Proceeding of the 2004 American Control Conference
Boston, Massachusetts June 30 - July 2, 2004

Neural Network Control of a Class of Nonlinear Systems with Actuator Saturation

Wenzhi Gao and Rastko R. Selmic

Abstract— **In this paper, neural net (NN)-based actuator saturation compensation scheme for the nonlinear systems in Brunovsky canonical form is presented. The scheme that leads to stability, command following and disturbance rejection is rigorously proved, and verified using a nonlinear system of "pendulum type". On-line weights tuning law, the overall closed-loop performance, and the boundness of the NN weights are derived and guaranteed based on Lyapunov approach. The actuator saturation is assumed to be unknown, and the compensator is inserted into a feedforward path. The simulation results indicate that the proposed scheme can effectively compensate for the saturation nonlinearity in the presence of system uncertainty.**

I. INTRODUCTION

Saturation, deadzone, backlash, and hysteresis, are the most common actuator nonlinearities in practical control systems. Saturation nonlinearity is unavoidable in most actuators. Categories of saturation nonlinearities include constraints of the magnitude and the rate of actuator inputs. When an actuator has reached such an input limit, it is said to be "saturated", since efforts to further increase the actuator output would not result in any variation in the output. Due to the non-analytic nature of the actuator nonlinear dynamics and the fact that the exact actuator nonlinear functions, namely operation uncertainty, are unknown, such systems present a challenge to the control design engineer [13], and provide an application field for adaptive control, sliding control and neural network-based control. Proportional-derivative (PD) controller has observed limit cycles if the saturation exists.

To tackle this problem, Astrom and Wittenmark [1] developed the general actuator saturation compensator scheme; Hanus and Peng [6] addressed a controller based on the conditional technique; Walgama and Sternby [19] developed an observer-based anti-windup compensator; Niu [14] designed a robust anti-windup controller based on the Lyapunov approach to accommodate the constraints and disturbance; Chan [3] investigated the actuator saturation stability issues related to the number of the integrators in the plant; Annaswamy *et al*. [2] addressed an adaptive controller to accommodate saturation constraints in the presence of time delays, which is applicable to 1st, 2nd and n-th order plants.

In some seminal recent work several rigorously derived adaptive schemes have been given for actuator nonlinearity compensation [18]. Compensation for non-symmetric deadzone is considered in [15], backlash compensation is addressed in [17], and hysteresis in [18].

Much has been written on intelligent control using neural networks (NNs) [12]. With the universal approximation property and learning capability, NNs have proven to be a powerful tool to control complex dynamic nonlinear systems with parameter uncertainty. The common control strategies with regards to NN are direct adaptive NN control method with guaranteed stability, indirect adaptive NN control based on identification [13], and dynamic inverse NN control [12]. In general, NN is used to estimate the unknown nonlinear dynamics and/or function and to compensate for them. Unlike the standard adaptive control schemes, NN can also cope with a nonlinear system that is linearly unparameterizable. Recently, several research results [4], [11] have used NN in feedback linearization schemes, incorporating the Lyapunov theory, to ensure the overall system stabilization, command following, and disturbance rejection.

Most saturation compensation approaches mentioned above focus on the linear plant and assume that the saturation is symmetric with the measurable actuator output. This paper proposes a NN-based scheme for saturation control for a class of nonlinear systems in the Brunovsky canonical form. The approach is applied to the n-th order feedback linearizable nonlinear plant, with a general model of actuator saturation assuming that the actuator output is not necessarily measurable. NN weights are tuned on-line, and the overall system performance is guaranteed using Lyapunov function approach. The convergence of the NN learning process and the boundness of the NN weights estimation error are all rigorously proven. The simulation results regarding to the nonlinear robot dynamics are provided.

Manuscript received September 17, 2003.
The authors are with the Electrical Engineering Department, Louisiana Tech University, Ruston, LA 71270 USA, (e-mail: wga002@latech.edu).

This paper is organized as follows. Section 2 provides the preliminary remarks and definitions. Section 3 presents the saturation nonlinearity and converted expressions. Section 4 addresses the Brunovsky canonical form nonlinear dynamics in the presence of saturation, the design process of out loop tracking controller and NN compensator, and the rigorous proof of the weights tuning laws. In Section 5, a simulation example of "pendulum type" nonlinear system with saturation nonlinearity is given. The conclusion is drawn in Section 6.

II. PRELIMINARY REMARKS AND DEFINITIONS

Let $\Re$ denote real numbers, $\Re^n$ denote the real n vector, and $\Re^{m\times n}$ denote the real $m\times n$ matrices. Let S be a compact simply connected set of $\Re^n$. With map $f: S\to\Re^m$, define $C(S)$ as the space such that f is continuous. The initial condition is $x_0\equiv x(t_0)$, let the equilibrium point x_e, and U_{x_e} be the neighborhood of x_e.

Definition 1 (Vector and Matrix Norms): By $\| \|$ is denoted any suitable vector norm. When it is required to be specific we denote the P-norm by $\| \|_P$. The supremum norm of $f(x)$, over S, is defined as

$$\sup_{x\in S}\|f(x)\|,\ f: S\to\Re^m \tag{1}$$

Given $A=[a_{ij}], B\in\Re^{m\times n}$ the Frobenius norm is defined by

$$\|A\|_F^2 = tr(A^T A)=\sum_{i,j} a_{ij}^2 \tag{2}$$

with $tr(\,)$ the trace. The Frobenius norm is compatible with the 2-norm so that $\|Ax\|_2\le\|A\|_F\|x\|_2$. The associated inner product is $<A,B>_F = tr(A^T, B)$, and $tr(AB)=tr(BA)$. Suppose A is positive definite, then for any $B\in\Re^{mxn}$, then

$$tr(BAB^T)\ge 0 \tag{3}$$

$$\frac{d}{dt}\{tr(A(x))\} = tr\left(\frac{dA(x)}{dt}\right) \tag{4}$$

Definition 2 (Uniformly Ultimate Boundness (UUB)) [11]: Consider the nonlinear system

$$\dot{x}=g(x,t) \tag{5}$$

with state $x(t)\in\Re^n$. The equilibrium point x_e is said to be uniformly ultimately bounded if there exists a compact set $S\subset\Re^n$, so that for all $x_0\in S$ there exists an $\varepsilon>0$, and a number $T(\varepsilon,x_0)$ such that $\|x(t)-x_e\|\le\varepsilon$ for all $t\ge t_0+T$. That is, after a transition period T, the state $x(t)$ remains within the ball of radius ε around x_e.

Consider two-layer NN, consisting of two layers of tunable weights. The hidden layer has L neurons, and the output layer has m neurons

$$y=W^T\sigma(V^T x+v_0)\ . \tag{6}$$

The multilayer NN is a nonlinear mapping from input space $\Re^n$ into output space $\Re^m$, where

$$V=[V_{ji}],\quad j=1,2,\cdots,n;\quad i=1,2,\cdots,L \tag{7}$$

$$W=[W_{ik}],\quad i=0,1,2,\cdots,L;\quad k=1,2,\cdots,m \tag{8}$$

$$x=[x_1,x_2,\cdots,x_n] \tag{9}$$

$$y=[y_1,y_2,\cdots,y_m] \tag{10}$$

$$v_0=[v_{01},v_{02},\cdots,v_{0L}]^T\ . \tag{11}$$

In order to include the thresholds in the matrix W, the vector activation function is defined as $\sigma(W)=[1,\sigma(W_1),\sigma(W_2),\cdots\sigma(W_L)]^T$. where $W\in\Re^L$. Tuning of the weights W then also includes tuning of the thresholds.

Many well-known results indicate that any sufficiently smooth function can be approximated arbitrary closely on a compact set using a two-layer NN with appropriate weights [5], Function $\sigma(\cdot)$ could be any continuous sigmoidal function [5]. NN *universal approximation property* defines that any continuous function can be approximated arbitrarily well using a linear combination of sigmoidal functions, namely,

$$f(x)=W^T\sigma(V^T x+v_0)+\varepsilon(x)\,, \tag{12}$$

where the $\varepsilon(x)$ is the NN approximation error. The reconstruction error is bounded on a compact set S by $\|\varepsilon(x)\|<\varepsilon_N$. Moreover, for any ε_N one can find a NN such that $\|\varepsilon(x)\|<\varepsilon_N$ for all $x\in S$.

The first layer weights V are selected randomly and will not be tuned. The second layer weights W are tunable. The approximation holds [7] for such NN, with approximation error convergence to zero of order $O\left(C/\sqrt{L}\right)$, where L is the number of the hidden layer nodes (basis functions), and C is independent of L. The approximating weights W are ideal target weights, and it is assumed that they are bounded so that $\|W\|_F\le W_M$.

III. ACTUATOR SATURATION

We study actuator saturation that appears in the nonlinear system plant, and the way of its compensation. Compensation technique is based on NN learning capabilities.

Figure 1 is the linear saturation $\tau=sat(u)$, where τ and u are scalars. In general, τ and u are vectors. Saturation operation limits are $\tau_{\min}$ and $\tau_{\max}$. In this paper, we assume that saturation nonlinearity is unknown.

Assuming ideal saturation, mathematically, the output of the actuator $\tau(t)$ is given by

$$\tau(t)=\begin{cases}\tau_{\max}: & u(t)\ge \tau_{\max}/m\\ mu(t): & \tau_{\min}/m<u(t)<\tau_{\max}/m\\ \tau_{\min}: & u(t)\le \tau_{\min}/m\end{cases}, \quad (13)$$

where $\tau_{\max}$ is the chosen positive, and $\tau_{\min}$ is the negative saturation limits. The control that can not be implemented by the actuator, denoted as $\delta(t)$, is given by

$$\delta(t)=\begin{cases}\tau_{\max}-u(t): & u(t)\ge \tau_{\max}/m\\ (m-1)u(t): & \tau_{\min}/m<u(t)<\tau_{\max}/m\\ \tau_{\min}-u(t): & u(t)\le \tau_{\min}/m\end{cases} \quad (14)$$

The nonlinear actuator saturation can be described using $\delta(t)$, see [2], [3], [9], [10]. In this paper, NN is used to approximate unknown function $\delta(t)$.

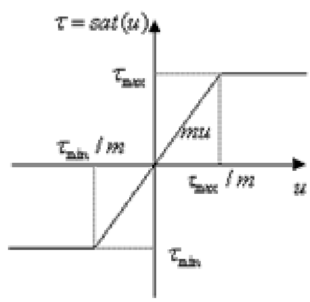

Figure 1. Saturation nonlinearity

IV. SATURATION COMPENSATION

A. Nonlinear System Dynamics

Consider the nonlinear systems with state space representation in the Brunovsky canonical form

$$\begin{aligned}\dot{x}_1&=x_2\\ \dot{x}_2&=x_3\\ &\vdots\\ \dot{x}_n&=f(x)+g(x)\tau\\ y&=x_1\end{aligned} \quad (15)$$

with $x=[x_1,x_2,\cdots,x_n]^T$, $f:\Re^n\rightarrow\Re$, an unknown smooth function; $g:\Re^n\rightarrow\Re$, a known smooth function; τ the control input.

Assumption 1: Function $g(x)$ is assumed to be known, such that $|g(x)|>\varepsilon$, where $\varepsilon>0,\varepsilon\in\Re$.

Define the desired state vector, $x_d(t)$, as

$$x_d(t)=[y_d,\dot{y}_d,\cdots,y_d^{(n-1)}]^T. \quad (16)$$

Assumption 2 (Bounded Desired Trajectory): The desired trajectory $x_d(t)$ is bounded and continuous, and $\|x_d(t)\|\le Q$ with Q known scalar bound.

B. Tracking Error Dynamics and Feedback Linearization

Define the state tracking error vector, $e(t)$ as

$$e(t)=x(t)-x_d(t). \quad (17)$$

Let us define a filtered tracking error as

$$r=K^Te, \quad (18)$$

where $K=[k_1,k_2,...k_{n-1},1]^T$ is appropriately chosen coefficient vector, so that $e\rightarrow 0$ exponentially as $r\rightarrow 0$ [16], [11]. Then, the time derivative of the filtered tracking error can be written as

$$\dot{r}=f(x)+g(x)\tau+Y_d, \quad (19)$$

where $Y_d=-y_d^{(n)}+\sum_{i=1}^{n-1}k_ie_{i+1}$.

Consider the saturation nonlinearity equation (14). The following n-th order nonlinear system dynamics

$$x_1^{(n)}=f(x)+g(x)\tau, \quad (20)$$

which is equivalent to (15), can be described as

$$x_1^{(n)}=f(x)+g(x)u+g(x)\delta, \quad (21)$$

in which the saturation will be treated as input system disturbance [2].

Similarly, in terms of the filtered tracking error above system dynamics can be described as follows

$$\dot{r}=f(x)+g(x)(u+\delta)+Y_d, \quad (22)$$

where Y_d is a known function of the tracking error and the desired function.

Choose the tracking control law as

$$w=\frac{1}{g(x)}(-\hat{f}-Y_d+v-K_vr), \quad (23)$$

where $\hat{f}$ is the fixed approximation of function $f(x)$. The functional estimation error is given by

$$\tilde{f}=f-\hat{f}. \quad (24)$$

Approximation $\hat{f}$ is fixed in this paper and will not be adapted. Robust term v is chosen for the disturbance rejection. The control law u then consists of the tracking controller with the saturation compensator, shown in the Figure 2 and given by

$$u=w-\hat{\varphi}, \quad (25)$$

where $\hat{\varphi}$ is the approximation of modified saturation nonlinear function $\delta(x)$.

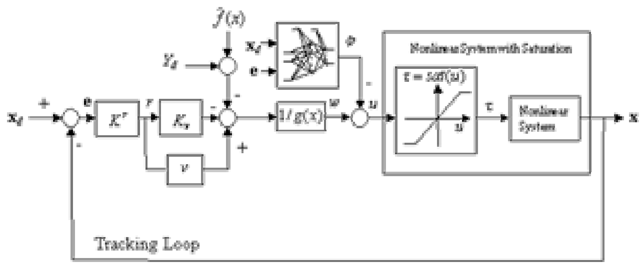

Figure 2. Nonlinear system with saturation compensator.

C. NN Saturation Compensator

Using the general NN function approximation property, there exists a NN that closely approximates the modified saturation nonlinear function $\delta(x)$

$$\delta = W^T \sigma(V^T x_{NN}) + \varepsilon \ . \qquad (26)$$

Implemented NN is actually an approximation of the ideal NN (26), and is given by

$$\hat{\varphi} = \hat{W}^T \sigma(V^T x_{NN}) \ , \qquad (27)$$

where the NN weights approximation error is

$$\tilde{W} = W - \hat{W} \ . \qquad (28)$$

Input to the NN saturation compensator is chosen as $x_{NN} \equiv [x_d, e]^T$.

Assumption 3 (Bounded Ideal NN Weights): The ideal NN weights W are bounded so that $\|W\| \le W_M$, with W_M known bounds.

Assumption 4 (Bounded Estimation Error): The estimate $\hat{f}(x)$ of a nonlinear unknown function $f(x)$ is assumed known, so that that the functional estimation error $\tilde{f}(x)$ satisfies

$$\left|\tilde{f}(x)\right| \le f_M(x) \qquad (29)$$

for some known function bounds $f_M(x)$ [8], [13].

Using control law (23) and (25), and substituting into (22), overall closed-loop error dynamics is given by

$$\dot{r} = \tilde{f} + g(x)\tilde{W}^T \sigma(V^T x_{NN}) + v - K_v r + \varepsilon \ . \qquad (30)$$

D. Weights Tuning Law for Guaranteed Tracking Performance

The purpose of NN saturation compensator is to design proper control laws and stable on-line NN weights update tuning rules, to guarantee the tracking performance of the overall closed loop systems under the unknown saturation nonlinearity. Moreover, an NN saturation compensator, if designed properly, should reduce the deleterious effect of saturation nonlinearity on the overall system performance.

Theorem 1 (Tuning of NN Compensator): Given the system in (30) and Assumptions 1-4, choose the tracking control law (23), plus the saturation compensator (25), (27), and the robustifying term as

$$v(t) = -f_M(x) sign(r) \ , \qquad (31)$$

where the $f_M(x)$ are bounds on the functional estimation error, and $sign(.)$ is standard sign function. Let the estimated NN weights be provided by the NN tuning algorithm

$$\dot{\hat{W}} = S\sigma(V^T x_{NN}) r \, g(x) - kS|r|\hat{W} \ , \qquad (32)$$

where

$S = S^T > 0$: any constant matrix representing the learning rate of the NN

k : small scalar positive design parameter.

By properly selecting the control gains and the design parameters, the filtered tracking error $r(t)$ and the NN weights $\hat{W}$ are UUB (Uniformly Ultimately Bounded).

Proof: Choose Lyapunov function candidate as

$$L = \frac{1}{2} r^2 + \frac{1}{2} tr(\tilde{W}^T S^{-1} \tilde{W}) \qquad (33)$$

Differentiating yields

$$\dot{L} = r\dot{r} + tr(\tilde{W}^T S^{-1} \dot{\tilde{W}}) \qquad (34)$$

Whence substitution from (30) yields,

$$\dot{L} = -K_v r^2 + r\tilde{f} + rg(x)\tilde{W}^T \sigma(V^T x_{NN}) + rv + r\varepsilon + tr(\tilde{W}^T S^{-1} \dot{\tilde{W}}) \qquad (35)$$
$$= -K_v r^2 + r(\tilde{f} + v + \varepsilon) + tr(\tilde{W}^T (S^{-1} \dot{\tilde{W}} + \sigma(V^T x_{NN}) r g(x)))$$

Applying the NN tuning rules, selected Lyapunov function is simplified to

$$\dot{L} = -K_v r^2 + r(\tilde{f} + v + \varepsilon) + k|r| tr(\tilde{W}^T \hat{W}) \qquad (36)$$

Using (31) one has

$$\dot{L} \le -K_v r^2 + k|r| tr(\tilde{W}^T (W - \tilde{W})) - |r| f_M + |r|\tilde{f} + |r|\varepsilon_N \qquad (37)$$

Using the inequality,

$$tr\left[\tilde{X}^T (X - \tilde{X})\right] \le \left\|\tilde{X}\right\|_F \left\|X\right\|_F - \left\|\tilde{X}\right\|_F^2 , \qquad (38)$$

the inequality (37) can be written as

$$\dot{L} \le -K_v |r|^2 + k|r|(\|\tilde{W}\|_F \|W\|_F - \|\tilde{W}\|_F^2) + |r|\varepsilon_N$$
$$\le |r| \left\{ -K_v |r| + k(\|\tilde{W}\|_F W_M - k\|\tilde{W}\|_F^2) + \varepsilon_N \right\} .$$
$$= |r| \left\{ -K_v |r| - k(\|\tilde{W}\|_F - \frac{1}{2} W_M)^2 + \frac{1}{4} k W_M^2 + \varepsilon_N \right\} ,$$

which is guaranteed to remain negative as long as

$$|r| \ge \frac{\frac{k}{4} W_M^2 + \varepsilon_N}{K_v} \qquad (39)$$

or

$$(k\|\tilde{W}\|_F^2 - \frac{1}{2}W_M)^2 \geq \frac{k}{4}W_M^2 + \varepsilon_N \;, \qquad (40)$$

which is equivalent to

$$\|\tilde{W}\|_F \geq \left(\frac{\sqrt{\frac{k}{4}W_M^2 + \varepsilon_N} + \frac{1}{2}W_M}{k} \right)^{\frac{1}{2}} . \qquad (41)$$

The following remarks are relevant.

Unsupervised Backpropagation Through Time With Extra Terms. The first term of (32) is modified version of the standard backpropagation algorithm. The k term corresponds to the *e*-modification, to guarantee bounded parameter estimates.

Bounds on the Tracking Error and NN Weights Estimation Errors. The right-hand side of inequality (39) can be taken as a practical bound on the tracking error in the sense that $r(t)$ will never stray far above it. Note that the stability radius may be decreased by any amount by increasing the PD gain K_v. It is noted that PD, PID, or any other standard controller does not posses this property when saturation nonlinearity is present in the system. Moreover, it is difficult to guarantee the stability of such a highly nonlinear system using only PD. Using the NN saturation compensation, stability of the system is proven, and the tracking error can be kept arbitrarily small by increasing the gain K_v. The NN weights errors are fundamentally bounded in terms of W_M. The tuning of parameter k offers a design tradeoff between the eventual relative magnitudes of $\|\tilde{W}\|_F$ and r.

NN Weights Initialization. The weights V are set to random values. It is shown in [7] that for such NN, termed random variable functional link (RVFL) NN, the approximation property holds. The weights W are initialized at zero. Then the PD loop in Figure 2 holds the system stable until the NN begins to learn.

Intelligent Anti-Windup Saturation Compensation. The proposed method utilizes an NN controller to compensate for the saturation nonlinearity effects. Initially, the NN controller "learns" and adjusts its weights to prevent the control signal from being saturated. After the initial learning period, which will be demonstrated below in the simulation, the NN signal effectively keeps the control signal within saturation bounds. Therefore, the proposed NN control scheme presents a form of *Intelligent Anti-Windup Saturation Compensation*.

V. SIMULATION OF NN SATURATION COMPENSATOR

The simulation was performed to verify the effectiveness of the proposed NN saturation compensator. We consider a "generalized pendulum" nonlinear system given by

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -5x_1^3 - 2x_2 + \tau \\ y &= x_1 \end{aligned} \qquad (42)$$

The control input is constrained by the saturation nonlinearity characterized by the parameters

$$\tau_{\max} = 5,\, \tau_{\min} = -5,\, m = 1 \;. \qquad (43)$$

The size of the NN affects the stability, performance, limitation of control efforts, and possible operating conditions. The slow convergence of the tracking error is usually due to the small network size. Moreover, if the network chosen size is too large, the computation burden increases. Common approach is to start with the smaller NN size, and gradually increase the number of hidden layer nodes until satisfactory performance is achieved.

In this paper, the NN has four, ten and one neurons at the input, hidden and output layers, respectively. The first-layer weights V are selected randomly [7], they are uniformly randomly distributed between –1 and +1. These weights represent the stiffness of the sigmoid activation function. The threshold weights for the first layer v_0 are uniformly randomly distributed between -15 and +15. The threshold weights represent the bias in activation functions' positions. The second layer weights W are initialized to zero or any random numbers, and the effect of the inaccurate initialization number can be retrieved by the on-line weights tuning law methodology.

Tracking loop controller parameters are chosen so that $K_v = 10$, $K = [2, 1]^T$. Initial conditions are $[0, 0]^T$, and desired trajectory is given by $x_1(t) = \sin(t), x_2(t) = \cos(t)$. The position tracking errors and the control input signal with and without NN saturation compensator are shown in Figure 3 and Figure 4.

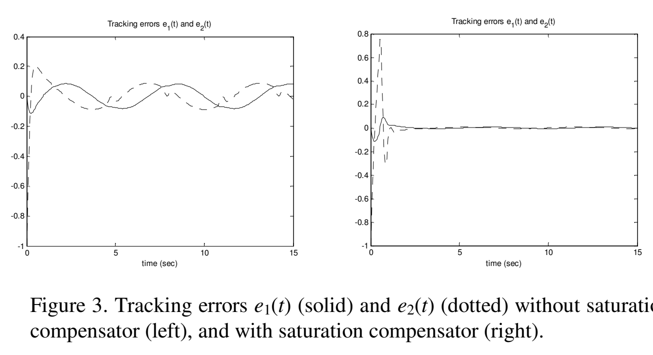

Figure 3. Tracking errors $e_1(t)$ (solid) and $e_2(t)$ (dotted) without saturation compensator (left), and with saturation compensator (right).

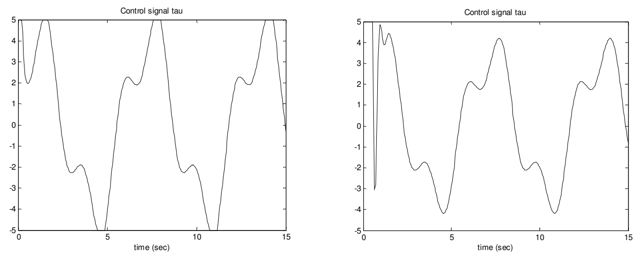

Figure 4. Control torque $\tau(t)$ without saturation compensator (left), and with saturation compensator (right).

The NN saturation compensator weights tuning parameters are chosen as

$$k = 0.0001\,, \quad S = 5\ . \tag{44}$$

From the above simulation, it is clear that the proposed scheme can effectively compensate the saturation nonlinearity in a class of nonlinear systems. Note also that after some initial time required for NN to learn the unknown saturation nonlinearity, the NN saturation compensator effectively prevents the control signal from reaching saturation limits. NN is trained using the filtered tracking error, trying to minimize the same. That is achieved by keeping control signal under saturation limit range. Design tradeoff is that intelligent NN saturation compensator requires extra controller complexity and extra computational power.

VI. CONCLUSIONS

Neural network-based saturation compensation signal is inserted into the actuator control signal, effectively preventing it from being saturated. The proposed NN saturation compensation scheme presents a form of intelligent anti-windup saturation where NN adjusts its output to prevent saturation of the control signal. Simulation results show that the proposed saturation compensation techniques can be effective for a feedback-linearizable class of nonlinear systems.

REFERENCES

[1] K. J. Astrom and B. Wittenmark, *Computer-Controlled Systems: Theory and Design* Prentice Hall; 3rd edition, November 20, 1996.

[2] A.M. Annaswamy, S. Evesque, S. Niculescu, and A.P. Dowling, "Adaptive Control of a Class of Time-delay Systems in the Presence of Saturation," *Adaptive Control of Nonsmooth Dynamic Systems*, Eds. G. Tao and F. Lewis, Springer-Verlag, New York, NY, 2001.

[3] C.W. Chan, and K. Hui "On the existence of globally stable actuator saturation compensators," *International journal of Control*, Vol. 69, No. 6, pp. 773-788, April 1998.

[4] F.-C. Chen, and H. K. Khalil, "Adaptive control of nonlinear systems using neural networks," *Int. J. Contr.*, vol. 55, no. 6, pp. 1299-1317, 1992.

[5] G. Cybenko, "Approximation by superpositions of a sigmoidal function," *Math. Contr. Signals, Syst.*, vol. 2, no. 4, pp. 303-314, 1989.

[6] R. Hanus and Y. Peng, "Conditioning Technique for Controller with Time-Delays," *IEEE Trans. Automat. Contr.*, vol. AC-37, pp. 689-692, May, 1992.

[7] B. Igelnik and Y. H. Pao, "Stochastic Choice of Basis Functions in Adaptive Function Approximation and the Functional-Link Net," *IEEE Trans. Neural Networks*, vol. 6, no. 6, pp. 1320-1329, Nov. 1995.

[8] A. Isidori, *Nonlinear Control Systems*, 2nd ed. Berlin, Germany: Springer_Verlag, 1989.

[9] E. N. Johnson, A. J. Calise, "Neural Network Adaptive Control of Systems with Input Saturation," *In American Control Conference*, Arlington, Virginia, June 2001.

[10] R. L. Kosut, "Design of linear systems with saturating linear control and bounded states". *IEEE Trans. Automat. Contr.*, 28(1), 121-124 1983.

[11] F. L. Lewis, S. Jagannathan, and A. Yesildirek, *Neural Network Control of Robot Manipulators and Nonlinear Systems*, Taylor and Francis, Philadelphia, PA, 1999.

[12] W. T. Miller, R. S. Sutton, and P. J. Werbos, *Neural Networks for Control.* Cambridge, MA: MIT press, 1990.

[13] K. S. Narendra, "Adaptive Control Using Neural Networks," *Neural Networks for Control*, pp. 115-142. ed. W. T. Miller, R. S. Sutton, P. J. Werbos, Cambridge: MIT Press, 1991.

[14] W. Niu and M. Tomizuka, "A Robust Anti-Windup Controller Design for Asymptotic Tracking of Motion Control System Subjected to Actuator Saturation," *The 37th IEEE Conference on Decision and Control*, Tampa, December 1998.

[15] R. R. Selmic and F. L. Lewis, "Deadzone compensation in motion control systems using neural networks," *IEEE Trans. Automat. Contr.*, vol. 45, no. 4, pp. 602-613, April 2000.

[16] J. E. Slotine and W. Li, *Applied Nonlinear Control*, Prentice Hall, 1991.

[17] G. Tao and P. V. Kokotovic, "Continuous-time adaptive control of systems with unknown backlash," *Trans. Automat. Control*, vol. 40, no. 6, pp. 1083-1087, June 1995.

[18] G. Tao and P. V. Kokotovic, *Adaptive Control of Systems With Actuator and Sensor Nonlinearities*, John Wiley & Sons, New York, 1996

[19] K. S. Walgama and J. Sternby, "Inherent observer property in a class of anti-windup compensator", *Int. J. Control*, 52, 3, 705-724 (1990).