On-Line Approximation Based Control of Uncertain Nonlinear Systems with Magnitude, Rate and Bandwidth Constraints on the States and Actuators[1]

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Abstract

This article presents a new method for on-line approximation based backstepping control in the presence of known magnitude, rate, or bandwidth constraints on the intermediate states or actuators. The presentation is based on developing the design and analysis for a second-order system — these results can be recursively extended to higher-order systems. The new results allow on-line learning to continue even when known magnitude, rate, or bandwidth constraints are in effect, even though those constraints do not allow the control objectives to be met for the duration of those constraints.

1 Introduction

A variety of feedback control approaches have been developed to deal with nonlinear systems, including feedback linearization [10], sliding mode control [14], and backstepping [16]. In their ideal form, both feedback linearization and backstepping rely on cancellation of known nonlinearities. To address the issue of uncertainty, several "robustifying" techniques have been developed: (i) *adaptive* methods deal with parametric uncertainty [12], where the nonlinearities are assumed to be known but some of the parameters that multiply these nonlinearities are unknown or uncertain; (ii) *robust* methods deal with the case where known upper bounds on the unknown nonlinearities are available [2] and therefore, they tend to be conservative, sometimes leading to high-gain feedback; (iii) robust adaptive methods combine parametric uncertainty and unknown nonlinearities with partially known bounds [20].

The above control techniques are based on the assumption that the plant nonlinearities are either known or can be bounded by some known functions. In many applications, such as control of high performance aircraft and uninhabited air vehicles (UAVs), some of the nonlinearities need to be approximated on-line. This may be due to modeling errors during the identification/modeling phase or, quite often, due to time-variations in the dynamics as a result of changes in the operating conditions or due to component wear or battle damage. To address the issue of unknown nonlinearities, various control system architectures have incorporated various on-line approximators of unknown nonlinearities [26]. Examples of such on-line approximators include sigmoidal neural networks, splines, radial basis functions, wavelets, etc.

The application of on-line approximation methods to nonlinear systems in a feedback framework yields a complex nonlinear closed-loop system, which is analyzed using Lyapunov stability methods. Typically, the feedback control law and the adaptive law for updating the network weights are derived by utilizing a Lyapunov function, whose time derivative is forced to have some desirable stability properties (for example, negative definiteness). Therefore, the stability of the closed-loop system is obtained during the synthesis of the adaptive control laws. Examples of this type of approach, which is referred to as Lyapunov synthesis method, include [3, 4, 5, 7, 8, 15, 18, 21, 23, 22, 27].

From a practical perspective, one of the key problems in feedback control systems is that the signal $u(t)$ generated

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by the control law cannot be implemented due to physical constraints. A common example of such constraint is input saturation, which imposes limitations on the magnitude of the control input. In some applications this problem is crucial especially in combination with nonlinear on-line approximation based control, which tends to be aggressive in seeking the desired tracking performance. In aircraft control applications, input saturation is caused by limitations in control surface deflections. For UAVs, the absence of humans in the air vehicle may allow more aggressive maneuvering, however the feedback control law has to deal both with unknown nonlinearities and input saturation. Another practical issue of significant importance in many applications, especially in backstepping where states are used as intermediate control variables, is constraints of the state variables. Such constraints include magnitude, rate and bandwidth limitations of the state variables.

Control signal rate and amplitude constraints in an adaptive linear control framework are addressed in, for example, [1, 6, 11, 13, 17, 19, 25]. One possible approach is to completely stop adaptation during saturation of the control input. While this ad-hoc method does prevent the tracking error induced by actuator constraints from corrupting parameter estimation, the stability properties of the closed-loop system cannot be established. Another approach that has been proposed, which we refer to as training signal hedging (TSH), see e.g. [1, 13], modifies the tracking error definition used in the parameter update laws. Finally, a third approach, referred to as pseudo-control hedging (PCH), alters the commanded input to the loop [11, 17]. The idea behind the PCH approach is to attenuate the command to the loop so that the generated control signal is implementable without saturation.

This article presents a robust adaptive backstepping design with state and actuator constraints. A novel aspect of the presented approach is the ability to accommodate magnitude, rate and bandwidth constraints on the actuators signals and each of the intermediate state variables of the backstepping procedure. The results are developed herein for two state scalar subsystems and can be recursively applied for application to systems of higher dimension. The control design framework prevents the presence of input/state constraints from corrupting the learning capabilities and memory of an on-line approximator in feedback control systems.

A second contribution of this article is a new method for addressing the derivative of intermediate control commands (i.e., state commands) in adaptive backstepping. In backstepping, a time derivative of the intermediate control command appears in each step of the backstepping procedure. These time derivatives cannot be directly computed when the plant model is unknown. Many approximate methods have been suggested. The method introduced herein handles these derivative terms directly and rigorously.

2 Standard On-Line Approximation Based Control Problem

In this section we first review the standard on-line approximation based control problem, consider the second-order system

$$\dot{x}_1 = f_1(x) + g_1(x)x_2 \tag{1}$$
$$\dot{x}_2 = f_2(x) + g_2(x)u \tag{2}$$

where $x = [x_1, x_2]^\top$ is the state and u is the control signal. The functions f_i, g_i for $i = 1, 2$ are (possibly nonlinear) Lipshitz functions that are not known. There is a desired trajectory $x_{1c}(t)$, with derivative $\dot{x}_{1c}(t)$, both of which lie in a region $\mathcal{D}$ for $t \geq 0$ and both signals are known[1]. The region $\mathcal{D}$ is the specified operation region of the system and is assumed to be convex and compact. Define the tracking errors

$$\tilde{x}_1 = x_1 - x_{1c}$$
$$\tilde{x}_2 = x_2 - x_{2c}$$

where x_{2c} will be defined by the backstepping controller.

Define approximations to the unknown functions f_i and g_i as

$$\hat{f}_i = \theta_{f_i}^\top \phi_{f_i}(x) \tag{3}$$
$$\hat{g}_i = \theta_{g_i}^\top \phi_{g_i}(x) \tag{4}$$

for $i = 1, 2$, and θ_{f_i} and θ_{g_i} will be estimated on-line. The vector functions ϕ_{f_i} and ϕ_{g_i} are the basis for the function approximation, which are assumed to be uniformly bounded. Many options are available as basis for function approximation: splines, radial basis functions, wavelets, etc. Herein, we are not concerned with the selection or motivation of a particular basis set. For simplicity, in this article we use linearly parameterized approximators. The case of nonlinearly parameterized approximators can also be considered by appropriate handling of the higher-order terms [21].

To simplify the initial analysis, we will assume that the number of basis elements is selected large enough so that there exists $\theta_{f_i}^*$ and $\theta_{g_i}^*$ such that

$$f_i = (\theta_{f_i}^*)^T \phi_{f_i}(x)$$
$$g_i = (\theta_{g_i}^*)^T \phi_{g_i}(x).$$

In practice, the best possible approximation may not be exact, resulting in a residual approximation error, which

[1] This assumption can be satisfied by passing a user specified signal x_{1c}^o through a second order relative degree one prefilter. See Figure 1.

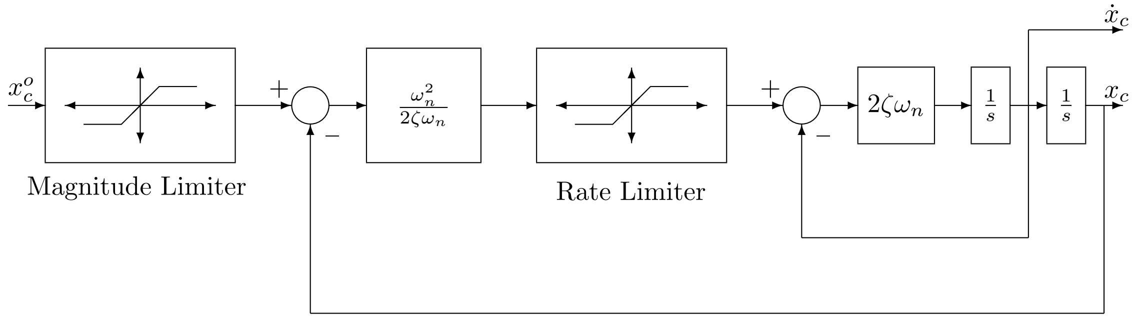

Figure 1: Magnitude, rate, and bandwidth limiting filter.

is referred to as Minimum Functional Approximation Error (MFAE). For simplicity, in this Section we assume zero MFAE. Note that the on-line approximation based control law will not depend on $\theta^*_{f_i}$ and $\theta^*_{g_i}$, and these values therefore need not be known. Define the parameter estimation errors

$$
\begin{aligned}
\tilde{\theta}_{f_i} &= \theta_{f_i} - \theta^*_{f_i} \\
\tilde{\theta}_{g_i} &= \theta_{g_i} - \theta^*_{g_i}.
\end{aligned}
$$

According to the standard backstepping procedure [16, 14], first consider the component subsystem described by (1). Assume that there exist a smooth feedback control $x_2 = \mu_1(x_1, \tilde{x}_1, \dot{x}_{1c}, \theta_{f_1}, \theta_{g_1})$ and adaptation laws of the form

$$
\begin{aligned}
\dot{\theta}_{f_1} &= \alpha_1(x_1, \tilde{x}_1) && (5)\\
\dot{\theta}_{g_1} &= \beta_1(x_1, \tilde{x}_1, \mu_1) && (6)
\end{aligned}
$$

and a smooth positive definite function $\mathcal{V}_1(\tilde{x}_1, \tilde{\theta}_{f_1}, \tilde{\theta}_{g_1})$ such that

$$
\frac{\partial \mathcal{V}_1}{\partial \tilde{x}_1}\left(\hat{f}_1 + \hat{g}_1\mu_1 - \dot{x}_{1c} + (f_1 - \hat{f}_1) + (g_1 - \hat{g}_1)\mu_1\right)
$$

$$
+\frac{\partial \mathcal{V}_1}{\partial \tilde{\theta}_f}\alpha_1(x_1, \tilde{x}_1) + \frac{\partial \mathcal{V}_1}{\partial \tilde{\theta}_g}\beta_1(x_1, \tilde{x}_1, \mu_1) \quad \leq \quad -W(\tilde{x}_1) \quad (7)
$$

where $W(\tilde{x}_1)$ is positive definite in $\tilde{x}_1$.

Example. An example of an on-line approximation based controller satisfying the above assumptions is

$$
\begin{aligned}
\mu_1 &= \frac{1}{\hat{g}_1}\left(-k_1\tilde{x}_1 - \hat{f}_1 + \dot{x}_{1c}\right) && (8)\\
\dot{\theta}_{f_1} &= \Gamma_{f_1}\tilde{x}_1\phi_{f_1} && (9)\\
\dot{\theta}_{g_1} &= \Gamma_{g_1}\tilde{x}_1\phi_{g_1}\mu_1 && (10)
\end{aligned}
$$

where $k_1 > 0$ is a scalar and Γ_{f_1}, Γ_{g_1} are positive definite matrices representing the learning rate. The adaptive law (10) needs to be modified using, for example, a projection modification [9] to ensure that $\hat{g}_1$ is bounded away from zero. In this example, the Lyapunov function $\mathcal{V}_1$ and the positive definite function $W(\tilde{x}_1)$ are given by

$$
\begin{aligned}
\mathcal{V}_1(\tilde{x}_1, \tilde{\theta}_{f_1}, \tilde{\theta}_{g_1}) &= \frac{1}{2}\left(\tilde{x}_1^2 + \tilde{\theta}_{f_1}^\top \Gamma_{f_1}^{-1}\tilde{\theta}_{f_1} + \tilde{\theta}_{g_1}^\top \Gamma_{g_1}^{-1}\tilde{\theta}_{g_1}\right)\\
W(\tilde{x}_1) &= \tilde{x}_1^2
\end{aligned}
$$

Through a recursive backstepping procedure, the control of the second subsystem (2) can be obtained [16, 14, 21]. Next, we consider the case where the state and/or control signals are constrained by physical limitations. We introduce some new tools for handling such systems.

3 State and Actuator Constrained Scalar Problem

When the state and actuators have physical limitations, the above approach may not be able to be successfully implemented. To address magnitude, rate, and bandwidth constraints on the state and control, define the following procedure:

1. Define

$$
\begin{aligned}
x^o_{2c} &= \mu_1(x_1, \tilde{x}_1, \dot{x}_{1c}, \theta_{f_1}, \theta_{g_1}) - \xi_2 && (11)\\
\dot{\xi}_1 &= -k_1\,\xi_1 + \hat{g}_1\left(x_{2c} - x^o_{2c}\right), && (12)
\end{aligned}
$$

   where ξ_2 will be defined in step 3. The signal x^o_{2c} is filtered to produce the magnitude, rate, and bandwidth limited command signal x_{2c} and its derivative $\dot{x}_{2c}$ that is within the operating envelope $\mathcal{D}$ of the system. Such a filter is shown in Figure 1.

2. Define the compensated tracking errors as

$$
\bar{x}_i = \tilde{x}_i - \xi_i, \text{ for } i = 1, 2. \qquad (13)
$$

3. Define[2]

$$
\begin{aligned}
u^o_c &= \frac{1}{\hat{g}_2}\left(-k_2\tilde{x}_2 + \dot{x}_{2c} - \hat{f}_2 - \hat{g}_1\bar{x}_1\right) && (14)\\
\dot{\xi}_2 &= -k_2\xi_2 + \hat{g}_2(u_c - u^o_c) && (15)
\end{aligned}
$$

[2] Note that u^o_c is computed using $\dot{x}_{2c}$, not $\dot{x}^o_{2c}$. The quantity $\dot{x}_{2c}$ is available as the output of the filter in step 1. The quantity $\dot{x}^o_{2c}$ is not available and cannot be computed, since the time derivative of μ_1 is not tractable.

where u_c^o is filtered to produce u_c which is within the magnitude, rate, and bandwidth limitations of the actuation system. Therefore, u_c is achievable by the actuators. We therefore assume that $u = u_c$.

4. Define the parameter update laws according to

$$\begin{aligned} \dot{\theta}_{f_1} &= \Gamma_{f_1}\phi_{f_1}\bar{x}_1 = \alpha_1(x,\bar{x}_1) && (16)\\ \dot{\theta}_{f_2} &= \Gamma_{f_2}\phi_{f_2}\bar{x}_2 && (17)\\ \dot{\theta}_{g_1} &= \Gamma_{g_1}\phi_{g_1}\bar{x}_1 x_2 = \beta_1(x,\bar{x}_1) && (18)\\ \dot{\theta}_{g_2} &= \Gamma_{g_2}\phi_{g_2}\bar{x}_2 u. && (19) \end{aligned}$$

Note that the parameter update laws for θ_{f_1} and θ_{g_1} are similar to the corresponding update laws (9), (10), derived in the standard on-line approximation based control problem with the tracking error $\tilde{x}_i$ being replaced by the compensated tracking error $\bar{x}_i$. As we will see later, the use of the compensated tracking error in the update laws is crucial in preventing state and actuator constraints in on-line approximation schemes from destroying their previously learned information.

In the definition of the control law (14), we again assume that the update law for θ_{g_2} uses some type of projection modification in order to ensure that $\hat{g}_2$ is bounded away from zero. An implicit assumption here (which is standard in the adaptive control literature [9]) is that the sign of g_1 and g_2 are known. Moreover, typically a known lower bound for $|g_1|$ and $|g_2|$ is assumed to be available. It is noted that this requirement, which is a consequence of the *stabilizability* problem of adaptive schemes, arises independent of the issue of handling magnitude, rate and bandwidth constraints of the state and the actuators, which is the main topic of this article.

Given the above procedure, we now analyze the stability of the control law subject to the physical limitations. The tracking error dynamics can be written as

$$\begin{aligned} \dot{\tilde{x}}_1 &= \hat{f}_1 + \hat{g}_1\, x_{2c}^o - \dot{x}_{1c} + (f_1 - \hat{f}_1)\\ &\quad +\hat{g}_1(x_{2c} - x_{2c}^o) + (g_1 x_2 - \hat{g}_1\, x_{2c})\\ &= \hat{f}_1 + \hat{g}_1\mu_1 - \dot{x}_{1c} - \hat{g}_1\, \xi_2 + (f_1 - \hat{f}_1) && (20)\\ &\quad +\hat{g}_1(x_{2c} - x_{2c}^o) + (g_1 x_2 - \hat{g}_1\, x_{2c})\\ \dot{\tilde{x}}_2 &= \hat{f}_2 + \hat{g}_2\, u_c^o - \dot{x}_{2c} + (f_2 - \hat{f}_2)\\ &\quad +\hat{g}_2(u_c - u_c^o) + (g_2 u - \hat{g}_2\, u_c)\\ &= -k_2\tilde{x}_2 - \hat{g}_1\bar{x}_1 + (f_2 - \hat{f}_2) && (21)\\ &\quad +\hat{g}_2(u_c - u_c^o) + (g_2 - \hat{g}_2)u. \end{aligned}$$

As defined in (12) and (15), the variables ξ_1, ξ_2 represent the filtered effect of the non-achievable portion of x_{2c} and the control signal u_c respectively. The variables $\bar{x}_i$ represent the compensated tracking errors, obtained after removing the corresponding non-achievable portion of x_{2c} and u_c. After some algebraic manipulation, the dynamics of the compensated tracking errors are described by

$$\begin{aligned} \dot{\bar{x}}_1 &= \hat{f}_1 + \hat{g}_1\, \mu_1 - \dot{x}_{1c} - \hat{g}_1\xi_2 + (f_1 - \hat{f}_1)\\ &\quad +(g_1 x_2 - \hat{g}_1 x_{2c}) + k_1\xi_1\\ &= \hat{f}_1 + \hat{g}_1\mu_1 + k_1\tilde{x}_1 - \dot{x}_{1c} - k_1\, \bar{x}_1 + (f_1 - \hat{f}_1)\\ &\quad +(g_1 - \hat{g}_1)x_2 + \hat{g}_1\bar{x}_2 && (22)\\ \dot{\bar{x}}_2 &= -k_2\bar{x}_2 - \hat{g}_1\bar{x}_1 + (f_2 - \hat{f}_2) + (g_2 - \hat{g}_2)\, u. && (23) \end{aligned}$$

Consider the following Lyapunov function candidate

$$\begin{aligned} \mathcal{V} &= \sum_{i=1}^{2} \mathcal{V}_i(\bar{x}_i, \tilde{\theta}_{f_i}, \tilde{\theta}_{g_i})\\ &= \sum_{i=1}^{2} \frac{1}{2}\left(\bar{x}_i^2 + \tilde{\theta}_{f_i}^\top \Gamma_{f_i}^{-1}\tilde{\theta}_{f_i} + \tilde{\theta}_{g_i}^\top \Gamma_{g_i}^{-1}\tilde{\theta}_{g_i}\right) && (24) \end{aligned}$$

The derivative of $\mathcal{V}$ along solutions of eqns. (16-19) and (22-23) is

$$\begin{aligned} \dot{\mathcal{V}}_1 &= \bar{x}_1[\hat{f}_1 + \hat{g}_1\mu_1 + k_1\tilde{x}_1 - \dot{x}_{1c} - k_1\, \bar{x}_1 + (f_1 - \hat{f}_1)\\ &\quad +(g_1 - \hat{g}_1)x_2 + \hat{g}_1\bar{x}_2] + \tilde{\theta}_{f_1}^\top\phi_{f_1}\bar{x}_1 + \tilde{\theta}_{g_1}^\top\phi_{g_1}\bar{x}_1 x_2\\ &= -k_1\bar{x}_1^2 + \bar{x}_1\hat{g}_1\bar{x}_2\\ \dot{\mathcal{V}}_2 &= \bar{x}_2\left(-k_2\bar{x}_2 - \hat{g}_1\bar{x}_1 + (f_2 - \hat{f}_2) + (g_2 - \hat{g}_2)u\right)\\ &\quad +\tilde{\theta}_{f_2}^\top\phi_{f_2}\bar{x}_2 + \tilde{\theta}_{g_2}^\top\phi_{g_2}\bar{x}_2 u\\ &= -k_2\bar{x}_2^2 - \bar{x}_2\hat{g}_1\bar{x}_1\\ \dot{\mathcal{V}} &= \dot{\mathcal{V}}_1 + \dot{\mathcal{V}}_2 = -k_1\bar{x}_1^2 - k_2\bar{x}_2^2. && (25) \end{aligned}$$

Since $\dot{\mathcal{V}}$ is negative semi-definite, the variables $\bar{x}_1$, $\bar{x}_2$, θ_{f_1}, θ_{f_2}, θ_{g_1}, θ_{g_2} are each bounded. Therefore, $\hat{f}_i$ and $\hat{g}_i$ are bounded for $i = 1,\ 2$. Since $\ddot{\mathcal{V}}$ is bounded, Barbalat's lemma implies that $\bar{x}_1$ and $\bar{x}_2$ each approach zero as t approaches infinity. Finally, the last line of the above analysis implies that $\bar{x}_1$ and $\bar{x}_2$ are each in $\mathcal{L}_2$, since

$$\begin{aligned} \dot{\mathcal{V}} &= -k_1\bar{x}_1^2 - k_2\bar{x}_2^2\\ \mathcal{V}(t) - \mathcal{V}(0) &= -\int_0^t k_1\bar{x}_1^2(\tau) + k_2\bar{x}_2^2(\tau)d\tau\\ \mathcal{V}(0) &\geq \int_0^\infty k_1\bar{x}_1^2(\tau) + k_2\bar{x}_2^2(\tau)d\tau. \end{aligned}$$

Therefore, we can summarize these results in the following theorem.

Theorem 1 *Given a system described as (1-2). Let the on-line approximation based control law of eqns. (5-6) solve the tracking problem for system*

$$\dot{x}_1 = f_1(x) + g_1(x)\mu_1$$

with Lyapunov function $\mathcal{V}_1$ satisfying (7). Then the on-line approximation based controller of (11-19), with physical contstraints, solves the tracking problem with Lyapunov function (24) satisfying (25), which guarantees:

1. $\bar{x}_1$*,* $\bar{x}_2$*,* θ_{f_1}*,* θ_{f_2}*,* θ_{g_1}*,* $\theta_{g_2} \in \mathcal{L}_\infty$*;*

2. $\bar{x}_1$ *and* $\bar{x}_2 \in \mathcal{L}_2$*;*

3. $\lim_{t\to\infty} \bar{x}_1(t) = 0$*;* $\lim_{t\to\infty} \bar{x}_2(t) = 0$*.*

Note that this theorem can be applied recursively (n-1) times to address a system with n states.

The goal of the derivation of this theorem was the ability to accommodate magnitude, rate, and bandwidth limitations of the physical system. This goal has been achieved in the sense that on-line approximation will continue to function correctly even when the physical limitations do not allow the the desired control signals to be implemented. Note that the theorem guarantees desirable properties for the compensated tracking errors $\bar{x}_i$, not the actual tracking errors $\tilde{x}_i$. During periods when the control signals are not physically achievable, the system is incapable of achieving at least one command x^o_{ic} for some i. Once the physical constraint is no longer in effect, $\tilde{x}_i \to \bar{x}_i$.

In addition to achieving the desired objective, the approach described above eliminates another complexity of the backstepping approach (see pp. 588-589 in [14]). Without the command filter introduced herein, implementation of the backstepping approach requires calculation of $\dot{\mu}_1$. When the model (f_1 and g_1) are known, this is possible, but cumbersome as the number of iterations of the Lemma 13.2 of [14] increases. When the model is not known, but estimated on-line, the calculation of $\dot{\mu}_1$ is usually very complicated since it involves the rate of change of the functions that are being approximated. Various authors address this term in different ways [16]. The theorem of this section handles this term rigorously using filtering techniques.

4 Conclusions

This article has presented an extension of the backstepping approach with on-line approximation. The primary goal of this extension is to allow on-line approximation to continue to function correctly, even when physical limitations of the system (magnitude, rate, or bandwidth limitations of the state or actuators) affect the system performance.

The presented approach also removes an implementation difficulty that previously existed for adaptive backstepping approaches and a computation inconvenience for nonadaptive backstepping. That difficulty was the computation of the time derivative of the commands to intermediate states.

Due to space constraints, we were not able to include an example in this article with the proper level of attention. Preliminary examples showing good performance are included in [6] and [19]. The method has been successfully applied to full six degree of freedom flight control for an unmanned vehicle (in a medium fidelity simulation). An article describing those results is under review.

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