Proceeding of the 2004 American Control Conference
Boston, Massachusetts June 30 - July 2, 2004



# Improved Bounded Real Lemma for Continuous-Time Stochastic Systems with Polytopic Uncertainties

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Abstract - By decoupling product terms between the Lyapunov matrix and system matrices, this paper proposes a parameter-dependent bounded real lemma (BRL) for continuous-time stochastic systems with polytopic uncertainties. This new criterion is shown, via a numerical example, to be much less conservative than previous result in the quadratic framework. In addition, the obtained result is further extended to cope with stochastic systems with time-varying state delay. All of the improved BRLs are expressed as strict linear matrix inequality (LMI) conditions, which can be easily tested by using standard numerical software.

## I. Introduction

The concept of quadratic stability [2] has been largely used in robust control theory, which greatly facilitates the stability test of uncertain linear systems. However, the analysis and synthesis based on the quadratic stability notion have been well recognized to be conservative, since it uses a single Lyapunov function to satisfy certain restrictions for the whole uncertain domain. Therefore, recently many researchers tried to use parameter-dependent Lyapunov functions to reduce the conservativeness of quadratic framework.

A breadthrough toward this direction was made in [3], where parameter-dependent stability was proposed for discete-time systems with polytopic uncertainties. This new stability eliminates product terms between the Lyapunov matrix and system matrices by the introduction of an additional slack matrix variable, which enables us to obtain parameter-dependent stability when used for polytopic uncertain systems. This idea was further extended in [4] to cope with analysis and synthesis problems with $H_2$ and $H_\infty$ performances. Subsequent works of similar idea have also been reported (see, for instance, [1], [6], [11], [12] and the references therein). It is worth mentioning that the realization of this idea is relatively easier for discrete-time systems, but for continuous-time systems, usually special techniques have to be employed.

In the present work, we make an attempt to investigate the parameter-dependent stability issue for continuous-time systems subject to stochastic pertubation. New linear matrix inequality (LMI) based bounded real lemma (BRL) is proposed, which also eliminates product terms between the Lyapunov matrix and system matrices. This new BRL is shown, via a numerical example, to be much less conservative than the result in the quadratic framework. In addition, the obtained result is further extended to cope with stochastic systems with time-varying state delay. All of the improved BRLs are expressed as strict LMI conditions, which can be easily tested by using standard numerical software.

The notations used throughout the paper are fairly standard. The superscript "$T$" stands for matrix transposition; $\mathbb{R}^n$ denotes the $n$-dimensional Euclidean space and the notation $P > 0$ means that $P$ is real symmetric and positive definite; $L_2[0,\infty)$ is the space of square-integrable vector functions over $[0,\infty)$; the notation $|\cdot|$ refers to the Euclidean vector norm. In symmetric block matrices or long matrix expressions, we use an asterisk $(*)$ to represent a term that is induced by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. In addition, $\mathbb{E}\{\cdot\}$ denotes the expectation operator.

## II. Main Result

Consider the following stochastic system $\mathcal{S}_c$:

$$\mathcal{S}_c: \qquad \begin{aligned} dx(t) &= [Ax(t)+B\omega(t)]\,dt \\ &\quad + [Mx(t)+N\omega(t)]\,dv(t) \\ z(t) &= Cx(t)+D\omega(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector; $z(t) \in \mathbb{R}^p$ is the objective signal to be attenuated; $\omega(t) \in \mathbb{R}^l$ is the disturbance input which belongs to $L_2[0,\infty)$; and $v(t)$ is a one-dimensional (1-D) Brownian motion satisfying $\mathbb{E}\{dv(t)\} = 0$, $\mathbb{E}\{dv(t)^2\} = dt$.

We first introduce the following definitions which will be essential for our derivation [8].

Definition 1: The stochastic system $\mathcal{S}_c$ in (1) with $\omega(t) = 0$ is said to be mean-square asymptotically stable if $\lim_{t\to\infty} \mathbb{E}\left\{|x(t)|^2\right\} = 0$ for any initial condition.

Definition 2: Given a scalar $\gamma > 0$, the stochastic system $\mathcal{S}_c$ in (1) is said to be mean-square asymptotically stable with an $H_\infty$ disturbance attenuation $\gamma$ if it is mean-square asymptotically stable and under zero initial condition, $\|z(t)\|_E < \gamma \|\omega(t)\|_2$ for all nonzero $\omega(t) \in L_2[0,\infty)$, where

$$\begin{aligned}\|z(t)\|_E &:= \sqrt{\mathbb{E}\left\{\int_0^\infty |x(t)|^2\right\}} \\ \|\omega(t)\|_2 &:= \sqrt{\int_0^\infty |\omega(t)|^2}\end{aligned}$$

Then, the standard BRL for the stochastic system $\mathcal{S}_c$ in (1) can be given as follows [8].

Lemma 1: The stochastic system $\mathcal{S}_c$ in (1) is mean-square asymptotically stable with an $H_\infty$ disturbance attenuation $\gamma$ if and only if there exists a $n \times n$ matrix $P > 0$ satisfying

$$\begin{bmatrix} -P & 0 & PM & PN \\ * & -I & C & D \\ * & * & PA + A^T P & PB \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0 \qquad (2)$$

It is worth noting that the condition (2) in Lemma 1 contains product terms between the Lyapunov matrix $P$ and system matrices $(A, B, M, N, C, D)$. When the BRL in Lemma 1 is extended to cope with stochastic system with polytopic uncertainties, that is, the system matrices $(A, B, M, N, C, D)$ are not exactly known but reside in a given polytope described by

$$\Omega := (A, B, M, N, C, D) \in \mathcal{R} \qquad (3)$$

where

$$\mathcal{R} := \left\{ \Omega(\lambda) \middle| \Omega(\lambda) = \sum_{i=1}^{s} \lambda_i \Omega_i; \sum_{i=1}^{s} \lambda_i = 1, \lambda_i \geq 0 \right\}$$

and $\Omega_i := (A_i, B_i, M_i, N_i, C_i, D_i)$ denotes the vertices of the polytope $\mathcal{R}$, we obtain the following corollary.

Corollary 1: The stochastic system $\mathcal{S}_c$ in (1) with polytopic uncertainties (3) is robustly mean-square asymptotically stable with an $H_\infty$ disturbance attenuation $\gamma$ if there exists a $n \times n$ matrix $P > 0$ satisfying (2) for $i = 1, \ldots, s$, where the matrices $A, B, M, N, C, D$ are taken with $A_i, B_i, M_i, N_i, C_i, D_i$ respectively.

Remark 1: Corollary 1 extend Lemma 1 to polytopic uncertain case. The polytopic uncertainty described by (3) has been widely used in the problems of robust control and filtering for uncertain systems (see, for instance, [10], [7] and the references therein), and many practical systems possess parameter uncertainties which can be either exactly modeled or overbounded by the polytope $\mathcal{R}$. As can be seen above, due to the product terms between the Lyapunov matrix $P$ and system matrices in (2), a BRL in the quadratic framework is obtained when extending Lemma 1 to polytopic uncertain case. In this extension, the Lyapunov matrix $P$ is set to be fixed, which is required to satisfy each vertex of the polytope $\mathcal{R}$. Following the work [1], [3], [12], we present an improved version of (2) in the following theorem.

Theorem 1: The stochastic system $\mathcal{S}_c$ in (1) is mean-square asymptotically stable with an $H_\infty$ disturbance attenuation $\gamma$ if and only if there exist $n \times n$ matrices $P > 0, F, G, V$ satisfying

$$\begin{bmatrix} P - V - V^T & 0 & V^T M & 0 & V^T N \\ * & -I & C & 0 & D \\ * & * & F^T A + A^T F & P - F^T + A^T G & F^T B \\ * & * & * & -G^T - G & G^T B \\ * & * & * & * & -\gamma^2 I \end{bmatrix} < 0 \qquad (4)$$

To prove the theorem, we need the following lemma (see, for instance, [5], [9]).

Lemma 2: Let $W = W^T \in \mathbb{R}^{n \times n}$, $\Psi \in \mathbb{R}^{n \times m}$ and $\Phi \in \mathbb{R}^{k \times n}$ be given matrices, and suppose rank$(\Psi) < n$, and rank$(\Phi) < n$. Consider the problem of finding some matrix $\mathcal{G}$ satisfying

$$W + \Psi \mathcal{G} \Phi + (\Psi \mathcal{G} \Phi)^T < 0 \qquad (5)$$

Then, (5) is solvable for $\mathcal{G}$ if and only if

$$\Psi^{\perp} W \Psi^{\perp T} < 0, \; \Phi^{T\perp} W \Phi^{T\perp T} < 0 \qquad (6)$$

Proof of Theorem 1. First by following similar lines as in the proof of Theorem 1 in [4], LMI (4) is equivalent to

$$\begin{bmatrix} -P & 0 & PM & 0 & PN \\ * & -I & C & 0 & D \\ * & * & F^T A + A^T F & P - F^T + A^T G & F^T B \\ * & * & * & -G^T - G & G^T B \\ * & * & * & * & -\gamma^2 I \end{bmatrix} < 0 \qquad (7)$$

Rewrite (7) in the form of (5), where

$$W = \begin{bmatrix} -P & 0 & PM & 0 & PN \\ 0 & -I & C & 0 & D \\ M^T P & C^T & 0 & P & 0 \\ 0 & 0 & P & 0 & 0 \\ N^T P & D^T & 0 & 0 & -\gamma^2 I \end{bmatrix}, \Psi = \begin{bmatrix} 0 \\ 0 \\ A^T \\ -I \\ B^T \end{bmatrix},$$

$$\Phi = \begin{bmatrix} 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \end{bmatrix}, \mathcal{G} = \begin{bmatrix} F & G \end{bmatrix}$$

Note that

$$\Psi^{\perp} = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & I & A^T & 0 \\ 0 & 0 & 0 & B^T & I \end{bmatrix}, \; \Phi^{T\perp} = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix}$$

Then, by using Lemma 2, (5) is solvable for $\mathcal{G}$ if and only if (2) holds, then the proof is completed. □

Remark 2: From the above proof, we can see that LMI (4) in Theorem 1 is actually equivalent to LMI (2). The advantage of LMI (4) lies in the fact that by introducing three additional slack variables $F, G, V$, it eliminates the product terms between the Lyapunov matrix $P$ and system matrices. This decoupling will enable us to obtain an improved BRL for stochastic systems with polytopic uncertainties, yielding the following corollary.

Corollary 2: The stochastic system $\mathcal{S}_c$ in (1) with polytopic uncertainties (3) is robustly mean-square asymptotically stable with an $H_\infty$ disturbance attenuation $\gamma$ if there exist $n \times n$ matrices $P_i > 0, F, G, V$ satisfying (4) for $i = 1, \ldots, s$, where the matrices $P, A, B, M, N, C, D$ are taken with $P_i, A_i, B_i, M_i, N_i, C_i, D_i$ respectively.

Remark 3: Corollary 2 can be called parameter-dependent BRL, since it requires different Lyapunov matrices $P_i$ for each vertex of the polytope $\mathcal{R}$. As is mentioned above, this idea stems from [3], following which many results have been proposed for different systems.

By following similar lines as in the proof of Theorem 1, the improved stability condition for stochastic systems can be given as follows.

Corollary 3: The following stochastic system

$$dx(t) = Ax(t)dt + Mx(t)dv(t) \tag{8}$$

is mean-square asymptotically stable if and only if there exist $n \times n$ matrices $P > 0, F, G, V$ satisfying

$$\begin{bmatrix} P - V - V^T & V^T M & 0 \\ * & F^T A + A^T F & P - F^T + A^T G \\ * & * & -G^T - G \end{bmatrix} < 0$$

In the following, we will present an illustrative example to show the less conservativeness of Corollary 2 than Corollary 1.

Example 1. Consider system $\mathcal{S}_c$ in (1) with the following matrices:

$$\begin{aligned} A &= \begin{bmatrix} 0 & 1 \\ -1+g & -1-g \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\ M &= \begin{bmatrix} 0 & 0.5+g \\ 0 & 0 \end{bmatrix}, N = \begin{bmatrix} 0.5-g \\ 0 \end{bmatrix}, \\ C &= \begin{bmatrix} 1 & -2 \end{bmatrix}, D = 0 \end{aligned}$$

where $g$ is an uncertain parameter that is known to reside in the interval $\begin{bmatrix} -g_1 & g_1 \end{bmatrix}$. Our purpose is to check the $H_\infty$ performance of the above uncertain stochastic system. The guaranteed minimum $\gamma^*$ obtained by applying Corollary 2 and Corollary 1 for different $g_1$ are listed in Table 1.

| different cases | $g_1 = 0$ | $g_1 = 0.4$ | $g_1 = 0.6$ |
|---|---|---|---|
| $\gamma^*$ by Corollary 2 | 3.8845 | 5.2791 | 6.4307 |
| $\gamma^*$ by Corollary 1 | 3.8871 | 8.1355 | 70.9889 |

Table 1. Calculation Results of Example 1

The obtained results in Table 1 show that the parameter-dependent approach (Corollary 2) is much less conservative than the quadratic approach (Corollary 1).

## III. Extension to Time-Delay Case

In this section, we extend the above results to time-delay case. Consider the following stochastic time-delay system $\mathcal{S}_d$:

$$\begin{aligned} \mathcal{S}_d : dx(t) &= [Ax(t) + A_d x(t-d(t)) + B\omega(t)]\, dt \\ &\quad + [Mx(t) + M_d x(t-d(t)) + N\omega(t)]\, dv(t) \\ z(t) &= Cx(t) + C_d x(t-d(t)) + D\omega(t) \end{aligned} \tag{9}$$

where $x(t)$, $z(t)$, $\omega(t)$ and $v(t)$ have the same meanings and dimensions as in the above section. $d(t)$ is a time-varying delay satisfying $0 < d(t) \leq \bar{d} < \infty$, $\dot{d}(t) \leq \tau < 1$, where $\bar{d}$ and $\tau$ are real constant scalars.

Then, the following lemma provides a BRL for the stochastic time-delay system $\mathcal{S}_d$ in (9) [13].

Lemma 3: The stochastic time-delay system $\mathcal{S}_d$ in (9) is mean-square asymptotically stable with an $H_\infty$ disturbance attenuation $\gamma$ if there exist $n \times n$ matrices $P > 0$ and $Q > 0$ satisfying

$$\begin{bmatrix} -P & 0 & PM & PM_d & PN \\ * & -I & C & C_d & D \\ * & * & Q + A^T P + PA & PA_d & PB \\ * & * & * & -(1-\tau)Q & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix} < 0 \tag{10}$$

By extending Lemma 3 to stochastic time-delay system with polytopic uncertainties, that is, the system matrices $(A, A_d, B, M, M_d, N, C, C_d, D)$ are not exactly known but reside in a given polytope described by

$$\Sigma := (A, A_d, B, M, M_d, N, C, C_d, D) \in \mathcal{R}_d \tag{11}$$

where

$$\mathcal{R}_d := \left\{ \Sigma(\lambda) \middle| \Sigma(\lambda) = \sum_{i=1}^{s} \lambda_i \Sigma_i; \sum_{i=1}^{s} \lambda_i = 1, \lambda_i \geq 0 \right\}$$

and $\Sigma_i := (A_i, A_{di}, B_i, M_i, M_{di}, N_i, C_i, C_{di}, D_i)$ denotes the vertices of the polytope $\mathcal{R}_d$, we obtain the following Lemma.

Corollary 4: The stochastic time-delay system $\mathcal{S}_d$ in (9) with polytopic uncertainties (11) is robustly mean-square asymptotically stable with an $H_\infty$ disturbance attenuation $\gamma$ if there exist $n \times n$ matrices $P > 0$ and $Q > 0$ satisfying (10) for $i = 1, \ldots, s$, where the matrices $A, A_d, B, M, M_d, N, C, C_d, D$ are taken with $A_i, A_{di}, B_i, M_i, M_{di}, N_i, C_i, C_{di}, D_i$ respectively.

Then, the following theorem presents an improved version of Lemma 3.

Theorem 2: The stochastic time-delay system $\mathcal{S}_d$ in (9) is mean-square asymptotically stable with an $H_\infty$ disturbance attenuation $\gamma$ if there exist $n \times n$ matrices

$P > 0, Q > 0, F, G, V$ satisfying

$$\left[\begin{array}{cccccc} P-V-V^T & 0 & V^TM & 0 & V^TM_d & V^TN \\ * & -I & C & 0 & C_d & D \\ * & * & F^TA+A^TF+Q & P-F^T+A^TG & F^TA_d & F^TB \\ * & * & * & -G^T-G & G^TA_d & G^TB \\ * & * & * & * & -(1-\tau)Q & 0 \\ * & * & * & * & * & -\gamma^2 I \end{array}\right] < 0 \quad (12)$$

Proof. First by following similar lines as in the proof of Theorem 1 in [4], LMI (12) is equivalent to

$$\left[\begin{array}{cccccc} -P & 0 & PM & 0 & PM_d & PN \\ * & -I & C & 0 & C_d & D \\ * & * & F^TA+A^TF+Q & P-F^T+A^TG & F^TA_d & F^TB \\ * & * & * & -G^T-G & G^TA_d & G^TB \\ * & * & * & * & -(1-\tau)Q & 0 \\ * & * & * & * & * & -\gamma^2 I \end{array}\right] < 0 \quad (13)$$

Rewrite (12) in the following form (5), where

$$W = \left[\begin{array}{cccccc} -P & 0 & PM & 0 & PM_d & PN \\ 0 & -I & C & 0 & C_d & D \\ M^TP & C^T & Q & P & 0 & 0 \\ 0 & 0 & P & 0 & 0 & 0 \\ M_d^TP & C_d^T & 0 & 0 & -(1-\tau)Q & 0 \\ N^TP & D^T & 0 & 0 & 0 & -\gamma^2 I \end{array}\right],$$

$$\Psi = \left[\begin{array}{c} 0 \\ 0 \\ A^T \\ -I \\ A_d^T \\ B^T \end{array}\right], \quad \Phi = \left[\begin{array}{cccccc} 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 \end{array}\right],$$

$$\mathcal{G} = \left[\begin{array}{cc} F & G \end{array}\right]$$

Note that

$$\Psi^{\perp} = \left[\begin{array}{cccccc} I & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & I & A^T & 0 & 0 \\ 0 & 0 & 0 & A_d^T & I & 0 \\ 0 & 0 & 0 & B^T & 0 & I \end{array}\right],$$

$$\Phi^{T\perp} = \left[\begin{array}{cccccc} I & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & I \end{array}\right]$$

Then, by using Lemma 2, (5) is solvable for $\mathcal{G}$ if and only if (10) holds, then the proof is completed. □

Then we readily have the following corollary.

Corollary 5: The stochastic time-delay system $\mathcal{S}_d$ in (9) with polytopic uncertainties (11) is robustly mean-square asymptotically stable with an $H_\infty$ disturbance attenuation $\gamma$ if there exist $n \times n$ matrices $P_i > 0$, $Q_i > 0$, $F, G, V$ satisfying (12) for $i = 1, \ldots, s$, where the matrices $P$, $Q$, $A$, $A_d$, $B$, $M$, $M_d$, $N$, $C$, $C_d$, $D$ are taken with $P_i$, $Q_i$, $A_i$, $A_{di}$, $B_i$, $M_i$, $M_{di}$, $N_i$, $C_i$, $C_{di}$, $D_i$ respectively.

Example 2. Consider system $\mathcal{S}_d$ in (9) with the following matrices:

$$A = \left[\begin{array}{cc} 0 & 3+0.5\rho \\ -4 & -5 \end{array}\right], \quad A_d = \left[\begin{array}{cc} -0.1 & 0 \\ 0.2 & -0.2+0.3\sigma \end{array}\right],$$

$$B = \left[\begin{array}{c} -0.5 \\ 1 \end{array}\right], \quad N = \left[\begin{array}{c} 0.5+0.2\sigma \\ 1 \end{array}\right],$$

$$M = M_d = \left[\begin{array}{cc} 0.5 & 0 \\ 0 & 0.5+0.2\rho \end{array}\right],$$

$$C = \left[\begin{array}{cc} 1 & 2 \end{array}\right], \quad C_d = \left[\begin{array}{cc} 0.5 & 0.3 \end{array}\right], \quad D = 1$$

where $\rho$ and $\sigma$ are uncertain real parameters satisfying $|\rho| \le \rho_1$, $|\sigma| \le \sigma_1$. Our purpose is to check the $H_\infty$ performance of the above uncertain stochastic system. The guaranteed minimum $\gamma^*$ obtained by applying Corollary 5 and Corollary 4 for different $\rho_1$ and $\sigma_1$ are listed in Table 2.

| different cases | $\rho_1 = \sigma_1 = 2$ | $\rho_1 = \sigma_1 = 2.5$ |
|---|---|---|
| $\gamma^*$ by Corollary 5 | 6.0669 | 8.5766 |
| $\gamma^*$ by Corollary 4 | 8.6465 | 69.1348 |

Table 2. Calculation Results of Example 2

The obtained results in Table 2 show again that the parameter-dependent approach (Corollary 5) is much less conservative than the quadratic approach (Corollary 4).

## IV. Concluding Remarks

This note investigates the parameter-dependent stability issue for stochastic systems with polytopic uncertainties. By decoupling product terms between the Lyapunov matrix and system matrices, improved bounded real lemma is proposed, which is shown, via a numerical example, to be less conservative than the result in the quadratic framework. In addition, the obtained result is further extended to cope with stochastic time-delay systems. All of the improved BRLs are expressed as strict LMI conditions, which can be easily tested by using standard numerical software.

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