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POWER SYSTEM SINGULARLY PERTURBED DISCONTINUOUS CONTROL

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Abstract—**The synchronous generators have a natural different time scale dynamics. That is why for modeling and control design in such systems the methods of singular perturbations are widely used. In this paper the possibilities of sliding mode control design for synchronous generators are analyzed. With this aim the concept of singular perturbation is revised in order to use it for relay control system with a *discontinuous slow-motion integral manifold.* Obtained results are used for variable structure control of synchronous generator**.

Index Terms—**sliding mode control, singular perturbations, nonlinear systems**

I. INTRODUCTION

Simplifications of plant models is a classical tool for electric power systems control design, and the most typical way is the singular perturbation approach (see [1], [2], [3], [4]). From the other hand, a fruitful and relatively simple approach, especially when we are dealing with nonlinear plants subjected to perturbations, is based on Variable Structure Control technique with sliding mode [5]. However the usage discontinuous (relay) control to a plant model with the singular perturbation leads to some problems. Classical methods of singular perturbation (see [6], and [3]) are based on the spectrum separation and consequently these approaches need the smoothness of the models and control law. That is why the classical methods of singular perturbations are not valid for Singularly Perturbed Relay Control Systems (SPRCS).

The decomposition methods for SPRCS were developed by [7], [8], [9], [10],[11], [12]. Some control algorithms for SPRCS was developed also in [12].The present paper discusses the advantages and possibilities of sliding mode control design for nonlinear SPRCS describing the synchronous generator dynamics. For this we use a Two Step Control Design (TSCD) procedure:

I. Eliminate the stator dynamics via singular perturbation methods and derive the reduced (6th order) model describing the slow mechanical and rotor fluxes dynamics.

II. Design a sliding mode excitation control law using block control technique [13].

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So the order of the original SPRCS is reduced in two steps: elimination of the fast dynamics and then the reduction of the slow dynamics via sliding domain.

This paper is organized as follows. Section II introduces the basic equations of the synchronous generator. In Section III the concepts of singularly perturbed models with relay control are justified. In section IV the singular perturbation approach is applied to design a synchronous generator controller. Simulation results are shown in Section V.

II. SYNCHRONOUS GENERATOR MODELS

A. Basic Equations

The mechanical equilibrium equations for a synchronous generator are given by

$$\frac{d\delta}{dt} = \omega - \omega_b \tag{1}$$

$$\frac{d\omega}{dt} = \frac{\omega_b}{2H}\left(T_m - T_e\right) \tag{2}$$

where δ is the power angle (rad.), ω is the angular velocity (rad./sec.), ω_b is the synchronous angular velocity (rad./sec.), H is the inertia constant (sec.), T_m is the mechanical torque (p.u.), and T_e is the electromechanical torque (p.u.). The equilibrium equations affected by the Park transformations, are expressed as

$$V = Ri + \omega G\varphi + \frac{d\varphi}{d\bar{t}} \tag{3}$$

$$\varphi = Li \tag{4}$$

where $\bar{t} = \omega_b t$, ω_b is the base angular velocity, $\bar{t}$ is the time in p.u., t is the time in seconds, $i = \left[i_d, i_q, i_f, i_g, i_{kd}, i_{kq}\right]^T$, $V = \left[V_d, V_q, V_f, 0, 0, 0\right]^T$, $\varphi = \left[\varphi_d, \varphi_q, \varphi_f, \varphi_g, \varphi_{kd}, \varphi_{kq}\right]^T$

$$R = \begin{bmatrix} -r_s & & & & & \\ & -r_s & & 0 & & \\ & & r_f & & & \\ & & & r_g & & \\ & 0 & & & r_{kd} & \\ & & & & & r_{kq} \end{bmatrix} \quad G = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$L = \begin{bmatrix} -L_d & 0 & L_{md} & 0 & L_{md} & 0 \\ 0 & -L_q & 0 & L_{mq} & 0 & L_{mq} \\ -L_{md} & 0 & L_f & 0 & L_{md} & 0 \\ 0 & -L_{mq} & 0 & L_g & 0 & L_{mq} \\ -L_{md} & 0 & L_{md} & 0 & L_{kd} & 0 \\ 0 & -L_{mq} & 0 & L_{mq} & 0 & L_{kq} \end{bmatrix}.$$

V means a voltage, i means a current, φ means a flux linkage, r means a resistance, L means an inductance, and

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the subscripts means: s is an astator, d is a direct axis circuit, q is a quadrature axis circuit, f is a field excitation circuit, g is a quadrature field circuit, kd is a direct axis damper, kq is a quadrature axis damper, md is a direct magnetizing, mq is a quadrature magnetizing.

The equation for the electromechanical torque in terms of the currents and fluxes, is governed by

$$T_e = \varphi_d i_q - \varphi_q i_d \quad (5)$$

and the excitator dynamics is represented by

$$\dot{V}_f = -a_f V_f + b_f u \quad (6)$$

where a_f and b_f are the excitator parameters and u is the control input.

B. Complete Model

From (1) to (5), we obtain the following model of synchronous generator of the 8th order:

$$\dot{x} = \begin{bmatrix} \dot{x}^1 \\ \dot{x}^2 \\ \dot{V}_f \end{bmatrix} = \begin{bmatrix} F_1(x^1, x^2, z) \\ F_2(x^1, x^2, z) \\ 0 \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \\ -a_f \end{bmatrix} V_f + \begin{bmatrix} 0 \\ 0 \\ b_f \end{bmatrix} u \quad (7)$$

$$\mu \dot{z} = F_3(x^1, x^2, z, V_f) \quad (8)$$

where $x^1 = (x_1, x_2, x_3)^T$, $x^2 = (x_4, x_5, x_6)^T$, $z = (z_1, z_2)^T$, $x_1 = \delta$, $x_2 = \omega$, $x_3 = \varphi_f$, $x_4 = \varphi_g$, $x_5 = \varphi_{kd}$, $x_6 = \varphi_{kq}$, $z_1 = i_d$, $z_2 = i_q$, $\mu = 1/\omega_b$,

$$F_1 = \begin{bmatrix} x_2 - \omega_s \\ d_m T_m - (a_{21}x_3 z_2 + a_{22}x_4 z_1 + a_{23}x_5 z_2 + a_{24}x_6 z_1 + a_{25}z_1 z_2) \\ a_{31}x_3 + a_{32}x_5 + a_{33}z_1 \end{bmatrix}$$

$$F_2 = \begin{bmatrix} b_{11}x_4 + b_{12}x_6 + b_{13}z_2 \\ b_{21}x_3 + b_{22}x_5 + b_{23}z_1 \\ b_{31}x_4 + b_{32}x_6 + b_{33}z_2 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 0 \\ b_3 \end{bmatrix},$$

$$F_3 = \begin{bmatrix} c_{11}x_2 x_4 + c_{12}x_2 x_6 + c_{13}x_2 z_2 + c_{14}z_1 + c_{15}V_d^{\infty} + c_{16}V_f \\ c_{21}x_2 x_3 + c_{22}x_2 x_5 + c_{23}x_2 z_1 + c_{24}z_2 + c_{25}V_q^{\infty} \end{bmatrix}.$$

The coefficients of (7)-(8) depend on the plant parameters.

III. SINGULARLY PERTURBED APPROACH

A. Singularly Perturbed Model

In this paper we are dealing with the singularly perturbed model having the form:

$$\frac{dx}{dt} = f(x, z, \mu, u), \qquad x(0) = x_0 \quad (9)$$

$$\mu \frac{dz}{dt} = g(x, z, \mu, u), \qquad z(0) = z_0 \quad (10)$$

where $x \in R^n$, $z \in R^m$, $u \in R$, $\mu \in R$; f and g are smooth functions of their argument and linear on z and u, $\mu > 0$ is a small parameter, and u is

$$|u| \le u_0 \text{ with } u_0 > 0. \quad (11)$$

B. Control Design Procedure

The sliding mode control design procedure for original system (9), (10) consists of two steps.

Step 1. Setting $\mu = 0$ makes instantaneous the fast dynamics (10)

$$0 = g(x, z, 0, u). \quad (12)$$

Let us consider a smooth isolated solution of equation (12)

$$\bar{z} = h(x, u) \quad (13)$$

where $\bar{z}$ presents the quasi-steady state. Substituting (13) in (9) we obtain the reduced order model (ROM)

$$\frac{d\bar{x}}{dt} = f(\bar{x}, h(\bar{x}, u), 0, u) \quad (14)$$

where $\bar{x}(t)$ defines the solution of (14) for a fixed control $u(\bar{x})$.

Step2. Design a nonlinear sliding surface $s(\bar{x}) = 0$, $s \in R$ for the system (14), such that the solution of the equation

$$\frac{ds}{dt} = \bar{G}f(\bar{x}, h(\bar{x}, u_{eq}), 0, u_{eq}) = 0$$

with respect to the equivalent control, $u_{eq}(\bar{x})$ [5], does exist, and the sliding mode equation (SME)

$$\frac{d\bar{x}}{dt} = f(\bar{x}, h^*(\bar{x}), 0, u_{eq}(\bar{x})), \quad h^*(\bar{x}) = h(\bar{x}, u_{eq}(\bar{x})) \quad (15)$$

$$s(\bar{x}) = 0 \quad (16)$$

has the desired properties. Second, taking into account (16), it is selected a discontinuous control

$$u(x) = \begin{cases} u^+(x) & if \quad s(x) > 0, \\ u^-(x) & if \quad s(x) < 0, \end{cases} \qquad \begin{matrix} |u^+(x)| \le u_0, \\ |u^-(x)| \le u_0 \end{matrix} \quad (17)$$

that makes the sliding surface (16) to be attractive.

Note that one of the vector $\bar{x}$ components can be expressed from (16) as a function of other ($n-1$) components. Therefore, in fact, SME (15) has the order ($n-1$). So, the order of the original system (9)-(10) is reduced first, by using the motion separation due to different time scale, and second, via sliding mode.

To justify the proposed control design (TSCD) procedure (see steps 1 and 2), first we will analyze the behavior of the original SPRCS (9), (10) and (17) when the state vector reaches the switching surface, and then investigate the entrance of SPRCS solutions into the sliding mode domain (see subsection C). Finally, the stability condition for original SPRCS will be derived (see subsection D).

C. Analysis of the Reaching Phase for SPRCS Solutions

In this subsection we will study the behavior of the original SPRCS out from sliding mode domain. If a solution of the SPRCS is not crossing the discontinuity surface (16) it can be analyzed by classical method of singular perturbations (see [6], and [3]). From the other hand, the specific feature of SPRCS describing the behavior of synchronous machines is that the equations of slow variables depend on the relay control (17). We will show that in this case we can use the reduced order model to describe the SPRCS. Doing so, we have to describe specific features of SPRCS for both domains $s > 0$ and $s < 0$.

Moreover, it is necessary to verify the attraction condition for the switching point.

Denote the domains of definition for variables z and x as Z and X. The discontinuity surface $s(x)=0$ divides the domains X and Z into the parts defined as X^- and Z^- for $s<0$, and X^+ and Z^+ for $s>0$, respectively; and define the system structure as

$$f^+(x,z,\mu)=f\left(x,z,\mu,u^+(x)\right),$$

$$g^+(x,z,\mu)=g\left(x,z,\mu,u^+(x)\right) \quad \text{for } s\ge 0$$

$$f^-(x,z,\mu)=f\left(x,z,\mu,u^-(x)\right),$$

$$g^-(x,z,\mu)=g\left(x,z,\mu,u^-(x)\right) \quad \text{for } s\le 0$$

with $f^+,\ g^+ \in C^2\left[\bar{X}^+\times\bar{Z}^+\times[0,\mu_0]\right]$,

$f^-,\ g^- \in C^2\left[\bar{X}^-\times\bar{Z}^-\times[0,\mu_0]\right]$.

C.1 SPRCS in the domain $s<0$.

Denote

$$\frac{ds^-}{dt}(x,z,\mu)=Gf^-(x,z,\mu),\quad \frac{ds^+}{dt}(x,z,\mu)=Gf^+(x,z,\mu)$$

Suppose that $x_0\in X^-$, $z_0\in Z^-$. It is natural to assume that for the original system (9), (10) and (17) the following conditions of the Tikhonov theorem (see, for example, [6]) hold:

[a1] The function $\bar{z}^-=h^-(\bar{x})$ is an isolated solution of $0=g^-(x,z,0)$ for all $x\in X^-$.

[a2] The Cauchy problem for slow dynamics

$$\frac{d\bar{x}^-}{dt}=f^-\left(\bar{x}^-,h^-\left(\bar{x}^-\right),0\right),\quad \bar{x}^-(0)=x_0 \qquad (18)$$

has a unique solution $\bar{x}^-(t)$ on $[0,\bar{t}_s]$, where $\bar{t}_s$ is the switching point i.e. the smallest root of equation $s\left(\bar{x}^-(\bar{t}_s)\right)=0$.

[a3] The equilibrium point $\Pi^-z=0$ of the system

$$\frac{d\left(\Pi^-z\right)}{d\tau^-}=g^-\left(\bar{x}^-(t),\Pi^-z+h^-\left(\bar{x}^-(t)\right),0\right)$$

is asymptotically stable, where $\Pi^-z=z-h^-\left(\bar{x}^-(t)\right)$, $\tau^-=t/\mu$, moreover, for all $[0,\bar{t}_s]$

$$\text{Re Spec}\ \frac{\partial g^-}{\partial\Pi^-z}\left(\bar{x}^-(t),h^-(\bar{x}^-(t)),0\right)<-\alpha<0.$$

Define $\Psi(h^-)$ as the domain of attraction of the equilibrium point $\Pi^-z=0$, and suppose that

[a4] The initial value for fast variables belongs to the attraction domain, i.e. $z_0\in\Psi(h^-)$.

[a5] The trajectory of the reduced system (15) reaches the switching surface $s(x)=0$, without tangential touch, i.e.

$$\frac{d\,s^-}{dt}=\bar{G}f^-\left(\bar{x}^-(\bar{t}_s),h^-\left(\bar{x}^-(\bar{t}_s)\right),0\right)>0.$$

From Vasil'eva theorem [6] it follows that for sufficiently small μ there exists a time moment $t=t_s(\mu)$ such that for the slow coordinate of the original SPRCS we have $s\left(x(t_s(\mu),\mu)\right)=0$, i.e. a solution of the original SPRCS will reach the switching surface.

The following lemma is true [10]:

Lemma 1. *Suppose that the original SPRCS (9), (10) and (17) satisfies the conditions [a1]-[a5]. Then there exist small* $\mu_0>0$ *and* $\delta_0>0$, $t_s(\mu)$ *such that for all* $\mu\in[0,\mu_0]$ *there is a unique solution* $\left(x(t,\mu),z(t,\mu)\right)$ *of Cauchy problem (9) and (10) on* $[0,t_s(\mu)]$, *and*

$$\lim_{\mu\to 0}x(t,\mu)=\bar{x}(t)=\bar{x}^-(t)\ \ \textit{for}\ t\in[0,t_s(\mu)],$$

$$\lim_{\mu\to 0}z(t,\mu)=h^-\left(\bar{x}^-(t)\right)\ \text{for}\ t\in[\delta_0,t_s(\mu)\text{-}\delta_0],\quad \delta_0>0.$$

Remark 1. In the same way, we can prove that it is possible to use the equations for slow motions in the case when a solution of (9), (10) and (17) leaves the domain $X^+\times Z^+$ and reaches the switching surface [10].

C.2 Transition into sliding domain

The behavior of the original SPRCS (9), (10) and (17) into the sliding domain, is described. Denote the domains

$$S_0=\left\{x:\frac{ds^-}{dt}\left(\bar{x},h(\bar{x}),0\right)>0,\ \frac{ds^+}{dt}\left(\bar{x},h(\bar{x}),0\right)<0\right\},$$

$$S_\mu=\left\{(x,z,\mu):\frac{ds^-}{dt}(x,z,\mu)>0,\ \frac{ds^+}{dt}(x,z,\mu)<0\right\}.$$

Suppose that the control resources achieve the following sliding mode existence conditions [5]:

[c1] $$\frac{d\,s^-}{dt}\left(\bar{x}^-(\bar{t}_s),h^-\left(\bar{x}^-(\bar{t}_s)\right),0\right)>0,$$

$$\frac{d\,s^+}{dt}\left(\bar{x}^-(\bar{t}_s),h^-\left(\bar{x}^-(\bar{t}_s)\right),0\right)<0.$$

Now from the Tikhonov theorem it follows that for sufficiently small μ

$$\frac{ds^-}{dt}=Gf^-\left(x(t_s(\mu)),z(t_s(\mu)),\mu\right)>0 \quad \text{and}$$

$$\frac{ds^+}{dt}=Gf^+\left(x(t_s(\mu)),z(t_s(\mu)),\mu\right)<0.$$

This means that a solution of the original system (9), (10) and (17) enters into the sliding domain S_μ without tangential motions. Therefore, we can consider the coordinate of the switching point $\left(x(t_s(\mu),\mu),z(t_s(\mu),\mu)\right)$ as the initial condition for SPRCS into S_μ. Hence, a solution of the Cauchy problem (9), (10) with (17) into S_μ is described by the following system [5]:

$$\frac{dx^*}{dt}=f\left(x^*,z^*,\mu,u_{eq}(x^*,z^*,\mu)\right) \qquad (19)$$

$$\mu \frac{dz^*}{dt} = g\left(x^*, z^*, \mu, u_{eq}(x^*, z^*, \mu)\right) \qquad (20)$$

where $x^*(t_s(\mu),\mu) = x(t_s(\mu),\mu)$, $z^*(t_s(\mu),\mu) = z(t_s(\mu),\mu)$, $s(x^*) = 0$, $t \in [t_0(\mu), T]$, $x^* \in R^{n-1}$, $z^* \in R^m$, $u \in R$, $\mu \in [0, \mu_0]$, and $u_{eq}(x^*, z^*, \mu)$ is the equivalent control calculated as a solution of

$$\frac{ds}{dt} = Gf\left(x^*, z^*, \mu, u_{eq}\right) = 0 , \; s(x^*) = 0 .$$

Similar to the above case (subsection *C.1*) we suppose that for the system (19)-(20) the following conditions hold:

[c2] The function $\bar{z}^* = h^*\left(\bar{x}^*\right)$ is an isolated solution of $0 = g\left(x^*, z^*, 0, u_{eq}(x^*, z^*, 0)\right)$ for all $x \in S_0$.

[c3] The reduced (by $\mu = 0$) sliding mode equation

$$\frac{d\bar{x}^*}{dt} = f\left(\bar{x}^*, h^*(\bar{x}^*), 0, \bar{u}_{eq}(\bar{x}^*)\right), \quad \bar{x}^*(\bar{t}_s) = x_0^*$$

with $\bar{u}_{eq}(\bar{x}^*) = u_{eq}(\bar{x}^*, h^*(\bar{x}^*), 0)$ has a unique solution $\bar{x}^*(t)$ on $[\bar{t}_s, T]$, and $\bar{x}^*(t) \in S_0$ for all $t \in [\bar{t}_s, T]$.

[c4] The equilibrium point $\Pi^* z = 0$ of the system

$$\frac{d\left(\Pi^* z\right)}{d\tau^*} = g\left(\bar{x}^*(t), \Pi^* z + h^*\left(\bar{x}^*(t)\right), 0\right),$$

where $\Pi^* z = z - h^*\left(\bar{x}^*(t)\right)$, $\tau^* = t - \bar{t}_s/\mu$, is asymptotically stable, moreover, for all $t \in [\bar{t}_s, T]$

$$\operatorname{Re}\ \operatorname{Spec}\ \frac{\partial g^*}{\partial \Pi^* z}\left(\bar{x}^*(t), h^*(\bar{x}^*(t)), 0\right) < -\alpha < 0, \;\; \alpha > 0 .$$

Define $\Psi(h^*)$ as the domain of attraction of the equilibrium point $\Pi^* z = 0$, suppose that

[c5] The initial value of the jump for fast variables at the switching point belongs to the attraction domain, i.e.

$$h^-\left(\bar{x}^-(\bar{t}_s)\right) - h^*\left(\bar{x}^-(\bar{t}_s)\right) \in \Psi(h^*) .$$

The following lemma is true [11]:

Lemma 2. *Suppose that the original SPRCS (9), (10) and (17) satisfies the conditions [a1]-[a4] and [c1]-[c4]. Then there exist a small* $\mu_0 > 0$ *and* $\delta_0 > 0$ *such that for all* $\mu \in [0, \mu_0]$ *there is a unique solution* $\left(x(t,\mu), z(t,\mu)\right)$ *of (9), (10) and (17) on* $[0, T]$ *and*

1) $\lim_{\mu \to 0} x^*(t,\mu) = \bar{x}^*(t)$ *for* $t \in [t_s(\mu), T]$,

2) $\lim_{\mu \to 0} u_{eq}\left(x(t,\mu), z(t,\mu), \mu\right) = \bar{u}_{eq}\left(\bar{x}^*(t)\right), \;\; t \in [\bar{t}_s + \delta_0, T]$,

3) $\lim_{\mu \to 0} x(t,\mu) = \bar{x}(t) = \begin{cases} \bar{x}^-(t) & for\ t \in [0, t_s(\mu)] \\ \bar{x}^*(t) & for\ t \in [t_s(\mu), T] \end{cases}$,

4) $\lim_{\mu \to 0} z(t,\mu) = h^-\left(\bar{x}^-(t)\right)$ *for* $t \in [\delta_0, \bar{t}_s - \delta_0]$,

$\lim_{\mu \to 0} z(t,\mu) = h^*\left(\bar{x}^*(t)\right)$ *for* $t \in [\bar{t}_s + \delta_0, T]$.

Note if a solution of (9), (10) and (17) will leave the sliding domain, then it will not affect the zero approximation of the fast and the slow dynamics equations, since the slow motion integral manifold is continuous [10].

D. Stability Analysis

Consider the case, when the original SPRCS has the equilibrium into S_μ . Solving (20) for $u^*_{eq}\left(x(t,\mu), z(t,\mu)\right)$ and substituting it in (19), we obtain the *smooth* algebraic - differential singularly perturbed system described the sliding mode dynamics. From the equation of the sliding surface, taking into account that $G \neq 0$ one can express one coordinate or x as a function of other $(n-1)$ coordinates. Then a sliding mode dynamics is governed by the following singularly perturbed $(n+m-1)th$ order system:

$$\frac{dx^\otimes}{dt} = f^\otimes\left(x^\otimes, z^\otimes, \mu\right), \;\; \mu \frac{dz^\otimes}{dt} = g^\otimes\left(x^\otimes, z^\otimes, \mu\right), \;\; (21)$$

where the vector $x^\otimes \in R^{n-1}$ consists of the ($n-1$) independent coordinates of x , $z^\otimes = z$, $g^\otimes$,and $f^\otimes \in R^{n-1}$ are the values of g and the corresponding component of f computed at $u = u_{eq}\left(x^\otimes, z^\otimes, \mu\right)$. For the case of synchronous machine equations, $g^\otimes\left(\bar{x}^\otimes, h^\otimes\left(\bar{x}^\otimes\right), 0\right) = 0$ has a unique solution $\bar{z}^\otimes = h^\otimes\left(\bar{x}^\otimes\right)$, then the slow dynamics in (21) are described by the system

$$\frac{d\bar{x}^\otimes}{dt} = \bar{f}^\otimes\left(\bar{x}^\otimes\right) = f^\otimes\left(\bar{x}^\otimes, h^\otimes\left(\bar{x}^\otimes\right), 0\right)$$

$$0 = g^\otimes\left(\bar{x}^\otimes, h^\otimes\left(\bar{x}^\otimes\right), 0\right). \qquad (22)$$

Let us denote $x_{eq}{}^\otimes$ as the equilibrium point of (22). Then from Klimushchev – Krasovskii theorem [14] it follows that the equilibrium point of system (21) is uniformly asymptotically stable for $\mu \in [0, \mu_0]$, if the matrices $\frac{\partial \bar{f}^\otimes}{\partial x^\otimes}\left(x_{eq}{}^\otimes, h^\otimes\left(x_{eq}{}^\otimes\right), 0\right)$ and $\frac{\partial g^\otimes}{\partial z^\otimes}\left(x_{eq}{}^\otimes, h^\otimes\left(x_{eq}{}^\otimes\right), 0\right)$ in (22) are Hurwitz.

Now we can conclude that in order to verify correctness of the proposed control design procedure it is sufficient to check the conditions presented in the subsections B. - D.

IV. CONTROL OF GENERATOR

In this section we will derive a reduced model and a discontinuous control law for the generator.

A. Reduced Model of Synchronous Machine

The fast dynamics (8) rewritten as

$$\mu \; \dot{z}_1 = c_{11} x_2 x_4 + c_{12} x_2 x_6 + c_{13} x_2 z_2 + c_{14} z_1 + c_{15} V_d^\infty + c_{16} V_f \qquad (23)$$

$$\mu \; \dot{z}_2 = c_{21} x_2 x_3 + c_{22} x_2 x_5 + c_{23} x_2 z_1 + c_{24} z_2 + c_{25} V_q^\infty \qquad (24)$$

can be neglected by making $\mu = 0$, that is

$$0 = A_R z + F_R + B_R V_f \qquad (25)$$

where $A_R = \begin{bmatrix} c_{14} & c_{13\mu} \\ c_{23\mu} & c_{24} \end{bmatrix}$, $F_R = \begin{bmatrix} c_{11\mu}x_4 + c_{12\mu}x_6 + c_{15}V_d^{\infty} \\ c_{21\mu}x_3 + c_{22\mu}x_5 + c_{25}V_q^{\infty} \end{bmatrix}$, $B_R = \begin{bmatrix} c_{16} \\ 0 \end{bmatrix}$, and $rank\ A_R = 2$. So, a solution of (25) for z_1 and z_2 is calculated as

$$z = -A_R^{-1}F_R - A_R^{-1}B_R V_f := h(x^1, x^2, V_g, V_f) := \begin{bmatrix} h_1(x^1, x^2, V_g, V_f) \\ h_2(x^1, x^2, V_g, V_f) \end{bmatrix} \quad (26)$$

Substituting (26) in (7) gives the following reduced (6^{th} order) model:

$$\begin{bmatrix} \dot{x}^1 \\ \dot{V}_f \end{bmatrix} = \begin{bmatrix} \bar{F}_1(x^1, x^2, T_m) \\ 0 \end{bmatrix} + \begin{bmatrix} A_1 \\ -a_f \end{bmatrix} V_f + \begin{bmatrix} 0 \\ b_f \end{bmatrix} u \quad (27)$$

$$\dot{x}^2 = \bar{F}_2(x^1, x^2, V_f)$$

where $\bar{F}_2 = A_{21}x_3 + A_{22}x^2 + A_{23}V_g + A_{24}V_f$,

$A_{21} = \begin{bmatrix} b_{41} \\ b_{51} \\ b_{61} \end{bmatrix}$, $A_{22} = \begin{bmatrix} b_{42} & b_{43} & b_{44} \\ b_{52} & b_{53} & b_{54} \\ b_{62} & b_{63} & b_{64} \end{bmatrix}$, $A_{23} = \begin{bmatrix} b_{45} & b_{46} \\ b_{55} & b_{56} \\ b_{65} & b_{66} \end{bmatrix}$,

$A_2 = \begin{bmatrix} b_{47} \\ b_{57} \\ b_{67} \end{bmatrix}$ and $A_1 = \begin{bmatrix} 0 \\ -a_{25}d_{17} - a_{26}d_{27} \\ a_{33}d_{17} + b_3 \end{bmatrix}$.

The coefficients of (27) depend on the plant parameters.

B. Angular Speed Control

The system (27) has the Nonlinear Block Controllable Form with internal dynamics. Therefore in order to design the nonlinear sliding surface we use the block control technique [13]. To satisfy the control objective, namely: rotor angle stability enhancement, we define the control error as

$$\varsigma_2 = x_2 - \omega_b. \quad (28)$$

Then taking the time derivative of (28) along the trajectories of (27), gives

$$\dot{\varsigma}_2 = f_2(x^1, x^2, V_g, T_m) + b_2(x^1, x^2)x_3 + b_{f2}V_f \quad (29)$$

where

$f_2 = d_m T_m - \left(a_{22}x_4 h_1(\cdot) + a_{23}x_5 h_2(\cdot) + a_{24}x_6 h_1(\cdot) + a_{25}h_1(\cdot)h_2(\cdot)\right)$, $b_2 = a_{21}h_2(\cdot)$, and $b_{f2} = (-a_{25}d_{17} - a_{26}d_{27})$ are positive functions of the time. To eliminate the old dynamics in (29) and introduce a new one we put

$$x_3 = -b_2(\cdot)^{-1}\left[f_2(\cdot) + b_{f2}V_f + k_0\varsigma_2 - s\right], \quad k_0 > 0 \quad (30)$$

Then using (30) the switching surface can be defined as

$$s = b_2(\cdot)x_3 + f_2(\cdot) + b_{f2}V_f + k_0(x_2 - \omega_B) = 0 \quad (31)$$

The projection motion on the subspace s_ω can be derived using (31) and (30) of the form

$$\dot{s} = f_s(x^1, x^2, V_g, V_f, T_m) + b_s u$$

where f_s is a bounded function, $b_s = a_{47}b_f$, and is a positive function of the time.

C. Stability Analysis.

C.1 Sliding mode stability.

Under the following condition:

$$b_s u_0 \geq \left| f_s(x^1, x^2, V_g, T_m) \right|$$

the proposed relay control law

$$V_f = -u_0 sign(s_\omega), \qquad u_0 > 0 \quad (32)$$

ensures the convergence of the state to the surface (31) in a finite time.

C.2 Sliding dynamics stability

Once the sliding mode motion is achieved, this motion is governed by the reduced order (5^{th} order) system:

$$\dot{x}_1 = \varsigma_2, \quad \dot{\varsigma}_2 = -k_0\varsigma_2, \quad \dot{x}_3 = a_{31}x_3 + V_{feq} \quad (33)$$

$$\dot{x}^2, = \bar{F}_2(x^1, x^2, h_1(x^1, x^2, V_g, V_f), h_2(x^1, x^2, V_g, V_f)) \quad (34)$$

where the two first equations (33) describing the linearized mechanical dynamics, has the desired eigenvalue $-k_0$, while the third equation and (34) represent the internal excitator and rotor flux dynamics. This system can be rewritten as linear system with nonvanishing perturbation

$$\dot{x}_3 = a_{31}x_3 + V_{feq}, \quad \dot{x}^2, = A_{sm}x^2 + f\left(x^1, x^2, V_g, V_f, T_m\right)$$

where $A_{sm} = \begin{bmatrix} b_{11} + b_{13}d_{22} & b_{13}d_{23} & b_{12} + b_{13}d_{24} \\ b_{23}d_{12} & b_{22} + b_{23}d_{13} & b_{23}d_{14} \\ b_{31} + b_{33}d_{22} & b_{33}d_{23} & b_{32} + b_{33}d_{24} \end{bmatrix}$, and V_{feq} and $f\left(x^1, x^2, V_g, V_f, T_m\right)$ are bounded functions. The matrix A_{sm} is Hurwitz (see section V) and $a_{31} > 0$ therefore the *zero dynamics* on the invariant subspace $x_1 = 0,\ \varsigma_2 = 0$ and $s(x) = 0$ is stable. Hence, the solution $x^2(t)$ is ultimately bounded [15], the control error ς_2 tends exponentially to zero, and the angle x_1 tends to a constant steady state δ_{ss}.

C.3 Fast dynamics stability

Define $\Pi z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} - \begin{bmatrix} h_1(\bar{x}(0)) \\ h_2(\bar{x}(0)) \end{bmatrix}$, then the fast dynamics is governed by

$$\begin{bmatrix} \dfrac{d\Pi z_1}{d\tau} \\ \dfrac{d\Pi z_2}{d\tau} \end{bmatrix} = \begin{bmatrix} c_{14} & c_{13\mu} \\ c_{23\mu} & c_{24} \end{bmatrix} \begin{bmatrix} \Pi z_1 \\ \Pi z_2 \end{bmatrix},$$

$\begin{bmatrix} c_{14} & c_{13\mu} \\ c_{23\mu} & c_{24} \end{bmatrix}$ is Hurwitz, hence the equilibrium point $\Pi^{-}z = z - h^{-}(\bar{x}(0)) = 0$ of the system is exponentially stable. So all the assumptions described in section III are satisfied.

V. SIMULATION RESULTS

The proposed control algorithm was tested on the complete eight order model of synchronous generator connected through a transmission line to an infinite bus, Fig.1.

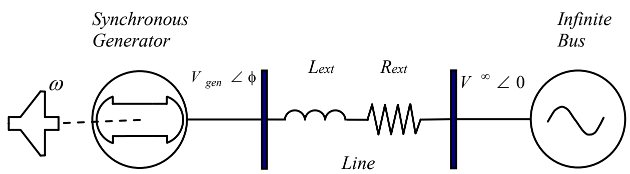

Fig. 1. Single machine with infinite bus.

The parameters of the synchronous machine and external network in p.u. are:

$T^{'}_{do} = 8.0$ sec. $T^{'}_{qo} = 1.0$sec. $T^{''}_{do} = 0.03$sec. $T^{''}_{qo} = 0.07$sec.

$L_d = 1.81$, $L^{'}_d = 0.3$, $L^{''}_d = 0.23$, $L_q = 1.76$, $L^{'}_q = 0.6$, $L_{ext} = 0.1$, $R_{ext} = 0.001$. From this we obtain the parameters of model (9)-(10), and (17). The controller gains was adjusted to k_0=10. The eigenvalues of (34) was calculated as, $\lambda_4 = -38.77$, $\lambda_5 = -0.5024$ and $\lambda_6 = -27.04$. Figures 2-4 depict results under a three-phase short circuit (150 ms. long) simulated at the transformer terminals.

These Figures reveal some important aspects:

1 State variables hastily reach a steady state condition after small and large disturbances, exhibiting the stability of the closed-loop system.

2 The terminal voltage recovers their steady state value after the short circuit

VI CONCLUSIONS

In this paper the possibility of usage a sliding mode control algorithms for nonlinear SPRCS describing a power system dynamics is analyzed. For this system the following two steps control design (TSCD) is proposed: firstly, the natural two scale properties of synchronous generator are used to obtain the reduced order model, and then the sliding mode control algorithm ensuring the desired behavior of the generator, is designed. The effectiveness of proposed algorithm is illustrated by simulations.

REFERENCES

[1] P.W. Sauer and P.V. Kokotovic, "An integral manifold approach to reduced order dynamic modeling of synchronous machines", *IEEE Transactions on Power Systems*, vol. 3, pp. 17-23, 1998.

[2] P-W. Sauer and M.A. Pai, "*Power system dynamics and stability"*, New Jersey: Prentice Hall, 1998.

[3] P.V. Kokotovic, H.K. Khalil and J. O'Reilly, "Singular Perturbation Methods in Control: Analysis and Design ", London: Academic Press, 1986.

[4] P.C.Krause, "*Analysis of Electric Machinery*", NY: McGraw Hill, 1986.

[5] V.I. Utkin, "*Sliding Mode in Control and Optimization",* London: Springer Verlag, 1992.

[6] A.B. Vasil`eva, V.F. Butozov and L.V. Kalachev, "*The boundary Function Method for Singular Perturbation Problems*", Philadelphia: SIAM, 1995.

[7] B.S. Heck and A. H. Haddad, "Singular perturbation analysis of linear system with scalar quantized control", *Automatica*, vol. 24, pp. 755-764, 1989.

[8] B.S. Heck, "Sliding mode control for singularly perturbed system", *International Journal of Control*, vol 53, pp. 985-1001, 1991.

[9] W.-C. Su, "Sliding surface design for singularly perturbed system", *International Journal of control*, vol. 72, pp. 990-995, 1999.

[10] L.M. Fridman, "Singularly Perturbed Analysis of Chattering in Relay Control Systems", *IEEE Transactions on Automatic Control*, vol. 47, No. 12 pp. 2079-2084, 2002.

[11] L.M. Fridman, "Slow periodic motions with internal sliding modes in variable structure systems", *International Journal of Control,* vol. 75 No. 7, pp. 524-537, 2002.

[12] M. Innocenti, L. Greco and L. Pollini, "Sliding mode control for two-time scale systems: stability issues", *Automatica*, vol. 39(2), pp. 273-280, 2003.

[13] A.G. Loukianov, "Nonlinear Block Control with Sliding Mode", *Automation and Remote Control*, vol. 59 (7), pp. 916-933, 1998.

[14] A.I. Klimushchev and N.N. Krasovskii, "Uniform asymptotic stability of systems of differential equations with a small parameter in the derivative terms", *J. Appl. Math. Mech*., vol. 25, pp. 1011-1025, 1962.

[15] H. K. Khalil, "Nonlinear systems", New Jersey: Prentice Hall, 1996.

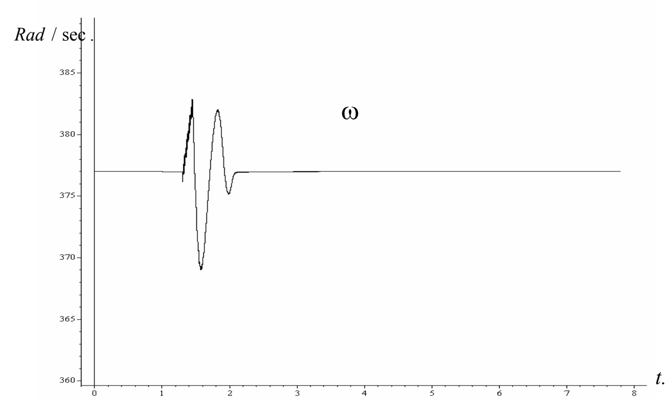

Fig. 2. Rotor angular velocity affected by a 0.15 sec. short circuit.

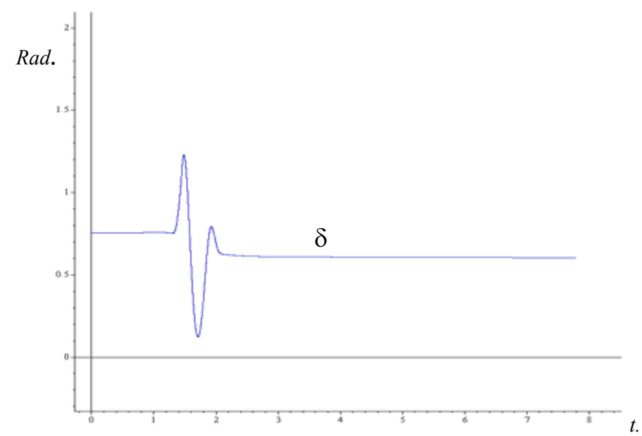

Fig. 3. Power angle affected by a 0.15 sec. short circuit.

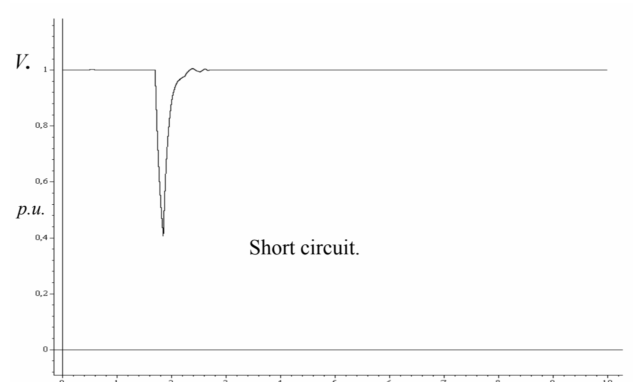

Fig. 4. Generator voltage affected by a 0.15 sec. short circuit.