tracking problem of the inter-distance signal $d^r(t)$. With this structure, the controller and the reference model can be defined independently. Thereby, the reference model will include the comfort and safety specifications, and it could be seen as an exosystem describing a *reference vehicle* dynamics. In that way the controller can be designed to optimally reject other systems disturbances specific to the sensors characteristics as well as other disturbance input torques such a side wind, road slopes, and vehicle internal actuator dynamics.

The figure 2 describes the system under study. The *leader vehicle* is represented as a massless point with longitudinal coordinate x_2. The *reference vehicle*, is located at a distance d^r (reference distance) from the leader vehicle, and it is represented by the coordinate x_1^r.

In order to characterize different safety levels, three zones are defined:

- *Green Zone* $d^r > d_o$. The inter-distance d is larger than the safe nominal inter-distance d_o (at nominal velocity). This is a safe operation region,
- *Orange Zone* $d_o \geq d^r > d_c$. The necessary inter-distance to avoid collision if a possible infinite braking is detected in the leader vehicle.
- *Red Zone* $d^r \leq d_c$. Where d_c is the minimal inter-distance.

u and w are the accelerations of the reference and the leader vehicles, respectively. It is assumed that the velocity and the acceleration of the leader vehicle can be estimated from suitable sensors. Finally, the constraints imposed by safety and comfort can be set as bounds on reference vehicle states and its time-derivatives. These constraints are summarized in Table I, where d_c, V_{max}, B_{max}, A_{max}, and J_{max} are positive constants. Nevertheless, these bounds may be dependent on the other road external factors as well. In this study, we assume that they are invariant.

Assume that the reference vehicle dynamics is a second order one, i.e.

$$\ddot{x}_1^r = u \tag{1}$$

Then, the dynamics of the inter-distance $d^r = x_2 - x_1^r$ writes as

$$\ddot{d}^r = \ddot{x}_2 - u \tag{2}$$

Introducing the shift coordinate $\tilde{d} \triangleq d_0 - d^r$, as being the inter-distance error with respect to the (constant) nominal inter-distance magnitude d_0. The dynamics of this error coordinate is

$$\ddot{\tilde{d}} = u - \ddot{x}_2 \tag{3}$$

The problem is then to find a suitable structure of u such that all the solutions of (3), for a given set of initial conditions (at the moment when orange zone is started), are consistent with the constraints indicated in Table I. To this aim, we search for nonlinear functions of $u = u(\tilde{d}, \dot{\tilde{d}})$. This is investigated in the next section.

III. INTER-DISTANCE REFERENCE MODEL.

The particular proposed structure for u allows the equation (3) to be re-interpreted as an equation describing the physics of an unit mass moving in the free space if $\tilde{d} < 0$, and "constrained" to a compliant surface for $\tilde{d} \geq 0$. This implies to have two different laws for u, i.e.

$$u = \begin{cases} u_1(\tilde{d}, \dot{\tilde{d}}) & \tilde{d} < 0 \\ u_2(\tilde{d}, \dot{\tilde{d}}) & \tilde{d} \geq 0 \end{cases} \tag{4}$$

where we assume continuity between these two structures, i.e. $\frac{\partial u_1}{\partial \tilde{d}}|_{\tilde{d}=0} = \frac{\partial u_2}{\partial \tilde{d}}|_{\tilde{d}=0}$; we assume also that in $\tilde{d} < 0$ (green zone), the initial conditions permit to the reference vehicle goes into the constrained zone (orange zone); Then, we will discuss only control structures for the "constrained" case.

A. Model for the constrained case ($\tilde{d} \geq 0$).

This case can be studied by making a parallel with the problem of compliant contacts. In particular we can get inspiration for the nonlinear models resulting from the theory of elasticity and mechanic of the contacts proposed by Hertz in 1881. He has proposed a model of the form $u_2 = -k\tilde{d}^n$, $\forall \tilde{d} \geq 0$, where n accounts for contact surface topology. However, the model has the major inconvenient of being non-dissipative, producing a oscillatory effect that may induce a non feasible *negative* vehicle velocity. To cope with this problem, Hunt and Crosseley [8], and then Marhefka and Orin [9] have introduced a non-linear damper/spring model of the general form $u_2 = -c|\tilde{d}|^n \dot{\tilde{d}} - k\tilde{d}^n$, $\forall \tilde{d} \geq 0$. Then, the forces are proportional to the penetration of the object into the surface. One of the advantages of this model is that in connection with (3), it is possible to compute the integral curves associated to the autonomous nonlinear differential equation.

In the "virtual contact" (orange) zone, we may want that the vehicle velocity behaves monotonically in the forward direction. For this, we can remove the spring-term in the damper/spring model discusses previously, and let u_1 be defined as

$$u_2 = -c|\tilde{d}|^n \dot{\tilde{d}}, \quad \forall \tilde{d} \geq 0 \tag{5}$$

which lead to the following equations

$$\ddot{\tilde{d}} = -c|\tilde{d}|^n \dot{\tilde{d}} - \ddot{x}_2 \tag{6}$$

Due to the necessity of eliminate the excess in kinetic energy that the vehicle has once it enters in the orange zone, it is then natural to only use a dissipation term to avoid collisions. Note that the goal of this structure is not to regulate back the reference vehicle to $\tilde{d} = 0$, but to stop the vehicle before it reaches the critical distance d_c, while respecting the imposed constraints.

Consider for simplicity $t = 0$ the time at which the orange zone is reached. Let Ω_0^{orange} be defined as

$$\Omega_0^{orange} = \left\{ \dot{x}_1^r(0), \tilde{d}(0) : \dot{x}_1^r(0) \leq V_{max}, \tilde{d}(0) = 0 \right\}$$

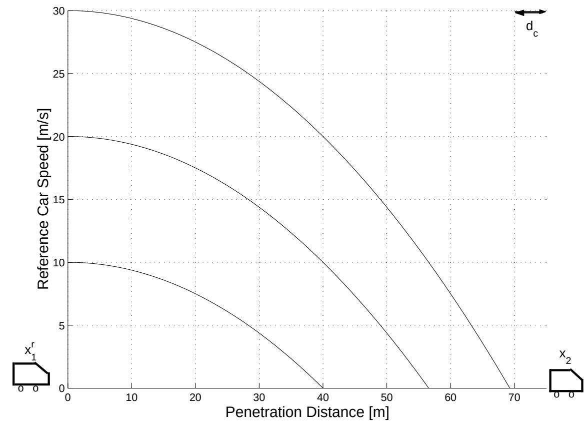

Fig. 3. Speed vs. Penetration Distance for different initial velocities. ($c = 0.0125$, $d_o = 75m$ and $d_c = 5m$).

the set of admissible initial state values at the crossing point $\tilde{d} = 0$. Now, the problem is then to find a gain c such that the restrictions in Table I will be satisfied for all possible solutions of (6) starting in Ω_0^{orange}.

Note that Equation (6) can be solved analytically. For $n = 1$, we have,

$$\dot{\tilde{d}}(t) = -\frac{c}{2}\tilde{d}(t)^2 - \dot{x}_2(t) + \beta \tag{7}$$

with $\beta = \dot{x}_1^r(0) + \frac{c}{2}\tilde{d}^2(0) = \dot{x}_1^r(0)$. Upon substitution of the relation $\dot{x}_1^r(t) = \dot{\tilde{d}}(t) + \dot{x}_2(t)$ in (7) one can obtain an explicit relation between the reference vehicle velocity and the "penetration" distance, i.e.

$$\dot{x}_1^r(t) = -\frac{c}{2}\tilde{d}(t)^2 + \dot{x}_1^r(0) \tag{8}$$

From this expression, we can find a c such that for all $0 \leq \dot{x}_1^r(0) \leq V_{max}$, the critical distance d_c is not attained. From:

$$\tilde{d}(t) = \sqrt{\frac{2(\dot{x}_1^r(0) - \dot{x}_1^r(t))}{c}} \tag{9}$$

the maximum penetration distance $\tilde{d}_{max}$ can be computed as $\tilde{d}_{max} = \sqrt{\frac{2\bar{\beta}}{c}}$; ($\bar{\beta} \triangleq \max_{\forall t}\{\dot{x}_1^r(0) - \dot{x}_1^r(t)\} = \dot{x}_1^r(0)$). Making $\tilde{d}_{max} \leq d_o - d_c$, we have,

$$\tilde{d}_{max} = \sqrt{\frac{2\dot{x}_1^r(0)}{c}} \leq d_o - d_c \tag{10}$$

which provides a first inequality for c, i.e.

$$\mathcal{C}_1 : \quad c \geq \frac{2\dot{x}_1^r(0)}{(d_o - d_c)^2} \tag{11}$$

Figure 3 displays the integral curves (8) for different initial reference vehicle velocities. The constant c is computed to ensure that the vehicle inter-distance d^r is larger than d_c for all $\dot{x}_1^r(0) \leq V_{max}$ and $\tilde{d}(0) = 0$.

By taking time-derivatives from (8), and proceeding in the same way, we can obtain expressions for the maximum braking $\ddot{x}^{r-}_{1max}$, positive acceleration $\ddot{x}^{r+}_{1max}$, and jerk $\dddot{x}^{r}_{1max}$,

$$|\ddot{x}^{r-}_{1max}| = \frac{2}{3}\dot{x}^r_1(0)\sqrt{\frac{2\dot{x}^r_1(0)c}{3}} \leq B_{max} \tag{12}$$

$$|\ddot{x}^{r+}_{1max}| \leq c\tilde{d}_{max}|\ddot{x}^+_2| \leq A_{max} \tag{13}$$

$$|\dddot{x}^{r}_{1max}| = \max(c(\dot{x}^r_1(0))^2, c\tilde{d}_{max}|\ddot{x}_2|) \leq J_{max} \tag{14}$$

where $\ddot{x}^+_2$ corresponds to the positive acceleration in leader vehicle. Figure 4 shows solutions of (6) by different values of c; Notice for example that high values of c yield high values in braking and jerk magnitudes. Relations (12), (13) and (14) yield three more inequalities providing upper bounds for c, i.e.

$$\mathcal{C}_2 \quad : \quad c \leq (\frac{27}{8})\frac{B^2_{max}}{\dot{x}^r_1(0)^3} \tag{15}$$

$$\mathcal{C}_3 \quad : \quad c \leq \frac{A_{max}}{\tilde{d}_{max}|\ddot{x}_2|} \tag{16}$$

$$\mathcal{C}_4 \quad : \quad c \leq \frac{J_{max}}{\max((\dot{x}^r_1(0))^2, \tilde{d}_{max}|\ddot{x}_2|)} \tag{17}$$

Since $\mathcal{C}_1$ and $\mathcal{C}_2$ are associated with safety specifications, they are seen as hard constraints, whereas $\mathcal{C}_3$ and $\mathcal{C}_4$ are associated with comfort and theses are seen as a soft constraints[2]. In the orange zone, the priority is given to safety, then to the constraints $\mathcal{C}_1$ and $\mathcal{C}_2$.

The problem can thus be formulated as finding the minimum value of c , subject to the set of constraints $\mathcal{C}_1$ and $\mathcal{C}_2$.

Therefore, a sufficient condition for c to exist is that $\mathcal{C}_1$ and $\mathcal{C}_2$ holds, i.e.

$$\frac{2\dot{x}^r_1(0)}{(d_o - d_c)^2} \leq (\frac{27}{8})\frac{B^2_{max}}{\dot{x}^r_1(0)^3} \tag{18}$$

which together with $\dot{x}^r_1(0) \leq V_{max}$, implies that specifications should at least meet the following relation

$$d_o \geq \sqrt{\left(\frac{16}{27}\right)\frac{V^2_{max}}{B_{max}}} + d_c \tag{19}$$

where d_0 and d_c are design parameters to be selected according to (19). If (19) holds, then we can define c from $\mathcal{C}_2$, as:

$$c = \frac{27B^2_{max}}{8V^3_{max}} \tag{20}$$

[2]Note that the model has not many degree or freedom and therefore, if the priority is given to safety, then comfort specifications can not be arbitrarily chosen.

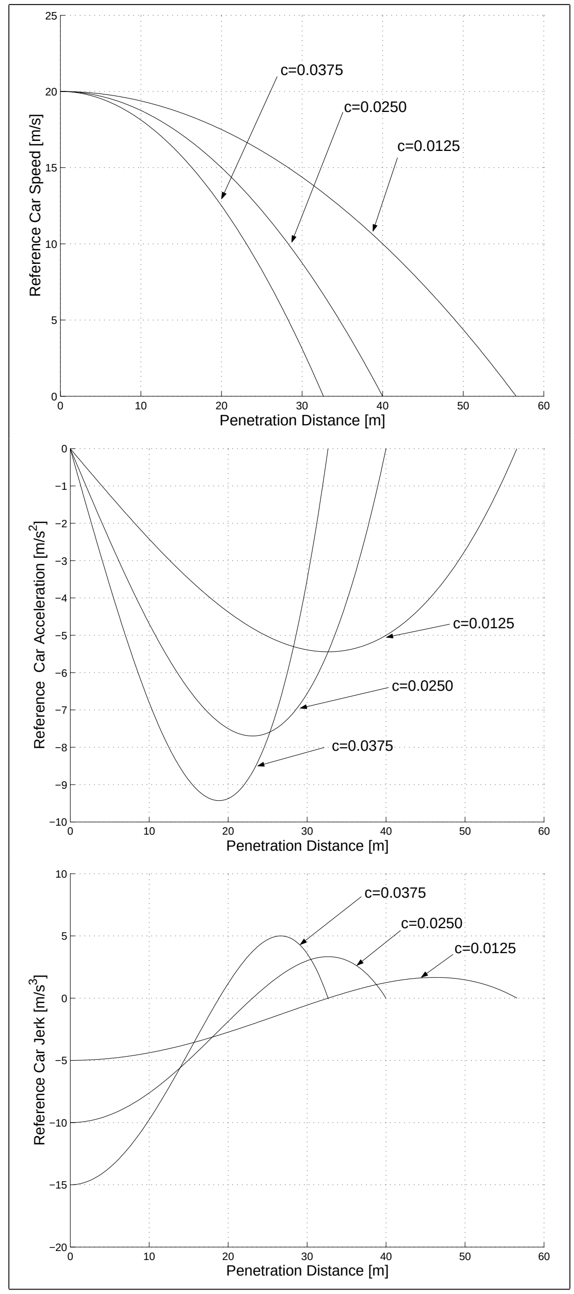

Fig. 4. Speed, Acceleration and Jerk vs. Penetration Distance for same initial conditions $\dot{x}^r_1(0) = 20m/s$ and different c values.

That means to choice the smallest c that generate safe distance, respecting the maximum braking capacity. Notice that the model gives an important relation between reference vehicle velocity and safe distance for a given braking capacity B_{max}. Figure 5 illustrates this relation.

Fixing c by equation (20), the positive maximum reference vehicle acceleration will be bounded by:

$$\ddot{x}^{r+}_{1max} \leq A_{max} \tag{21}$$

which suggests from (13) that the positive leader vehicle acceleration meets: $|\ddot{x}^+_2| \leq (\frac{8}{27}\sqrt{\frac{27}{16}}\frac{V_{max}}{B_{max}})A_{max}$.

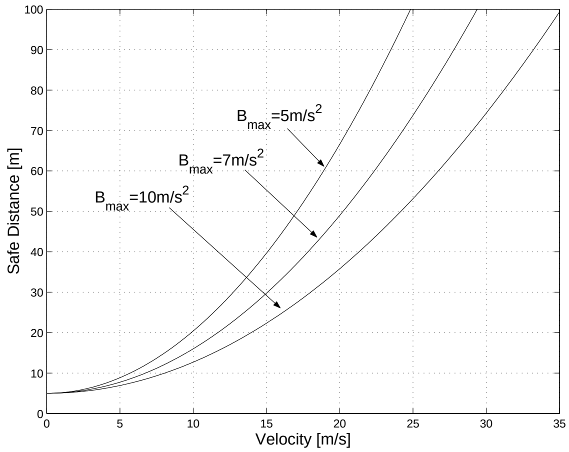

Fig. 5. Safe Distance vs. Velocity, with $d_c = 5m$ for different braking capacities.

while, the maximum reference vehicle jerk will be bounded by:

$$|\dddot{x}^r_{1max}| \leq \frac{27}{8}\frac{B^2_{max}}{V_{max}} \tag{22}$$

which suggests from (14) that the leader vehicle braking meets: $|\ddot{x}^-_2| \leq \sqrt{\frac{27}{16}}B_{max}$.

Otherwise the vehicle reference comfort would be not guaranteed.

B. Analysis for different values of n

Until now we have analyzed the model with $n = 1$ in (6); proceeding in the same way, but considering the parameter n, we obtain a more general expression of (19):

$$d_o \geq \left[\frac{n^n(n+1)^{2(n+1)}}{(2n+1)^{2n+1}}\right]^{\frac{1}{n+1}} \frac{V^2_{max}}{B_{max}} + d_c \tag{23}$$

Similarly, if condition (23) is satisfied, then there exists c such that the maximum braking value B_{max} is respected and inter-distance is always larger or equal than minimal inter-distance d_c, (for all initial speed smaller or equal to V_{max}). In addition, (23) suggests the existence of a minimum value for d_o in function of n. Figure 6 illustrates this. Although reducing n gives a smaller safe distance d_o, the comfort may be affected; Figure 7 shows a numerical plot of the maximum jerk values with respect to n, assuming $\max_{\forall t}\{\dot{x}^r_1(0) - \dot{x}_2(t)\} \leq V_{max}$, and $-B_{2max} \leq \ddot{x}_2(t) \leq A_{2max}$ in (24); where B_{2max} and A_{2max} are positive constants.

$$\begin{aligned}\dddot{x}^r_1 = -c\tilde{d}^n[\tfrac{c^2\tilde{d}^{2n+1}}{n+1} - c(\dot{x}^r_1(0) - \dot{x}_2)\tilde{d}^n - \ddot{x}_2] \\ -cn\tilde{d}^{n-1}(-\tfrac{c\tilde{d}^{n+1}}{n+1} + \dot{x}^r_1(0) - \dot{x}_2)^2\end{aligned} \tag{24}$$

Notice from figures 6 and 7, that $n = 1$ could be a suitable value.

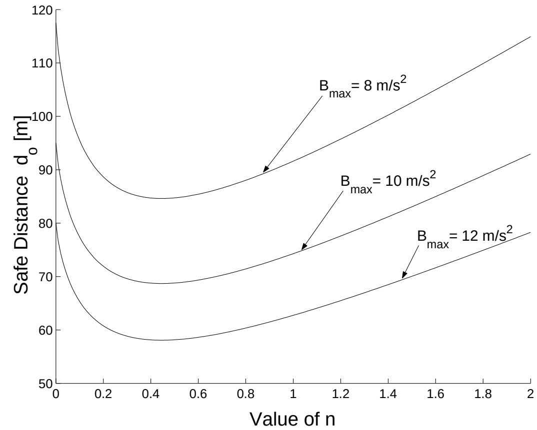

Fig. 6. Safe distance d_o with respect to n, for $V_{max} = 30m/s^2$.

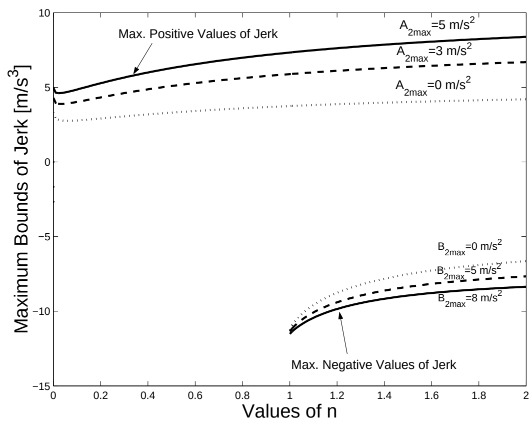

Fig. 7. Maximum jerk values with respect to n. For $n < 1$ jerk goes to minus infinite.

IV. STUDY CASE

To illustrate the behavior of the proposed inter-distance model, we have designed a profile that include car-following, hard-stop and stop-and-go scenarios.

The simulations have been done considering $V_{max} = 30m/s$, $B_{max} = 10m/s$, and $d_c = 5m$ (given $d_0 = 75m$ and $c = 0.0125$). Initial conditions are $x^r_1(0) = 0m$, $x_2(0) = 85m$, $\dot{x}^r_1(0) = 30m/s$, and $\dot{x}_2(0) = 20m/s$. The dotted lines in the figure 8 correspond to the curves produced by the simulated leader vehicle.

When the reference vehicle comes near to the leader vehicle, the velocity is adapted with comfortable deceleration and the reference vehicle is positioned to a safe distance; unexpectedly, at $t = 25s$ the leader vehicle is stopped with elevate braking value (approximately $10m/s^2$), while the reference vehicle obtains completed stop before critical distance $d_c = 5m$ with a braking smaller than $6m/s^2$.

After, the leader vehicle is accelerated and decelerated (stop-and-go) with usual acceleration values but elevate jerk; however, the reference vehicle is maintained to a safe distance, and a bounded jerk ($< 3m/s^3$).

Note how the vehicular inter-distance is adequate with respect to the different levels of velocity, and never reference vehicle goes into the red zone; Accelerations and/or braking have always moderated magnitudes in according to each situations.

Animation of simulations are provided on: http://www.lag.ensieg.inpg/canudas/

V. CONCLUSIONS

A new reference model for safe longitudinal control has been presented. The model provides dynamics solutions which a priori verify safety specifications with bounded acceleration and jerk. The model has few parameters that can be also set to account for external factors such as the road conditions as the traffic load. In addition, the proposed model produce smooth signals and it can be used in longitudinal control for highways and urban routes, specially, in stop-and-go scenarios.

As for future work, it will be interesting to analyze the *string stability* problem for platoons, using the proposed reference model.

VI. ACKNOWLEDGMENTS

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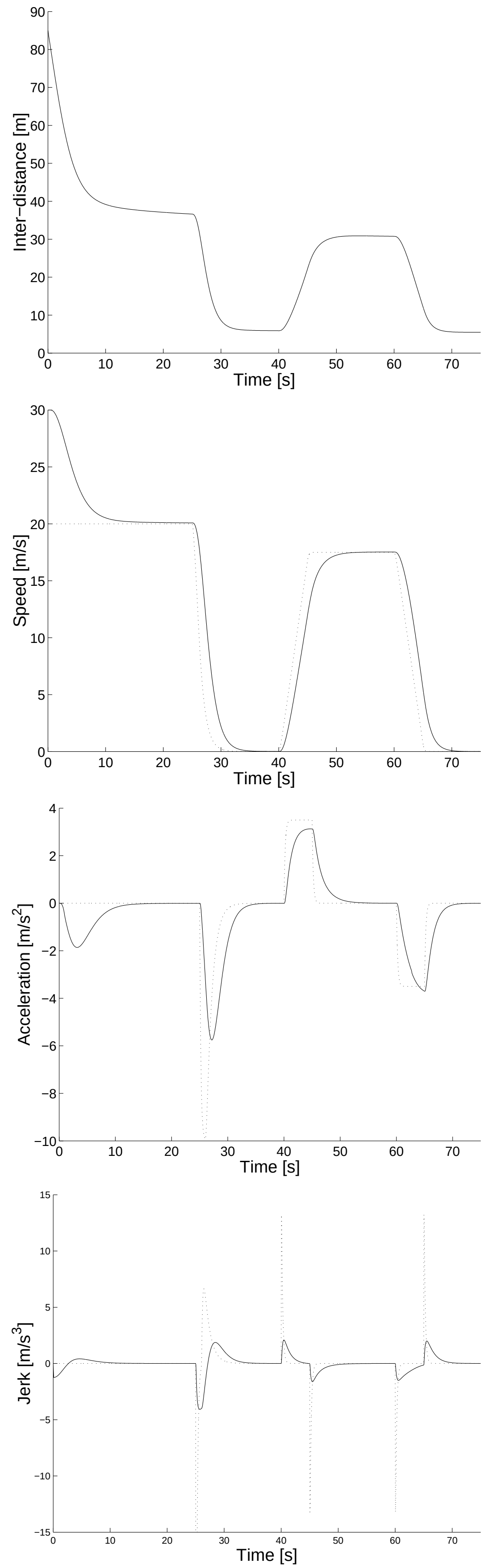

Fig. 8. Inter-distance, velocities, acceleration and jerk for a given leader profile.