Input Shaper Design in Convex Optimization Framework with Frequency Domain Constraints

Hong S. Bae
Department of Mechanical Engineering
Stanford University
Stanford, California 94305, USA
hongsang@stanford.edu

J. Christian Gerdes
Department of Mechanical Engineering
Stanford University
Stanford, California 94305, USA
gerdes@stanford.edu

***Abstract*— This paper demonstrates a method of introducing frequency domain constraints into the design of input shapers within a convex optimization framework. The magnitude bounds in the frequency domain are approximated to produce a set of linear constraints at discrete frequency points. The error bounds due to such approximation are easy to compute. This technique is applied to automated highway systems by including ride quality as an explicit frequency domain constraint.**

I. Introduction

Input shaping is a technique in which a reference command to a system is modified or shaped through convolution with an FIR filter (input shaper in Fig. 1). The purpose of this modification is to remove frequency content from the reference command that can produce oscillations in the closed-loop system. With properly chosen impulses, the effect can be very significant. In practice, input shaping has been used in areas where zero vibration of an object after a maneuver is required, for example, a reader arm in a hard disk drive or a cargo crane [9].

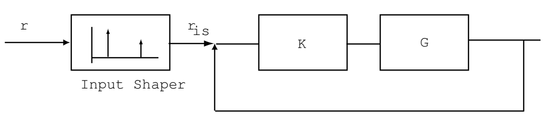

Fig. 1. Input shaping. Reference command is convolved with an input shaper.

Automated highway systems [13] are another interesting application that can benefit from input shaping. A platoon of automated vehicles can be thought of as a series of spring-mass-damper systems where vibration must be controlled for stable operation. For such systems, acceleration can be used for reference commands, but ideal signals must be modified to reflect performance limitations of a platoon of vehicles. A convex optimization approach to input shaper design that can capture time domain constraints such as engine saturation or maneuver end-point [8] is therefore quite useful. Previous work [1] has demonstrated that this approach can be successfully applied to automated highways.

These previous formulations for designing input shapers, however, have included only time domain constraints such as acceleration or position requirements. It is sometimes desirable to have constraints specified in the frequency domain, for instance limiting input shaper gain at high frequencies. In other cases, constraints can only be effectively expressed in the frequency domain. A good example in the context of automated highways is ride quality, which is strongly frequency dependent. This paper shows how such constraints can be added to the convex optimization framework.

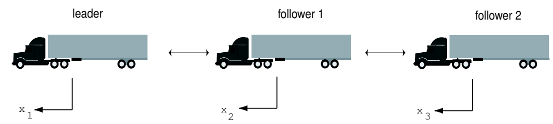

Fig. 2. A platoon of 3 vehicles.

This paper begins with a simple model for automated highway systems. Input shaping is then formulated as a convex optimization with time and frequency domain constraints. Finally, simulation results show how automated highway systems can benefit from smoother ride due to inclusion of ride quality constraint.

II. System Modeling and Implementation

An automated highway system of a platoon of three vehicles is used in this work [1]. Each vehicle in the platoon is modelled as a lumped mass, linear time invariant system with a first order actuator delay of 5 Hz. Vehicle parameters are assumed known with sufficient accuracy through parameter identification techniques [2]. The numerator is scaled for unity DC gain. In a transfer function form, the model for each vehicle is

$$G(s) = \mathcal{L}\left\{\frac{\ddot{x}(t)}{u(t)}\right\} = \frac{b}{s+b} \tag{1}$$

Each loop is closed with a simple constant gain controller. Controllers act on linear combinations of relative spacing errors and relative speeds with respect to the preceding vehicles. In other words, the leader (vehicle #1) is the only one that sees the reference command while the rest look only at current position and speed relative to the immediately preceding vehicle. In addition, no inter-vehicle communication is assumed. The gains are chosen for a string stable platoon. The state-space open loop model is described by

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \tag{2}$$

where the states, $x = [x_1\ v_1\ a_1\ x_2\ v_2\ a_2\ x_3\ v_3\ a_3]^T$, represent position, speed and acceleration of vehicle #1, #2 and

#3, respectively in Fig. 2. Output y is composed of all 9 states and 3 u's.

Closing the loops with control inputs for the leader

$$u_1 = a_r + K_1(V_r - \dot{x}_1) \quad (3)$$

and for second and third vehicles,

$$u_i = K_i\left[(\dot{x}_i - \dot{x}_{i-1}) + w_i\left(x_i - x_{i-1}\right)\right], \quad i = 2, 3 \quad (4)$$

the closed loop system is,

$$\begin{aligned} \dot{x} &= A_{cl}x + B_{cl}a_r \\ y &= C_{cl}x + D_{cl}a_r \end{aligned} \quad (5)$$

Vehicle parameters and controller gains are listed in Table I. Representing the system in the frequency domain,

Vehicle #	b	K	w
1	31.4	11	N/A
2	31.4	7	4
3	31.4	10	4

TABLE I

VEHICLE AND CONTROLLER PARAMETERS

transfer functions from reference acceleration, $a_{ref}(t)$, to control effort, $u(t)$, or vehicle acceleration, $\ddot{x}(t)$, are denoted respectively by,

$$\frac{U(s)}{A_{ref}(s)} \quad or \quad \frac{s^2X(s)}{A_{ref}(s)}$$

III. CONVEX OPTIMIZATION AND TIME DOMAIN CONSTRAINTS

A. Convex optimization framework

It has been shown that the objectives associated with input shaper design can be approximately reformulated as quasi-convex optimization problems in the discrete domain [12]. The input shaper design is then a special case of FIR filter design problem. In this framework, a filter with the minimum length and minimum number of non-zero impulses is designed to shape reference commands so that a system tracks the shaped reference command with little residual while satisfying performance constraints such as bounded control effort [8].

Defining an input shaper with N impulses (h_i) as,

$$H_N = \left[\begin{array}{cccc} h_0 & h_1 & \cdots & h_{N-1} \end{array}\right]^T \quad (6)$$

and an FIR filter form,

$$H(z) = \sum_{i=0}^{N-1} h_i z^{-i}, \quad (7)$$

the *modified* or *shaped* reference command for $k \leq N-1$ with the input shaper defined in (7) is

$$m_k = \left[\sum_{i=0}^{k} h_i\right] r \quad (8)$$

Discretizing the closed loop system (5) with T_s (a design variable) and propagating through time steps, the discrete system can be written in a notationally convenient form as

$$\begin{aligned} x_k &= Fx_{k-1} + Gm_{k-1} \\ &= \sum_{i=0}^{k-1}\left[\sum_{j=0}^{k-i-1} F^j G\right] h_i r \\ &= S_k H_N r \end{aligned} \quad (9)$$

where

$$S_k = \left[\sum_{j=0}^{k-1} F^j G \; \sum_{j=0}^{k-2} F^j G \; \cdots \; G \; \cdots \; 0_{N-k}\right] \quad (10)$$

B. Cost function

Performance goals and desired properties are formulated as linear constraints in the framework of convex optimization. In fact, this is simply linear programming since cost and constraint functions are linear.

In this work, the objective of an input shaper design is to find an FIR filter with the minimum number of non-zero impulses for a given N. This l_0-norm (non-convex) optimization is approximated with a weighted l_1-norm (convex) optimization. See [8] for details.

$$\min_{H_N} \; \|WH_N\|_1 \quad (11)$$

C. Constraint functions

A couple of the time domain objectives relevant to automated highways are listed below. Additional constraints can be found in [8], [1]. In the following, ϵ's are design variables that allow tight/loose control, depending on acceptable compromise between performance and cost. In addition, C_x extracts variables such as final acceleration (C_{af}) or control effort (C_u) from the state vector x.

1) Output constraint with small residual: Final states (accelerations) should be as close to the final desired states as possible, $x_N = X_{f,desired} = [a_1 \; a_2 \; a_3]^T_{final} = [1 \; 1 \; 1]^T$.

$$|\, x_N - X_f \,| = |\, C_{af} S_N H_N r - X_f \,| \leq \epsilon_x \quad (12)$$

2) Control magnitude constraint: Control effort should be bounded in order to avoid actuator saturation while reaching the final destination. This constraint is very useful in many applications where certain outputs would result in undesirable saturation and bounding control authority is crucial.

$$u_{min} \leq \; u_k = C_u S_k H_N r \; \leq u_{max} \quad (13)$$

IV. CONVEX OPTIMIZATION AND FREQUENCY DOMAIN CONSTRAINTS

It is well known that an input shaper with positive and negative impulses (as opposed to only positive impulses) may have magnitude amplification at high frequencies [6]. While this may be acceptable for real systems since most physical systems have a high frequency roll-off, a capability of frequency shaping is useful. This section summarizes how to incorporate frequency shaping when designing input shapers using convex optimization.

Expressing (7) in the frequency domain,

$$\begin{aligned} H(w) &= \sum_{i=0}^{N-1} h_i e^{-jw(iT_s)}, \quad j=\sqrt{-1} \\ &= \sum_{i=0}^{N-1} \{h_i \cos(iwT_s) - jh_i \sin(iwT_s)\} \end{aligned} \tag{14}$$

Some FIR filter design problems assume linear phase (symmetric impulses about the midpoint). This assumption simplifies the formulation of FIR filter design in a convex optimization framework since only real parts of (14) are used. However, input shapers in this work are not assumed to have a linear phase, which gives the input shaper more freedom to achieve the best performance.

Since the phase linearity is not assumed, manipulating the magnitude of the input shaper frequency response is essentially taking the magnitude of a series of complex numbers in (14). This makes formulating the filter magnitude shaping as a linear combination of h_i quite difficult. A number of techniques have been developed when the magnitude of a nonlinear phase FIR filter needs to be bounded in a form,

$$L(w) \le |H(w)| \le U(w), \tag{15}$$

such as reformulating the problem using power spectrum of $H(w)$ and retrieving impulse coefficients through spectral factorization [12]. However, most such formulations usually mean a nonlinear convex problem since they work with magnitude squared, instead of magnitude itself. Therefore, they cannot be added directly to the existing linear time domain constraints nor can such problems take advantage of the speed of a linear optimization solver [1].

However, methods for approximating these constraints in a linear form do exist [3], [11]. These methods, described in the following section, can be quite easily adapted to the input shaper problem.

A. Complex to real approximation

The magnitude of a complex number can be represented with a corresponding real number [4]. For any complex number $z = x + jy$,

$$|z| = \sqrt{x^2+y^2} = \max\{Re(z\eta) \mid \eta \in S\} \tag{16}$$

$$S = \{\eta \in \mathbf{C} \mid |\eta| = \mathbf{1}\} \tag{17}$$

Choosing $\eta = e^{j2\pi\theta}$ [3], for any complex number z,

$$|z| = \max_{\theta \in T}\{Re(ze^{j2\pi\theta})\} \tag{18}$$

$$T = \{\theta \mid -0.5 \le \theta \le 0.5\} \tag{19}$$

Suppose that an FIR filter with amplitude bounds over frequency points is desired. This is achieved by putting an upper bound on the magnitude of an FIR filter.

$$|H(w)| \le Z(w), \quad Z(w) \in \Re_+ \tag{20}$$

Since the frequency response of an FIR filter is a complex number at a given frequency, (20) can be written as

$$|H(w)| = \max_{\theta \in T}[Re\{H(w)e^{j2\pi\theta}\}] \le Z(w) \tag{21}$$

Solving (21) is a semi-infinite problem since the constraint is continuous. Discretizing the constraint by checking $2p$ points over the 2π radius range turns the problem into a semi-definite problem [3],

$$A(w) = \max_{\Delta_r}[Re\{H(w)e^{j\Delta_r}\}] \le Z(w) \tag{22}$$

$$\Delta_r = \frac{(r-1)}{2p}, \quad r = 1,2,\ldots,2p \quad (p \ge 2) \tag{23}$$

With the discretized constraints, the discretization error can be bounded as shown in [3], [10], giving

$$A(w) \le |H(w)| \le A(w)\sec\left(\frac{\pi}{2p}\right) \tag{24}$$

As p gets larger, the discretized constraints more closely resemble the continuous constraints. For example, $p = 4$ is used in this work giving an error magnitude bound of about 8% ($sec(\pi/2p) = 1.08$). However, p has to be chosen carefully as it adds complexity and cost in optimization process.

B. Frequency domain constraints in convex optimization framework

From (24),

$$\begin{aligned} |H(w_k)| &\le \\ & \max_{\Delta_r} Re\left\{ \sum_{i=0}^{N-1} h_i[\cos(w_k iT_s) - j\sin(w_k iT_s)] \cdot [\cos(\Delta_r) + j\sin(\Delta_r)] \right\} \sec\left(\frac{\pi}{2p}\right) \\ &= \max_{\Delta_r}\left[\sum_{i=0}^{N-1} h_i[\cos(w_k iT_s - \Delta_r)]\right]\sec\left(\frac{\pi}{2p}\right) \end{aligned} \tag{25}$$

where $k = 1, 2, \cdots, M$ is the number of frequency points for constraints to be matched at. As a rule of thumb, $M \approx 10N$ [12].

Note that the constraint in (25) is linear in h_i. Therefore, (25) and, hence, (21) at $w = w_k$ are satisfied, for all Δ_r if,

$$\sec\left(\frac{\pi}{2p}\right) C_k H_N \le Z(w_k)\mathbf{1} \tag{26}$$

where

$$\mathbf{1}^T = [1 \cdots 1]_{1 \times 2p}$$

$$C_k =$$

$$\begin{bmatrix} 1 & \cos(w_k T_s - \Delta_1) & \cdots & \cos(w_k(N-1)T_s - \Delta_1) \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \cos(w_k T_s - \Delta_{2p}) & \cdots & \cos(w_k(N-1)T_s - \Delta_{2p}) \end{bmatrix}_{2p \times N}$$

Then, for all frequency points, $w_k, \; k = 1, 2, \ldots, M$,

$$\sec\left(\frac{\pi}{2p}\right) \begin{bmatrix} C_1 \\ \vdots \\ C_k \\ \vdots \\ C_M \end{bmatrix} H_N \leq \begin{bmatrix} Z(w_1)\mathbf{1} \\ \vdots \\ Z(w_k)\mathbf{1} \\ \vdots \\ Z(w_M)\mathbf{1} \end{bmatrix} \quad (27)$$

Now, (27) can be easily added to the existing linear constraint matrices such as (12) and (13).

C. Frequency domain constraints for frequency shaping

In the previous section, frequency domain constraints have been added to modify the input shaper response in a desired manner. In other words, the magnitude of the input shaper can be forced to follow a certain frequency profile. The overall system response (plant as well as input shaper responses) can also be made to have a certain magnitude profile. A simple case would be the requirement that the magnitude of the overall system response be below one at all frequencies.

C_k in (27) represents the magnitude of the system at each frequency. The *system* is what we choose it to be: input shaper only, or overall system (input shaper and plant dynamics). If we change *gains* or put weights on C_k by introducing a value, we effectively change the magnitude of input shaper at each frequency point. In other words, instead of looking only at the magnitudes of input shaper frequency response, the overall system magnitude can be manipulated by working with $|Y_k|C_k$ which is the magnitude of input shaper frequency response, weighted by the magnitude of plant frequency response at frequency w_k. The new constraint in matrix form is then,

$$\begin{bmatrix} |Y_1|C_1 \\ \vdots \\ |Y_k|C_k \\ \vdots \\ |Y_M|C_M \end{bmatrix} H \leq \mathbf{1} \quad (28)$$

where, for example, to avoid actuator saturation in the automated highway system,

$$Y_k = \frac{U(w_k)}{A_{ref}(w_k)} \quad or \quad \frac{s^2 X(w_k)}{A_{ref}(w_k)}. \quad (29)$$

When $Y_k = U(s)/A_{ref}(s)$, the magnitude of the transfer function from reference acceleration ($a_{ref}(t)$) to control effort ($u(t)$) is a design objective. Similarly, the magnitude

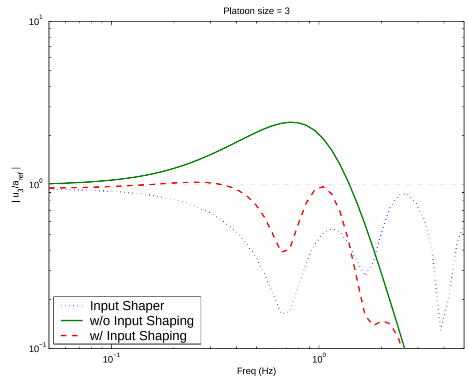

Fig. 3. $|U_3(s)/A_{ref}(s)|$ without (solid) and with (dashed) input shaping. A constant gain frequency constraint is used.

from reference acceleration to a vehicle acceleration is a constraint when $Y_k = s^2X(s)/A_{ref}(s)$ is used.

V. Simulation Results and Analysis

A platoon of three vehicles is analyzed with a step acceleration reference input. Transfer functions from acceleration reference input to control authorities, $U_i(s)/A_{ref}(s)$, are shown in solid in Fig. 3 and the subsequent frequency response plots. These transfer functions are normalized in the sense that maximum reference acceleration of the magnitude of one would produce maximum control authority of the same magnitude. Constraints on these transfer functions thus limit the control used in response to a given reference acceleration and can be employed to prevent saturation [1].

A. Unity gain constraint

Fig. 3 shows the input shaper response, $|H(s)|$, (dotted) and the overall system responses, $|U_3(s)/A_{ref}(s)|$, without (solid) and with (dashed) input shaping. The objective of the frequency constraint in the plot is to limit the magnitude of the overall system frequency response below one. The factor of $\sec(\pi/2p)$ in (27) is not used in this and the following simulations. Therefore, in Fig. 3, the magnitude of the overall system (dashed) matches within the magnitude error of 5%. Fig. 4 shows $|A_3(s)/A_{ref}(s)|$ that is identical to $|U_3(s)/A_{ref}(s)|$ since there are no additional dynamics between these responses.

The actuator responses are bounded in Fig. 5 by the frequency domain constraint, $|U_i(s)/A_{ref}(s)|$. This could also be accomplished with a time domain constraint [1]. In Fig. 5, the control effort (solid) is now bounded (dotted horizontal) with input shaping, compared to that without input shaping (dashed). Fig. 6 shows that the input shaper happens to have positive impulses only and the magnitude each impulse is below one (with 20 impulses and the sampling time of 0.1 sec).

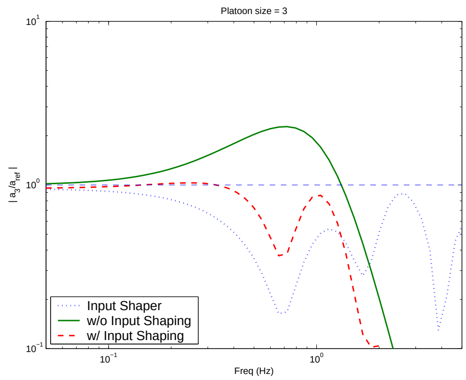

Fig. 4. $|A_3(s)/A_{ref}(s)|$ without (solid) and with (dashed) input shaping. A constant gain frequency constraint is used.

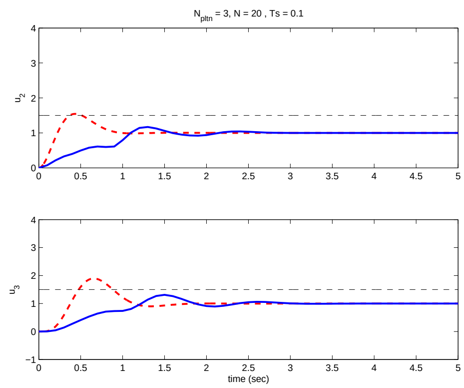

Fig. 5. Time responses of u_2 and u_3 before (dashed) and after (solid) input shaping with a constant gain frequency constraint.

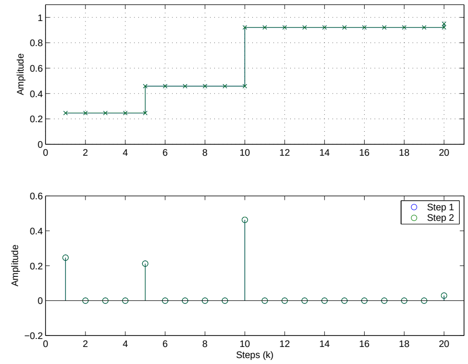

Fig. 6. Impulses with a constant gain frequency constraint.

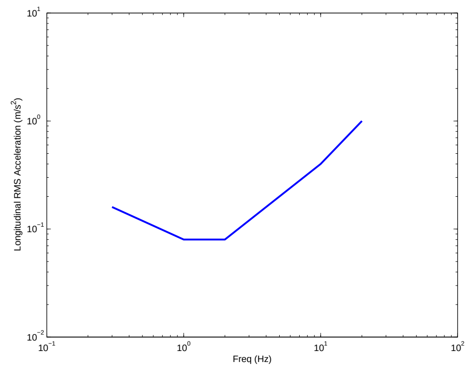

Fig. 7. Human tolerance limits for fore/aft vibrations.

B. Ride quality constraint

Ride quality or comfort is a subjective perception that is the cumulative effect of many factors such as seating position, interior volume, tactile inputs, duration of exposure and sound/visual vibration inputs. As a simple proxy, however, the human tolerance to fore/aft vibration can be used to give a measure of the impact of the truck acceleration on ride quality. Tolerance to vibration varies as a function of frequency, reflecting resonances in the torso in the 1-4 Hz range [7]. A sample curve representing human tolerance limits over frequency for fore/aft motion is shown in Fig. 7 [5].

This plot can be used to generate a weighting function in the form of a frequency domain constraint. For example, if the maximum reference acceleration for a heavy truck is 0.16g, the gain amplification between 1 Hz and 10 Hz should be less than 0.5 ($= 0.08g/0.16g$). Otherwise, the human tolerance limit would be violated (0.08g in that frequency range). Using the technique described in Section IV-C, the resulting frequency responses are shown along with the discretized frequency constraint for ride quality (thin dashed) in Fig. 8. Expressing such a constraint in time domain would be very hard and, therefore, formulation of such constraints in frequency domain is essential. Magnitude of the overall system response is now below the ride quality constraint within error bounds.

Actual impulses with the ride quality constraint are shown in Fig. 9. In addition, the actuator responses are bounded in Fig. 10 with input shaping. Compared to Fig. 5, the actuator responses are smoother with less overshoot or sudden variation which could be interpreted as better ride quality. Inclusion of the ride quality constraint has not increased the system response time since the input shaper length remains fixed.

VI. Conclusion

Constraints in previous input shaper design methods with convex optimization have been time domain based. When certain characteristics are desired that can only be represented in the frequency domain, such as ride quality in a vehicle, they can also be added to the input shaper design by approximate discretization. The usefulness of the frequency domain constraints in input shaper design in a convex optimization framework has been demonstrated with an automated highway system.

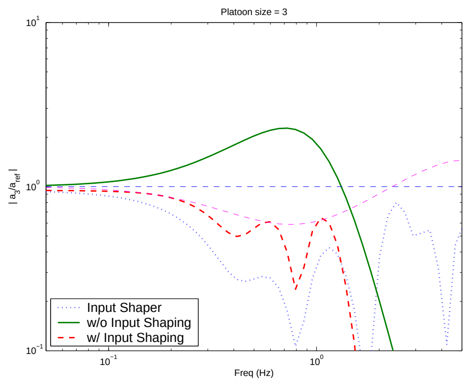

Fig. 8. $|A_3(s)/A_{ref}(s)|$ with ride quality constraint, before (solid) and after (dashed) input shaping. Thin dashed line shows ride quality constraint and dotted line shows input shaper itself.

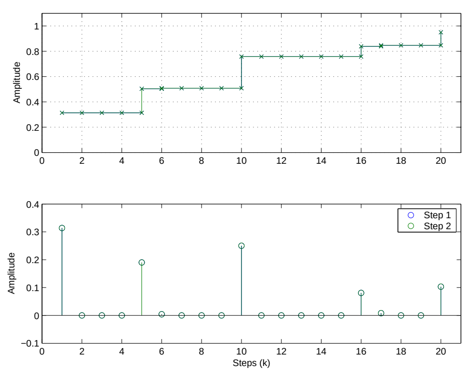

Fig. 9. Impulses with ride quality constraint.

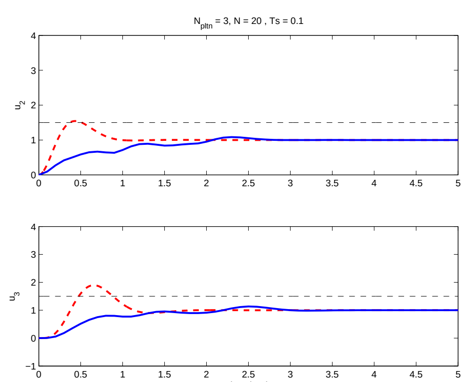

Fig. 10. Time responses of u_2 and u_3 before (dashed) and after (solid) input shaping with a frequency constraint.

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