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Dynamic Modeling and Sliding Mode Control of a Five-Link Biped during the Double Support Phase

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Abstract – In this paper, an improved approach to the dynamic modeling of a five-link biped and a sliding mode control algorithm for motion regulation are developed during the double support phase (DSP). By modifying the conventional definition of certain physical parameters of the biped system, it is shown that the procedure of the derivation of the dynamic equations and their final forms are significantly simplified. The dynamic model of the five-link biped during the DSP is first formulated as the motion of robot system under holonomic constraints, and then, the horizontal and vertical displacements of the hip and the orientation of the trunk are selected as independent generalized coordinates to describe the constraint system and to eliminate the constraint forces from the equation of motion. Based on the presented dynamic formulation, we develop a sliding mode controller for biped motion regulation. The stability and the robustness of the controller are investigated. The control scheme is evaluated by computer simulations. To the best of our knowledge, it is the first time that a robust sliding mode controller is developed for biped walking during the DSP. This work makes it possible to provide robust sliding mode control to a full range of biped walking and yield dexterity and versatility for performing specific gait patterns.

I. INTRODUCTION

The development of legged locomotion systems has recently received increased attention due to their higher mobility than conventional wheeled vehicles. A vast literature has contributed to biped locomotion on dynamic modeling and motion regulation. A general five-link biped model has been found in much literature [1-3,9-14] since such a model has the advantages that it has sufficiently low DOFs to establish the mathematical model yet still has enough DOFs to adequately describe the motion. However, we noticed that the procedure of the dynamic modeling and the detailed forms of the biped model are more complex and less structured [1,3,6,9] as compared with those of robot manipulators. In addition, most related papers concentrated on dynamic modeling and control of biped walking with only a single support phase (SSP) [1,2]. A double support phase (DSP) is often neglected or assumed to be instantaneous [3-5]. Motion of a biped robot with the DSP has the advantages in that it is more convenient to realize the stable motion and can fulfill more tasks than that only walking with the SSP. However, dynamic modeling of biped walking during the DSP is more challenging due to the involvement of the constraints and the constraint forces. Thus, an improved approach to the dynamic modeling for the five-link biped is needed.

In addition, it is difficult to control biped walking with the DSP. One difficulty lies in the involvement of the constraints in the highly nonlinear biped dynamic system. Motion of a biped robot during the DSP can be described as the motion of dynamic system under holonomic constraints with the condition that the contact points between the feet and the ground are fixed. Constraints introduce two difficulties in the solution of mechanical problems. One is that the coordinates are no longer independent. Another difficulty is that the constraint forces are not provided a prior but they are among the unknowns of the system which must be obtained from the solution we seek. As a result, controlling such constraint systems becomes challenging due to the complexity of the dynamics. Many published papers were found in the discussion of the control algorithms for constraint systems with low DOFs only [6-8].

Mitobe et al. [9] discussed the motion of a 4-DOF biped robot during the DSP and applied the computed torque control algorithm in the system. In their work, the position of center of gravity of the trunk were formulated as the reduced space independent generalized coordinates, and the control problem was defined as a trajectory tracking problem which the body of the biped was controlled to track a desired trajectory. In their controller, the desired constraint forces were designed as control input for controlling the constraint forces.

Sonoda et al. [10] introduced an approach to a 4-DOF biped control utilized redundancy in the DSP. They considered the biped robot during the DSP as a redundant manipulator, and set a general acceleration reference formulation to each joint given by a function with a null space vector (performance function), which decided robot's configuration. By choosing the performance function of the null space input in compliance with the aim of desired task, control of various configurations of the robot can be realized. The advantage of this strategy is that once the performance function for the desired task was applied to the designed reference formula, the configuration control can be realized. However, it is not always feasible to formulate performance functions for practical tasks.

Another difficulty to control biped walking is the inaccuracy of the parameters involved in the biped model. Sliding mode control as one of the most often used robust control techniques has been applied in biped walking with

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only the SSP [1,11], to remove the effect of poor performance due to modeling uncertainties. Chang and Hurmuzlu [11] has developed a sliding mode control law without reaching phase and applied it to a 5-link biped during the SSP. Tzafestas et al. [1] developed a robust sliding mode controller applied to a 5-link biped during the SSP and compared it to computed torque control. They both [1,11] proved that sliding mode control has good performance and is insensitive to parameter variations. However, it is more difficult when applying sliding mode control to a biped with the DSP than that of the SSP and no literature has been found on sliding mode control to the DSP. One of the reasons is that the angular displacements as the control variables are not independent. Another reason may be the complexity due to the constraint forces involved in such high-DOF nonlinear system together with the system parametric uncertainties.

In this paper, an improved approach to the dynamic modeling of a 5-link biped system is proposed and a sliding mode control algorithm is developed for biped motion regulation during the DSP, provided that the boundaries of the system uncertainties are known. In dynamic modeling, by modifying the conventional definition of certain physical parameters of the biped system, the procedure of the derivation of the dynamic equations and their final forms are significantly simplified. The dynamic model of the five-link biped during the DSP is first formulated as motion of robot system under holonomic constraints, and the hip position and the trunk orientation are then selected as independent generalized coordinates to describe the constraint system and to eliminate the constraint forces from the equation of motion. Based on the presented dynamic formulation, the sliding mode control algorithm is developed to this model with the desired joint angle profiles taken from [12]. The stability and the robustness of the controller in the DSP are analysed in this paper. Simulations are carried out to demonstrate the effectiveness of the control algorithm.

II. MODELING OF BIPED DURING THE DSP

In this part, the dynamic model of five-link biped walking during the DSP is presented. The biped structure is taken from Hurmuzlu's work [13] which has five links—a torso and two identical lower limbs with each limb having a thigh and a shank. Also the biped has two pelves at the hip, two knees between the thighs and the shanks, and two ankles at the tips of the two limbs. There is an actuator located at each joint and all of the joints are considered only rotating in the sagittal plane and friction free. We assume massless feet in the model. Although we neglect the dynamics of the feet, we assume that the biped can apply torques at the ankles. For the DSP, an actuator torque is applied at the leading ankle. The rear ankle does not possess a torque but can rotate by the effect of gravity. Figure 1 depicts a schematic representation of this biped model.

In this paper, our kinematic model of the biped system is different from those found in previous literature in that the biped system is considered as a successive open loop of kinematic chain from the supporting point to the free ends. The value of d_i indicating the location of the mass centre of each link i is measured as the distance between the mass centre of link i to joint i (which is the joint of each link i connecting to the previous link i-1, refer to figure 1), and θ_i is also denoted as the angle of each link i with respect to the vertical axis through joint i counter clockwise. While in previous literature on dynamic modeling of bipedal walking [1,3,14], d_i and θ_i have been both defined at its lower joint. Using our denotation, the dynamic models can be derived more conveniently and the final forms of the dynamic equations are significantly simplified as comparing with the above mentioned published works (refer to Appendix). Consequently, this modified kinematic model has greatly improved the efficiency of the derivation procedure and also made the computational programming much easier and less prone to mistakes.

In the DSP, since the contact position between the tips of the limbs and the ground is fixed, there exists a set of holonomic constraint equations:

$$\Phi(\theta) = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} x_e - x_b - L \\ y_e - y_b \end{bmatrix} = 0 \tag{1}$$

The dynamic model of a biped during the DSP can be expressed by:

$$\begin{cases} D(\theta)\ddot{\theta} + H(\theta,\dot{\theta})\dot{\theta} + G(\theta) = J^T(\theta)\lambda + T & \text{(2a)} \\ \Phi(\theta) = 0 & \text{(2b)} \end{cases}$$

where θ, $\dot{\theta}$, $\ddot{\theta}$, $T \in R^5$ denote the generalized coordinates, velocities, accelerations and torques, respectively. $D(\theta) \in R^{5\times 5}$ represents the positive definite and symmetric inertia matrix, $H(\theta,\dot{\theta}) \in R^{5\times 5}$ represents the centrifugal-Coriolis matrix, $G(\theta) \in R^5$ represents the gravity effects. λ is a 2×1 vector of Lagrange multipliers, and $J \in R^{2\times 5}$ is the Jacobian matrix: $J = \frac{\partial f_i}{\partial \theta_j}$. For simplification, we denote $\mathrm{H} = H(\theta,\dot{\theta})\dot{\theta} + G(\theta)$ hereafter. The detailed forms of D, H, G can be found in (A1-A3) in Appendix. The conciseness of these forms in (A1-A3) show the advantages of our modified kinematic model by comparison with other works [1,3,9].

According to Goldstein [15], for a dynamic system under holonomic constraints, it is possible to introduce a set of independent generalized coordinates to formulate the dynamic equations which can describe the constraint system without using the terms of constraint forces. Mitobe *et al*. [9] introduced the coordinates of the point at the bottom of the trunk as the generalized coordinates for their 4-DOF biped DSP model. For our 5 link biped model, the motion of the system in the DSP can be fully described by the hip and trunk motion and the set of independent generalized coordinates should be chosen in R^3. Here we select the hip position and the trunk orientation as the independent generalized coordinates,

$$p \in R^{3\times1} : p = \begin{pmatrix} x_h & y_h & \theta_3 \end{pmatrix}^T$$

where $(x_h, \ y_h)$ is the hip coordinate. The transformation from (θ_i) set to (p_j) set can be written as

$$p(\theta) = \begin{bmatrix} l_1 \sin\theta_1 + l_2 \sin\theta_2 \\ l_1 \cos\theta_1 + l_2 \cos\theta_2 \\ \theta_3 \end{bmatrix} \tag{3}$$

Furthermore, in order to use the independent coordinates to establish the dynamic model, we need to transform the (p_j) set to (θ_i) set. Firstly, by differentiating (3) twice with respect to time, yields

$$\dot{p} = R\dot{\theta} \tag{4}$$

$$\ddot{p} = \dot{R}\dot{\theta} + R\ddot{\theta} \tag{5}$$

where $R = \dfrac{\partial p}{\partial \theta} \in R^{3\times5}$. Moreover, in order to apply the constraint conditions in the dynamic derivation, (1) is differentiated twice with respect to time, to yield

$$\dot{\Phi} = J(\theta)\dot{\theta} = 0 \tag{6}$$

$$\ddot{\Phi} = \dot{J}(\theta)\dot{\theta} + J(\theta)\ddot{\theta} = 0 \tag{7}$$

Combining (4), (5) and (6), (7), we have

$$\begin{bmatrix} R \\ J \end{bmatrix}\dot{\theta} = \begin{bmatrix} \dot{p} \\ 0 \end{bmatrix} \tag{8}$$

$$\begin{bmatrix} R \\ J \end{bmatrix}\ddot{\theta} = \begin{bmatrix} \ddot{p} \\ 0 \end{bmatrix} - \begin{bmatrix} \dot{R} \\ \dot{J} \end{bmatrix}\dot{\theta} \tag{9}$$

It is obvious that $\begin{bmatrix} R \\ J \end{bmatrix} \in R^{5\times5}$ is a full-ranked matrix, thus is invertible. Since we have transformed the (p_j) set to (θ_i) set, we can rewrite the dynamic equation with (p_j) instead of (θ_i). Combining (8)-(9) and (2a), we have

$$\begin{bmatrix} \ddot{p} \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{R} \\ \dot{J} \end{bmatrix}\begin{bmatrix} R \\ J \end{bmatrix}^{-1}\begin{bmatrix} \dot{p} \\ 0 \end{bmatrix} + \begin{bmatrix} R \\ J \end{bmatrix}D^{-1}(T - \mathrm{H}) + \begin{bmatrix} R \\ J \end{bmatrix}D^{-1}J^T\lambda \tag{10}$$

which can be symbolically written as

$$\begin{bmatrix} \ddot{p} \\ 0 \end{bmatrix} = S_a\begin{bmatrix} \dot{p} \\ 0 \end{bmatrix} + S_b(T - \mathrm{H}) + S_c\lambda \tag{11}$$

where $S_a = \begin{bmatrix} S_{a11,3\times3} & S_{a12,3\times2} \\ S_{a21,2\times3} & S_{a21,2\times2} \end{bmatrix} = \begin{bmatrix} \dot{R}\begin{bmatrix} R \\ J \end{bmatrix}^{-1} \\ \dot{J}\begin{bmatrix} R \\ J \end{bmatrix}^{-1} \end{bmatrix}$,

$S_b = \begin{bmatrix} S_{b1,3\times5} \\ S_{b2,2\times5} \end{bmatrix} = \begin{bmatrix} RD^{-1} \\ JD^{-1} \end{bmatrix}$, $S_c = \begin{bmatrix} S_{c1,3\times2} \\ S_{c2,2\times2} \end{bmatrix} = \begin{bmatrix} RD^{-1}J^T \\ JD^{-1}J^T \end{bmatrix}$.

Note that S_{c2} is an invertible matrix. Thus, (11) can be expressed as

$$\begin{cases} \ddot{p} = S_{a11}\dot{p} + S_{b1}(T - \mathrm{H}) + S_{c1}\lambda & \text{(12a)} \\ 0 = S_{a21}\dot{p} + S_{b2}(T - \mathrm{H}) + S_{c2}\lambda & \text{(12b)} \end{cases}$$

The difference between (12) and (2) is that in (12b) the constraint forces are separated from the second order derivatives of the state vector. Thus they can be first solved from (12b) and then substituted into (12a), equation (12) becomes

$$\ddot{p} = B\dot{p} + C(T - \mathrm{H}) \tag{13}$$

where $B = S_{a11} - S_{c1}S_{c2}^{-1}S_{a21}$, $C = S_{b1} - S_{c1}S_{c2}^{-1}S_{b2}$.

Equation (13) will be used for a sliding mode controller design in next section.

III. SLIDING MODE CONTROLLER DESIGN

In (13), we have derived the dynamic equations for biped DSP without the constraint forces. This was under the assumption that the constraints, shown in (1), is always satisfied, i.e., both feet of the biped maintain firm contact on the ground without slipping and lifting during the DSP. However, this condition is not guaranteed a prior, because the nature of the contact depends on the constraint forces, which in turn vary depending on the motion of the biped robot [9]. Thus, in some work dealing with constrained systems [7-9], force/motion control algorithm is employed such that the constraint forces are maintained by force control and the constrained system keeps its prescribed motion with motion control. In this case, desired trajectories of both forces and motion need to be designed. Our control objective in this work is to provide stable motion for the biped robot during the DSP as far as the constraints are not violated when tracking some desired trajectories. Equation (13) is thus used for controlling the five-link biped during the DSP in this section. Conditions for satisfying the contact constraints will also be determined.

For the system described by (13), a sliding mode control algorithm can be developed to track the desired motion, maintain the constraint conditions and eliminate the effect of system parametric uncertainties. First, the following assumptions are made before establishing the control law:

1) The bounds of the parameter uncertainties of the biped system are known.
2) The state space variables are available for measurement.

'The parameters with uncertainties' are those physical parameters of the biped system, including *m*, *l*, *I* and *d*. We use the symbols φ_i as each physical parameters (*m*, *l*, *I*, *d*), $E_i \times 100\%$ ($0 \le E_i < 1$) as their uncertainties in the upper bounds, $\hat{\varphi}_i = (1 + E_i)\varphi_i$ as the corresponding estimated value of each parameter. Also, we use the symbol M_i as the matrices (*D*, *H*, G, *J*, etc) in (2), then, the estimated matrices are obtained by:

$$\hat{M}_i = \hat{M}_i(\hat{\varphi}_i) = M_i(\varphi_i) + \Delta M_i(E_i\varphi_i)$$

In our second order differential system described by (13), a time-varying sliding surface S is defined as $s(p,t)=0$, where

$$s=\dot{e}+2\Lambda e+\Lambda^2\int_0^t e(\tau)d\tau \quad (14)$$

where Λ is a 3×3 diagonal matrix of positive gains, e and $\dot{e}$ are the vectors of tracking errors.

By introducing the PD controller

$$u=\ddot{p}_d-K_d\dot{e}-K_p e \quad (15)$$

and the equivalent controller

$$\hat{T}=\hat{C}^-u-\hat{C}^-\hat{B}\dot{p}+\hat{\mathrm{H}} \quad (16)$$

the sliding mode controller is designed as

$$T_i=\hat{T}_i-\sum_{j=1}^{3}\left\{\hat{C}_{ij}^-\left[\mathrm{sgn}(s_j)\sum_{k=1}^{5}\left(\left|\hat{C}_{jk}\right|\eta_k+\sigma_k\right)\right]\right\}, i=1,2...5 \quad (17)$$

where $\eta\in R^3$ represents the system uncertainty bounds; $\hat{C}^-=\hat{C}^T(\hat{C}\hat{C}^T)^{-1}$ represents the pseudo-inverse matrix of $\hat{C}$ as $\hat{C}$ is not square.

In general, control law (17) is associated with a chattering problem due to the discontinuous switching function *sgn*(s) in the controller. It is undesirable in practice and should be eliminated. Slotine and Sastry [16] suggested a solution to this problem by replacing *sgn*(s) with a saturation function in a thin boundary layer neighboring the switching surface. Once the system trajectories enter the boundary layer, they will remain inside. Cai and Song [17] further discussed the control law by using a hyperbolic function *tanh*(s) which has the advantage of being smooth and it is adapted to our work:

$$T_i=\hat{T}_i-\sum_{j=1}^{3}\left\{\hat{C}_{ij}^-\left[\tanh(\alpha_j s_j)\sum_{k=1}^{5}\left(\left|\hat{C}_{jk}\right|\eta_k+\sigma_k\right)\right]\right\} \quad (18)$$

where $\alpha\in R^3$ and $\sigma\in R^5$ are constant vectors.

The constants of the entries in α can be determined by a sliding condition analysis through its necessary criteria for convergence. The quadratic Lyapunov function is considered as $V=\frac{1}{2}s^T s>0$, and the derivative of V along the solution trajectory of (13) is given by

$$\begin{aligned}\dot{V}&=s^T\dot{s}\\&=s^T(\ddot{p}-u)\\&=s^T(CT+B\dot{p}-C\mathrm{H}-u)\\&=s^T[C(\Delta_D-\hat{C}^-\tanh(\alpha s)(abs(\hat{C})\eta_D+\sigma))]\end{aligned} \quad (19)$$

here $\left[abs(\hat{C})\right]_{ij}=\left|\hat{C}_{ij}\right|$ denotes a 3×5 matrix, each entry being the absolute value of that of matrix $\hat{C}$.

For the parameter uncertainties less than 100%, we have $0<K_{\min}<\sum_{k=1}^{5}C_{ik}\hat{C}_{kj}^-<K_{\max}$. In order to guarantee $\dot{V}<0$, the following condition must be held

$$\tanh(\alpha_i|s_i|)>\frac{\sum_{k=1}^{5}\left|\hat{C}_{ik}\eta_{Dk}\right|}{k_{\min}\left(\sum_{k=1}^{5}\left|\hat{C}_{ik}\right|\eta_{Dk}+\sigma_i\right)}\underline{\underline{\Delta}}\nu_i\,;\ i=1,2,3 \quad (20)$$

which leads to

$$\alpha_i>\frac{1}{2\varepsilon_i}\ln\left(\frac{1+\nu_i}{1-\nu_i}\right) \quad (21)$$

When the width of the boundary layer ε_i is selected, α_i can be chosen according to (21).

IV. SIMULATION RESULTS

In this section, the five-link biped walking in the saggital plane is simulated to demonstrate the effectiveness of the control algorithm. The physical parameters of the biped model used in our simulation can be found in Table 1. The reference trajectory is taken from [12] with the average walking speed=1m/s, step length=0.68m, SSP time period=0.6s; DSP time period=0.1s. System parameter uncertainties are introduced as inertia terms within 40%, dimensions within 10%. Figure 2 shows the tracking errors of the horizontal and the vertical displacements at the hip and the angular displacement of the trunk. Figure 3 shows the actuate torques for the joints of one step in the DSP. Figure 4 shows the constraint forces between both tips and the ground. These results show that the control objective is achieved successfully in both ensuring the robustness against parameter uncertainties and guaranteeing the tracking accuracy. In addition, the constraint conditions are satisfied provided that the friction coefficient between the biped lower limb tips and the ground is greater than 0.4, which is not an overly restricted condition. Figure 5 shows the stick diagram of the biped walking for one full step. It shows that a stable gait can be produced by the proposed controller.

Table 1. Parameters of the biped robot

Link	m_i (kg)	I_i (kgm^2)	l_i (m)	d_i (m)
Torso	14.79	3.30×10^{-2}	0.486	0.282
Thigh	5.28	3.30×10^{-2}	0.302	0.236
Leg	2.23	3.30×10^{-2}	0.332	0.189

V. CONCLUSIONS

An approach to modeling and control of five-link biped walking with the DSP has been presented in this paper. By modifying the conventional definition of certain physical parameters of the biped system, the procedure of the derivation of the dynamic equations and the final forms of the equations are significantly simplified. It has greatly improved the efficiency of the derivation procedure and also made the programming much easier and is less prone to mistakes. The biped DSP model is formulated as motion of robot system under holonomic constraints and the hip and trunk motion is selected as a set of independent generalized coordinates to describe such a biped system. Based on the dynamic model and proposed trajectory [12], a control algorithm is developed to track the biped hip and trunk motion without force control and thus the control design is simplified by avoiding a separate force control law. The sliding mode controller is designed for the system in the presence of parameter uncertainties. To eliminate the chattering problem during the implementation of sliding mode control, the continuous control law is used to approximate the discontinuous controller. The simulation results show that in spite of the presence of system parameter uncertainties, the controller has a good performance in tracking reference signals. The results also show that the constraint condition can be satisfied with a moderate friction coefficient of contact surface. The control algorithm which allows the actuator joint torques to track Cartesian coordinates by introducing a non-squared transformation matrix has the potential to allow people having the freedom to distribute the joint torques, especially when some specific bounds for certain joint torques are imposed. The significance of this paper is to form a solid basis for the studies of a full range of motion planning and control of bipedal walking.

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APPENDIX: the detailed forms of (1)

The detailed forms of $D(\theta)$, $H(\theta,\dot{\theta})$, $G(\theta)$ in (1) are presented below:

$$D = [D]_{ij} = p_{ij}\cos(\theta_i - \theta_j) \tag{A1}$$

$$\underline{H} = [H]_{ij} = p_{ij}\sin(\theta_i - \theta_j)\dot{\theta}_j \tag{A2}$$

$$G = [G]_i = g_i \sin\theta_i \tag{A3}$$

where $i, j=1,2\ldots5$, p_{ij} and g_i are inertial terms defined as

$$p_{ij} = \begin{cases} I_i + m_i d_i^2 + a_i\left(\sum_{k=i+1}^{5} m_k\right) l_i^2 & j = i \\ a_i m_j d_j l_i + a_i a_j\left(\sum_{k=j+1}^{5} m_k\right) l_i l_j & j > i \\ p_{ji} & j < i \end{cases}$$

$$g_i = m_i d_i g + a_i\left(\sum_{k=i+1}^{5} m_k\right) l_i g$$

where $a_i=0$ when $i=3$, and $a_i=1$ when $i=1,2,4,5$. The concise form of (A1-A3) shows the great improvement of the dynamic modeling for the biped system as compared with previous works [1,3,9].

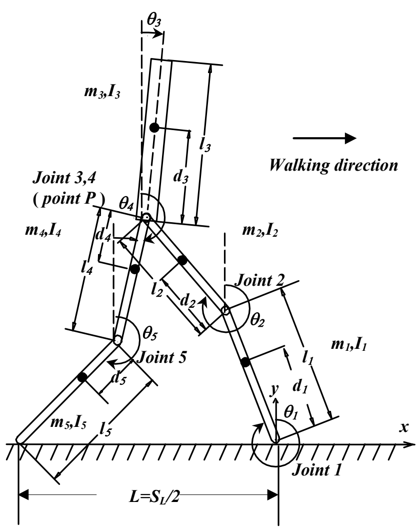

Figure 1 Biped model of DSP

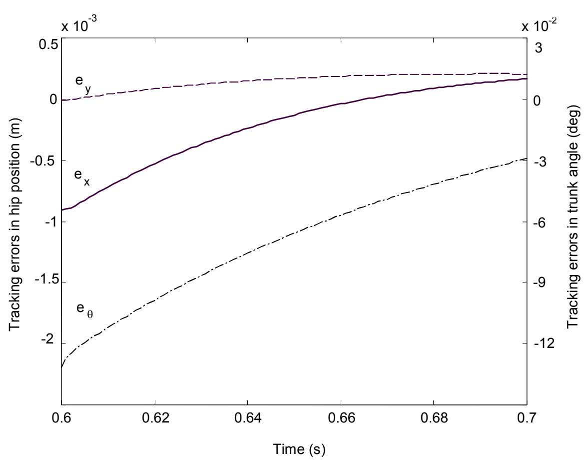

Figure 2 Tracking errors of the hip and trunk during DSP

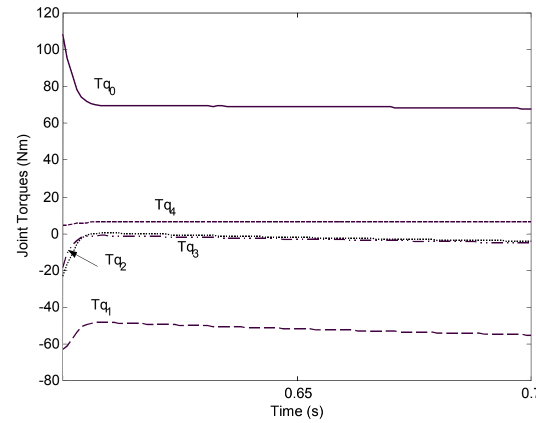

Figure 3 Actuator joint torques during DSP

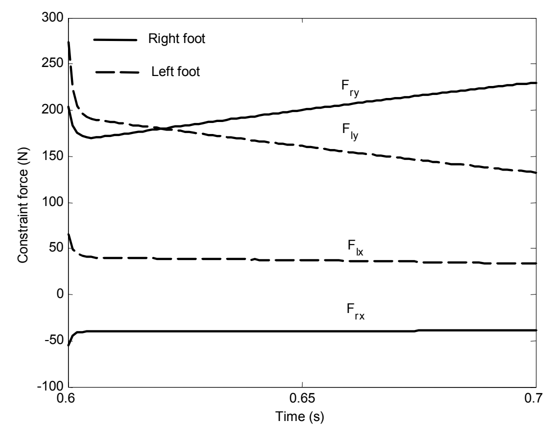

Figure 4 Constraint forces during DSP

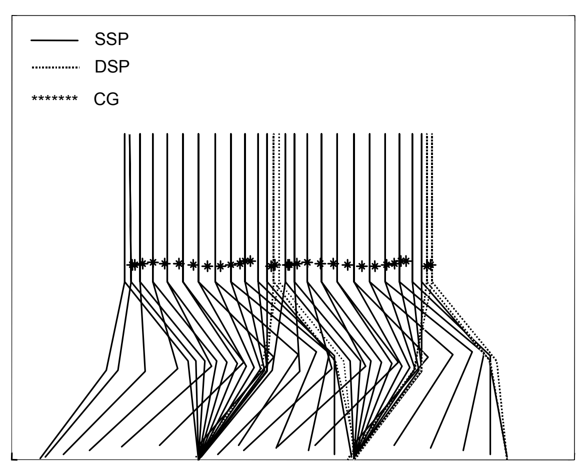

Figure 5 Stick diagram of the biped walking