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An Adaptive Observer for Fault Diagnosis in Nonlinear Discrete-Time Systems

Fabrizio Caccavale
Dipartimento di Ingegneria e Fisica dell'Ambiente
Università degli Studi della Basilicata
Viale dell'Ateneo Lucano 10,
85100 Potenza, ITALY
E-mail: caccavale@unibas.it

Luigi Villani
Dipartimento di Informatica e Sistemistica
Università degli Studi di Napoli Federico II
Via Claudio 21,
80125 Napoli, ITALY
E-mail: luigi.villani@unina.it

***Abstract*— This paper deals with the problem of Fault Diagnosis (FD) for a class of nonlinear systems in the presence of actuator failures. The scheme is based on a discrete-time diagnostic observer which computes an estimate of the system's state. To cope with the uncertainties and discretization errors, a discrete-time adaptive law is developed, based on a parametric model of the uncertain terms. A stability proof is developed to prove the global exponential stability of the system in the absence of faults. The effectiveness of the proposed approach is experimentally tested on a case study developed for an industrial mechanical manipulator.**

I. Introduction

The control system of a plant is often required not only to ensure stability and desired performance during normal operating conditions, but also to guarantee a suitable behavior at the occurrence of faults in the controlled plant, e.g. an actuator or a sensor failure. The early diagnosis of such failures could prevent the propagation of the fault effects to the critical components of the plant and permit an appropriate response of the control system. Therefore it is crucial to develop a Fault Diagnosis (FD) system capable to promptly detect the occurrence of failures (fault detection), recognize the location (fault isolation) and estimate the time evolution (fault identification) of the detected failures.

The research on FD systems has produced several contributions, especially for the case of linear systems [3], [7], [9]. Onthe other hand, fault detection methods for nonlinear systems can be roughly regrouped in three main classes: observer-based approaches, techniques based on model parameters estimation and algorithms based on learning methodologies [4], [11], [12], [13], [14], [15]. Recently, soft computing methods, integrating quantitative and qualitative modelling information, have been developed to improve the performance of FD schemes for uncertain systems (see, e.g., [10] and references therein).

Usually, the observer-based methods require a model of the system to be operated in parallel to the process (i.e., the so called diagnostic observer). Then, a set of variables sensitive to the occurrence of a failure (residuals) are computed as the difference between the measured output variables and those predicted via the diagnostic observer. Assuming an exact knowledge of the plant dynamics, the residuals should become nonzero when a fault occurs.

It should be pointed out, however, that perfect knowledge of the model is rarely a reasonable assumption; moreover, the discrete time implementation of the diagnostic observer may introduce additional errors. On the other hand, the robustness of the observer can be improved only at the expense of a reduced sensitivity to failures; hence, suitable trade-off solutions must be devised in the design of the FD system for a given plant.

In this paper, a discrete-time diagnostic observer is designed, where a term compensating for unmodeled dynamics, disturbances and noise is included. An adaptive strategy for computing the compensation term is designed by using the Lyapunov method. Then, the residuals are computed on the basis of the observer outputs.

A stability analysis is carried out to prove that, in the case of perfect parametrization of the unmodeled dynamics, the system is globally exponentially stable, provided that a suitable persistence of excitation condition is satisfied. This property ensures that the residuals remains ultimately uniformly bounded during normal operating conditions, also in the presence of bounded uncompensated disturbances(e.g., errors due to the discrete-time implementation of the observer).

The proposed FD scheme is experimentally tested on a six-degree-of-freedom industrial manipulator. The results demonstrate the effectiveness of the approach to achieve fault diagnosis for industrial gear-driven robots, for which the effects like backlash and friction are usually not negligible and difficult to model.

II. Modeling

Consider a nonlinear dynamical system characterized by the following discrete-time state-space model

$$\boldsymbol{x}(k+1) = \boldsymbol{A}\boldsymbol{x}(k) + \boldsymbol{h}(\boldsymbol{x}(k)) + \boldsymbol{B}(\boldsymbol{x}(k))\boldsymbol{u}(k) + \boldsymbol{\eta}(k, \boldsymbol{x}(k), \boldsymbol{u}(k)) \quad (1)$$

where $\boldsymbol{x}$ is the $(n \times 1)$ state vector, $\boldsymbol{u}$ is the $(m \times 1)$ vector of inputs. The term $\boldsymbol{\eta}$ collects all disturbances and uncertainties.

The above model may represent the discrete-time equivalent of a continuous model obtained, e.g., obtained via the well-known Euler method [5].

The class of failures considered in this work is that of *actuator* faults. This class of failures can be represented as an unknown additive disturbance on the *nominal* inputs to the system $\bar{\boldsymbol{u}}$. Hence, an actuator fault occurring at the k-th time step results in a faulty input given by

$$\boldsymbol{u}(k) = \bar{\boldsymbol{u}}(k) + \boldsymbol{\delta u}(k), \tag{2}$$

where $\boldsymbol{\delta u}(k)$ represents the time profile of the unknown fault.

Therefore, the nominal dynamics (1) in the presence of faults becomes

$$\begin{aligned}\boldsymbol{x}(k+1) = \boldsymbol{A}\boldsymbol{x}(k) + \boldsymbol{h}(\boldsymbol{x}(k)) + \boldsymbol{B}(\boldsymbol{x}(k))\bar{\boldsymbol{u}}(k) + \\ \boldsymbol{\eta}(k, \boldsymbol{x}(k), \bar{\boldsymbol{u}}(k), \boldsymbol{\theta}) + \boldsymbol{f}(k, \boldsymbol{x}(k)),\end{aligned} \tag{3}$$

where the fault vector $\boldsymbol{f}$ is given by:

$$\boldsymbol{f}(k, \boldsymbol{x}(k)) = \boldsymbol{B}(\boldsymbol{x}(k))\, \boldsymbol{\delta u}(k). \tag{4}$$

The uncertain term $\boldsymbol{\eta}(k, \boldsymbol{x}(k), \bar{\boldsymbol{u}}(k))$ is assumed to depend upon the nominal input and on the $(q \times 1)$ constant parameters vector $\boldsymbol{\theta}$. If $\boldsymbol{\eta}$ is linear in the parameters vector, it can be expressed as follows

$$\boldsymbol{\eta}(k, \boldsymbol{x}(k), \bar{\boldsymbol{u}}(k), \boldsymbol{\theta}) = \boldsymbol{\Omega}(k, \boldsymbol{x}(k), \bar{\boldsymbol{u}}(k))\, \boldsymbol{\theta}, \tag{5}$$

where the matrix $\boldsymbol{\Omega}$ is assumed to be known, while $\boldsymbol{\theta}$ is usually unknown (or partially known).

On the other hand, if $\boldsymbol{\eta}$ is not linear in the parameters and/or its structure is not exactly known, a good approximation can be obtained by resorting to the so-called on line interpolators [4], [11], [12], [13], [14], [15] (e.g., neural networks, splines). By choosing a linear-in-the-parameters interpolator structure, the uncertain term can be expressed as

$$\begin{aligned}\boldsymbol{\eta}(k, \boldsymbol{x}(k), \bar{\boldsymbol{u}}(k), \boldsymbol{\theta}) = \boldsymbol{\Omega}(k, \boldsymbol{x}(k), \bar{\boldsymbol{u}}(k))\, \boldsymbol{\theta} + \\ \boldsymbol{\epsilon}(k, \boldsymbol{x}(k), \bar{\boldsymbol{u}}(k)),\end{aligned} \tag{6}$$

where $\boldsymbol{\epsilon}$ represents the interpolation error. The following assumptions are made:

Assumption 1: The norm of the matrix $\boldsymbol{\Omega}(k, \boldsymbol{x}(k), \bar{\boldsymbol{u}}(k))$ is uniformly bounded by a constant $\Omega > 0$, i.e.:

$$\|\boldsymbol{\Omega}(k, \boldsymbol{x}(k), \bar{\boldsymbol{u}}(k))\| \leq \Omega.$$

Assumption 2: The norm of the interpolation error $\boldsymbol{\epsilon}(k, \boldsymbol{x}(k), \bar{\boldsymbol{u}}(k))$ is uniformly bounded by a constant $E > 0$, i.e.:

$$\|\boldsymbol{\epsilon}(k, \boldsymbol{x}(k), \bar{\boldsymbol{u}}(k))\| \leq E.$$

Notice that Assumption 1 can be satisfied for a wide class of functions $\boldsymbol{\Omega}$, by assuming that the the state $\boldsymbol{x}(k)$ and the input $\bar{\boldsymbol{u}}(k)$ of the system remains bounded even in the presence of failures. Moreover, Assumption 2 is often satisfied provided that a suitable interpolator structure is chosen (see, e.g., [6] and [8] for the approximation properties of neural networks).

III. Observer-based fault diagnosis

Assuming the whole state measurable, the proposed diagnostic observer has the following structure

$$\begin{aligned}\widehat{\boldsymbol{x}}(k+1) = \boldsymbol{A}\widehat{\boldsymbol{x}}(k) + \boldsymbol{h}(\boldsymbol{x}(k)) + \boldsymbol{B}(\boldsymbol{x}(k))\bar{\boldsymbol{u}}(k) + \\ \boldsymbol{K}_o\boldsymbol{e}(k) + \widehat{\boldsymbol{\eta}}(k, \boldsymbol{x}(k), \bar{\boldsymbol{u}}(k), \widehat{\boldsymbol{\theta}}(k))\end{aligned} \tag{7}$$

where $\boldsymbol{e} = \boldsymbol{x} - \widehat{\boldsymbol{x}}$ is the state estimation error, $\widehat{\boldsymbol{\eta}}$ represents an estimate of the uncertainties $\boldsymbol{\eta}$, $\widehat{\boldsymbol{\theta}}$ is an estimate of $\boldsymbol{\theta}$ and the matrix gain $\boldsymbol{K}_o$ is chosen such that $\boldsymbol{F} = \boldsymbol{A} - \boldsymbol{K}_o$ has all its eigenvalues in the unit circle.

It is worth remarking that the above observer should be seen as a diagnostic observer, i.e., a model which should reproduce the real behavior of the plant during its normal operations. Therefore, the primary goal of the observer is not the state estimation from input/output measurements, but the accurate reconstruction of the plant dynamics in real time, even in the presence of disturbances and modelling errors.

Therefore, in view of (3) and (7), the state estimation error dynamics is given by

$$\begin{aligned}\boldsymbol{e}(k+1) = \boldsymbol{F}\boldsymbol{e}(k) + \widetilde{\boldsymbol{\eta}}(k, \boldsymbol{x}(k), \bar{\boldsymbol{u}}(k), \boldsymbol{\theta}, \widehat{\boldsymbol{\theta}}(k)) + \\ \boldsymbol{f}(k, \boldsymbol{x}(k)),\end{aligned} \tag{8}$$

where

$$\begin{aligned}\widetilde{\boldsymbol{\eta}}(k, \boldsymbol{x}(k), \bar{\boldsymbol{u}}(k), \boldsymbol{\theta}, \widehat{\boldsymbol{\theta}}) = \boldsymbol{\eta}(k, \boldsymbol{x}(k), \bar{\boldsymbol{u}}(k), \boldsymbol{\theta}) - \\ \widehat{\boldsymbol{\eta}}(k, \boldsymbol{x}(k), \bar{\boldsymbol{u}}(k), \widehat{\boldsymbol{\theta}})\end{aligned} \tag{9}$$

represents the uncertainties estimation error.

Hereafter, for notation compactness, the explicit dependence of the functions upon $\boldsymbol{x}(k)$ and $\bar{\boldsymbol{u}}(k)$ will be omitted.

The residuals vector can be chosen as:

$$\boldsymbol{r}(k+1) = \boldsymbol{e}(k+1) - \boldsymbol{F}\boldsymbol{e}(k), \tag{10}$$

which can be rewritten as

$$\boldsymbol{r}(k+1) = \widetilde{\boldsymbol{\eta}}(k, \boldsymbol{\theta}, \widehat{\boldsymbol{\theta}}) + \boldsymbol{f}(k). \tag{11}$$

It can be recognized that the residuals vector is affected by the fault vector and the estimation error of the uncertain term, i.e. $\widetilde{\boldsymbol{\eta}}$. Hence, if an accurate estimation of $\boldsymbol{\eta}$ is achieved, the fault signature on the residual (i.e., its effect on the residuals) becomes more evident.

IV. Adaptive uncertainties estimation

If a parametric model of the uncertainties is available, an adaptive estimation algorithm of the unknown parameters can be set up. It is worth remarking that such a paradigm has been keenly exploited for adaptive fault identification (see, e.g., the work in [4], [11]–[15]). However, in this work the same concept is exploited in order to adaptively compensate for the uncertainties, so as to obtain small values of the residuals in the absence of faults.

In this case, the uncertain term can be indirectly evaluated through the estimation of $\boldsymbol{\theta}$. Namely, an adaptive update law for the parameters estimate $\boldsymbol{\theta}(k)$ can be chosen as

$$\widehat{\boldsymbol{\theta}}(k+1) = \widehat{\boldsymbol{\theta}}(k) + \boldsymbol{\Omega}^T(k)\boldsymbol{\Gamma}_\theta(k)\left(\boldsymbol{e}(k+1) - \boldsymbol{F}\boldsymbol{e}(k)\right). \tag{12}$$

The gain matrix $\boldsymbol{\Gamma}_\theta(k)$ is chosen as follows:

$$\boldsymbol{\Gamma}_\theta(k) = 2\left(\boldsymbol{\Omega}(k)\boldsymbol{\Omega}^T(k) + \boldsymbol{Q}_\theta\right)^{-1}, \qquad (13)$$

where $\boldsymbol{Q}_\theta$ is a positive definite symmetric matrix; hence, $\boldsymbol{\Gamma}_\theta(k)$ is symmetric and positive definite for all k. Notice that, by virtue of Assumption 1, $\boldsymbol{\Gamma}_\theta(k)$ can be lower bounded as

$$\|\boldsymbol{\Gamma}_\theta(k)\| \geq \frac{2}{\Omega^2 + Q_\theta} = \gamma > 0, \qquad (14)$$

where Q_θ is the largest eigenvalue of $\boldsymbol{Q}_\theta$.

The uncertainties estimation error (9), in view of (6) can be written as

$$\widetilde{\boldsymbol{\eta}}(k, \widetilde{\boldsymbol{\theta}}(k)) = \boldsymbol{\Omega}(k)\widetilde{\boldsymbol{\theta}}(k) + \boldsymbol{\epsilon}(k), \qquad (15)$$

where $\widetilde{\boldsymbol{\theta}}(k) = \boldsymbol{\theta} - \widehat{\boldsymbol{\theta}}(k)$ is the parameters estimation error.

By taking into account (8), (12) and (15), the state estimation error dynamics can be written as

$$\begin{cases} \boldsymbol{e}(k+1) = \boldsymbol{F}\boldsymbol{e}(k) + \boldsymbol{\Omega}(k)\widetilde{\boldsymbol{\theta}}(k) + \boldsymbol{f}(k) + \boldsymbol{\epsilon}(k) \\ \widetilde{\boldsymbol{\theta}}(k+1) = \boldsymbol{H}(k)\widetilde{\boldsymbol{\theta}}(k) - \boldsymbol{\Omega}^T(k)\boldsymbol{\Gamma}_\theta(k)\left(\boldsymbol{f}(k) + \boldsymbol{\epsilon}(k)\right), \end{cases} \qquad (16)$$

where the matrix $\boldsymbol{H}$ is given by

$$\boldsymbol{H}(k) = \boldsymbol{I}_p - \boldsymbol{\Omega}^T(k)\boldsymbol{\Gamma}_\theta(k)\boldsymbol{\Omega}(k), \qquad (17)$$

and $\boldsymbol{I}_p$ denotes the $(p \times p)$ identity matrix.

Therefore, the residuals vector becomes

$$\boldsymbol{r}(k+1) = \boldsymbol{\Omega}(k)\widetilde{\boldsymbol{\theta}}(k) + \boldsymbol{f}(k) + \boldsymbol{\epsilon}(k). \qquad (18)$$

V. Stability analysis

In order to analyze the convergence of the state estimation error, it is worth considering the estimation error dynamics in the absence of faults ($\boldsymbol{f} = \mathbf{0}$) and of interpolation errors ($\boldsymbol{\epsilon} = \mathbf{0}$)

$$\boldsymbol{e}(k+1) = \boldsymbol{F}\boldsymbol{e}(k) + \boldsymbol{\Omega}(k)\widetilde{\boldsymbol{\theta}}(k) \qquad (19)$$

$$\widetilde{\boldsymbol{\theta}}(k+1) = \boldsymbol{H}(k)\widetilde{\boldsymbol{\theta}}(k). \qquad (20)$$

Let

$$\boldsymbol{z} = \begin{bmatrix} \boldsymbol{e} \\ \widetilde{\boldsymbol{\theta}} \end{bmatrix},$$

the $((n+p) \times 1)$ state vector of the system (19),(20). The following theorem can be proven:

Theorem 1: Under Assumption 1, the equilibrium $\boldsymbol{z}_{eq} = [\,\boldsymbol{e}_{eq}^T \quad \widetilde{\boldsymbol{\theta}}_{eq}^T\,]^T = \mathbf{0}$ of the system (19),(20) is globally uniformly stable. Moreover, the estimation error $\boldsymbol{e}(k)$ converges asymptotically to $\mathbf{0}$.

Proof 1: Consider the positive definite Lyapunov function candidate:

$$V(k) = \boldsymbol{e}(k)^T\boldsymbol{P}_e\boldsymbol{e}(k) + p_\theta\widetilde{\boldsymbol{\theta}}^T(k)\widetilde{\boldsymbol{\theta}}(k), \qquad (21)$$

where p_θ is a positive constant and $\boldsymbol{P}_e$ is the solution to the Lyapunov equation

$$\boldsymbol{P}_e - \boldsymbol{F}^T\boldsymbol{P}_e\boldsymbol{F} = \boldsymbol{Q}_e$$

for a given symmetric and positive matrix $\boldsymbol{Q}_e$. Since $\boldsymbol{F}$ has all its eigenvalues inside the unit circle, the solution $\boldsymbol{P}_e$ exists and is symmetric and positive definite.

By taking into account (19)(20), the first difference of V, $\Delta V(k) = V(k+1) - V(k)$, is given by:

$$\begin{aligned} &\Delta V(k) = -\boldsymbol{e}(k)^T\boldsymbol{Q}_e\boldsymbol{e}(k) - \\ &\widetilde{\boldsymbol{\theta}}^T(k)(p_\theta\boldsymbol{I}_p - p_\theta\boldsymbol{H}^T(k)\boldsymbol{H}(k) - \boldsymbol{\Omega}^T(k)\boldsymbol{P}_e\boldsymbol{\Omega}(k))\widetilde{\boldsymbol{\theta}}(k) + \\ &2\boldsymbol{e}(k)^T\boldsymbol{F}^T\boldsymbol{P}_e\boldsymbol{\Omega}(k)\widetilde{\boldsymbol{\theta}}(k). \end{aligned} \qquad (22)$$

By exploiting the expression of matrix $\boldsymbol{H}$ in (17) and equation (13), it can be argued that

$$\begin{aligned} &\Delta V(k) = -\boldsymbol{e}(k)^T\boldsymbol{Q}_e\boldsymbol{e}(k) - \\ &\widetilde{\boldsymbol{\theta}}^T(k)\boldsymbol{\Omega}^T(k)(p_\theta\boldsymbol{\Gamma}_\theta^T(k)\boldsymbol{Q}_\theta\boldsymbol{\Gamma}_\theta(k) - \boldsymbol{P}_e)\boldsymbol{\Omega}(k)\widetilde{\boldsymbol{\theta}}(k) + \\ &2\boldsymbol{e}(k)^T\boldsymbol{F}^T\boldsymbol{P}_e\boldsymbol{\Omega}(k)\widetilde{\boldsymbol{\theta}}(k). \end{aligned} \qquad (23)$$

Moreover, from (15), written in the case $\boldsymbol{\epsilon} = \mathbf{0}$, the following equality holds:

$$\begin{aligned} &\Delta V(k) = -\boldsymbol{e}(k)^T\boldsymbol{Q}_e\boldsymbol{e}(k) - \\ &\widetilde{\boldsymbol{\eta}}^T(k)(p_\theta\boldsymbol{\Gamma}_\theta(k)\boldsymbol{Q}_\theta\boldsymbol{\Gamma}_\theta(k) - \boldsymbol{P}_e)\widetilde{\boldsymbol{\eta}}(k) + \\ &2\boldsymbol{e}(k)^T\boldsymbol{F}^T\boldsymbol{P}_e\widetilde{\boldsymbol{\eta}}(k), \end{aligned} \qquad (24)$$

where the dependence of $\widetilde{\boldsymbol{\eta}}$ upon the argument $\widetilde{\boldsymbol{\theta}}(k)$ has been skipped for notation compactness.

Function $\Delta V(k)$ can be upper bounded as

$$\begin{aligned} \Delta V(k) \leq -q_e\|\boldsymbol{e}(k)\|^2 - (p_\theta\gamma^2 q_\theta - P_e)\|\widetilde{\boldsymbol{\eta}}(k)\|^2 + \\ 2fP_e\|\boldsymbol{e}(k)\|\|\widetilde{\boldsymbol{\eta}}(k)\|, \end{aligned}$$

where q_θ is the smallest eigenvalue of $\boldsymbol{Q}_\theta$, q_e is the smallest eigenvalue of $\boldsymbol{Q}_e$, P_e is the largest eigenvalue of $\boldsymbol{P}_e$, and F is the norm of the matrix $\boldsymbol{F}$. Hence $\Delta V(k) \leq 0$, provided that p_θ satisfy the inequality

$$p_\theta > \frac{P_e q_e + F^2 P_e^2}{\gamma^2 q_\theta}. \qquad (25)$$

Notice that $\Delta V(k)$ is negative definite in the variables $\boldsymbol{e}(k)$, $\widetilde{\boldsymbol{\eta}}(k)$ but is only semi-definite in the variable $\boldsymbol{z}(k) = [\,\boldsymbol{e}^T(k) \quad \widetilde{\boldsymbol{\theta}}^T(k)\,]^T$, since $\boldsymbol{\Omega}(k)$ is not guaranteed to be full rank. Hence the equilibrium $\boldsymbol{e}_{eq} = \mathbf{0}$, $\widetilde{\boldsymbol{\theta}}_{eq} = \mathbf{0}$ of the system (19),(20) is globally uniformly stable. Since $V(k)$ in a decreasing and non-negative function, it converges to a constant value $V_\infty \geq 0$, as $k \to \infty$; hence, $\Delta V(k) \to 0$. This implies that both $\boldsymbol{e}(k)$ and $\widetilde{\boldsymbol{\theta}}(k)$ remain bounded for all k, and $\boldsymbol{e}(k) \to 0$, $\widetilde{\boldsymbol{\eta}}(k) \to 0$. $\triangle$

The following theorem proves the exponential convergence of both state and parameters estimation errors, when a suitable persistency of excitation property holds.

Theorem 2: If Assumption 1 holds and the following condition (*persistency of excitation*) is satisfied for all k and for some integer $N > 0$

$$\Lambda\boldsymbol{I}_p \geq \sum_{l=k}^{k+N-1} \boldsymbol{\Omega}^T(l)\boldsymbol{\Omega}(l) \geq \lambda\boldsymbol{I}_p, \quad \Lambda \geq \lambda > 0, \qquad (26)$$

the equilibrium $\boldsymbol{z}_{eq} = [\,\boldsymbol{e}_{eq}^T \quad \widetilde{\boldsymbol{\theta}}_{eq}^T\,]^T = \mathbf{0}$ of the system (19),(20) is globally uniformly exponentially stable.

Proof 2: Consider the equation (20) and the positive definite Lyapunov function candidate

$$V(k) = \widetilde{\boldsymbol{\theta}}(k)^T \widetilde{\boldsymbol{\theta}}(k). \tag{27}$$

By taking into account (20),(13), (17), the first difference of V, $\Delta V(k) = V(k+1) - V(k)$, is given by:

$$\Delta V(k) = \widetilde{\boldsymbol{\theta}}^T(k) \boldsymbol{C}^T(k) \boldsymbol{C}(k) \widetilde{\boldsymbol{\theta}}(k), \tag{28}$$

where

$$\boldsymbol{C}(k) = \boldsymbol{Q}_\theta^{1/2} \boldsymbol{\Gamma}(k) \boldsymbol{\Omega}(k). \tag{29}$$

As shown in [1], the persistency of excitation condition (26) implies that there exists a constant $0 < \alpha < 1$ such that

$$\sum_{l=k}^{k+N-1} \Delta V(l) \leq -\alpha V(k). \tag{30}$$

It is worth noticing that the above inequality may be satisfied even for a constant $\alpha^* > 1$; however, in this case the inequality still holds for any positive $\alpha < 1$, since $-\alpha^* V \leq -\alpha V$.

Since

$$\sum_{l=k}^{k+N-1} \Delta V(l) = V(k+N) - V(k),$$

inequality (30) implies that

$$V(k+N) \leq (1-\alpha) V(k), \quad \forall k \geq k_0.$$

For any $k \geq k_0$, let $L > 0$ the minimum integer value such that $k \leq k_0 + (L+1)N$. Then, taking into account that $\Delta V(k) \leq 0$, the previous inequality yields

$$\begin{aligned} V(k) \leq V(k_0 + LN) &\leq (1-\alpha) V(k_0 + (L-1)N) \\ &\leq (1-\alpha)^2 V(k_0 + (L-2)N) \\ &\vdots \\ &\leq (1-\alpha)^L V(k_0) \\ &\leq \frac{1}{1-\alpha} (1-\alpha)^{(k-k_0)/N} V(k_0). \end{aligned}$$

Therefore

$$\|\widetilde{\boldsymbol{\theta}}(k)\| \leq \sqrt{\frac{1}{(1-\alpha)}} (1-\alpha)^{(k-k_0)/2N} \|\widetilde{\boldsymbol{\theta}}(k_0)\|$$

which proves the global uniform exponential convergence of $\widetilde{\boldsymbol{\theta}}(k)$. Since $\boldsymbol{\Omega}(k)$ is bounded, by virtue of (19) the global uniform exponential convergence of $\boldsymbol{e}(k)$ follows. $\triangle$

From exponential stability of the equilibrium of the unperturbed system (19),(20) it can be inferred ultimate uniform boundedness of the solution of the error dynamics in the presence of bounded interpolation errors.

VI. Fault detection, isolation and identification

Once the residuals vector $\boldsymbol{r}(k)$ is computed at each step, a fault is declared if each component of $\boldsymbol{r}(k)$ exceeds a suitably selected threshold ρ_i, i.e.,

$$|r_i(k)| > \rho_i \qquad i = 1, \ldots, n. \tag{31}$$

The *a priori* selection of each threshold should be based on the expressions of the residuals vector (18). Namely, proper setting of the thresholds requires an accurate knowledge of the uncertainties influence on the residuals. However, this approach often leads to extremely conservative results. Therefore, an empirical approach may be pursued to set the residuals thresholds in alternative to (or in combination with) the approach based on the *a priori* knowledge of the uncertainties. Namely, a number of experiments in the absence of faults may be performed and the corresponding residuals recorded; then, the thresholds can be set on the basis of the maximum absolute values of each component of the residuals vector. Of course, the experimental trials should be chosen following the worst-case criterium for the residuals, i.e., the uncertainties influence should be the maximum possible.

On the other hand, a complete fault diagnosis scheme should ensure not only the early and reliable detection of the failures, but also the isolation of the fault, i.e., the localization of the failure. The expressions of the residuals vector (18) clearly show that the signatures of the faults reflect the structure of the fault vector $\boldsymbol{f}$. Hence, different faults correspond to distinct fault signatures on the residuals: this implies that a reliable fault isolation can be ensured.

The problem of fault identification (i.e., the determination of the fault time evolution as accurately as possible) is a difficult task, since only the combined effect of uncertainties and faults can be estimated and not the two contributions separately. In other words, the uncertainties and the faults affect the estimation error dynamics in the same way, thus making impossible a clear distinction between faults and uncertainties influence. Therefore, the best estimate of the fault vector can be obtained only by taking $\widehat{\boldsymbol{f}} = \widehat{\boldsymbol{\eta}}$. In detail:

- before a fault is declared (i.e., all the components of $\boldsymbol{r}(k)$ are below the chosen thresholds), $\widehat{\boldsymbol{f}}(k, \boldsymbol{x}(k))$ is set to the null vector;
- after the detection of a fault (i.e., some components of $\boldsymbol{r}(k)$ exceed the corresponding thresholds), the corresponding components of the fault vector are set equal to those of $\widehat{\boldsymbol{\eta}}(k, \boldsymbol{x}(k), \bar{\boldsymbol{u}}(k))$.

Then, after the detection an estimate of the fault $\boldsymbol{\delta u}$ can be determined from $\widehat{\boldsymbol{f}}$ if the relation $\boldsymbol{f} = \boldsymbol{B}\,\boldsymbol{\delta u}$ is left-invertible.

VII. Case study

In this section an experimental case study is developed to test the effectiveness of the proposed approach. A conventional industrial l-degrees-of-freedom mechanical manipulator is chosen as test bed. The set-up is based on the industrial manipulator Comau SMART-3 S. The manipulator has a six-revolute-joint anthropomorphic geometry with nonnull shoulder and elbow offsets and non-spherical wrist. The joints are actuated by brushless motors via gear trains; shaft absolute resolvers provide motor position measurements. The robot is controlled by the C3G 9000 control unit. It is worth remarking that the SMART-3 S is a conventional industrial robot and not a research prototype; hence, all the typical drawbacks of industrial manipulators (e.g., joint friction, stiction and backlash due to the gear trains, disturbances on the torque delivered by the actuators, unmodeled elasticity of the joint shafts) are present. Various operational modes are available in the control unit, allowing the PC to interact with the original controller both at trajectory generation level and at joint control level. To implement model-based control schemes, the operational mode 4 is used in which the PC is in charge of computing the control algorithm and passing the references to the current servos through the communication link.

Details on the model and the motion control scheme are omitted for brevity and can be found in [2].

A fifth-order polynomial trajectory is imposed at each joint of the manipulator with null initial and final velocities and accelerations. The total (programmed) duration of the motion is 4 s. The commanded trajectory has been then executed. In order to safely emulate the presence of sensor and actuator faults, an additive signal has been superimposed to the measured experimental data off-line. Namely, the sequence $\boldsymbol{\delta u}(k)$ and has been simply added to the measured fault-free sequence $\boldsymbol{u}(k)$. In detail, two actuator faults have been considered affecting the driving torques generated by the actuators of the joints 3 (occurring at time $t_{fault} = 1$ s) and 5 (occurring at time $t_{fault} = 3$ s), with the following time profile:

$$\begin{cases} \delta u_3(k) = 60\,(1 - \mathrm{e}^{-(kT-1)/0.002}) & kT \geq 1 \\ \delta u_5(k) = 40\,(1 - \mathrm{e}^{-(kT-3)/0.08}) & kT \geq 3. \end{cases}$$

The first fault has to be considered as a an abrupt fault, while the second can be seen as an incipient fault.

The diagnostic observer has been implemented at a sampling rate of 500 Hz ($T = 2$ ms). The matrix gains have been chosen as

$$\boldsymbol{K}_o = \begin{bmatrix} \boldsymbol{K}_1 & T\boldsymbol{I}_n \\ \boldsymbol{O}_n & \boldsymbol{K}_2 \end{bmatrix},\ \boldsymbol{K}_1 = \boldsymbol{K}_2 = 0.1\,\boldsymbol{I}_3,$$

$$\boldsymbol{Q}_\theta^{-1} = 0.05 \cdot \mathrm{diag}\{0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 2, 2, 2, 2, 2, 3\}.$$

In order to perform a proper fault detection, suitably defined thresholds on the residuals has been selected. Thresholds setting has been achieved by measuring the residuals obtained in a set of fault-free trajectories under various operating conditions. The obtained numerical values of the thresholds can be found in [2].

As for the choice of the parametric model of the uncertainties, it is possible to resort to various approaches. A widely adopted choice is represented by the so-called online approximators [4],[11]–[15], e.g., RBF neural networks and polynomials. However, in the case of industrial robots, the uncertainties model is usually known (e.g., friction at low velocities, periodic torque disturbances), but the corresponding parameters are not known *a priori*. For the robot used in the experiments an accurate dynamic model is available except for the periodic torque disturbances. Hence, a realistic model of the uncertainties is given by ($i = 1, \ldots, 6$):

$$\begin{cases} \eta_i &= \theta_i, \\ \eta_{i+6} &= \theta_{7i} + \theta_{7i+1}\,\sin(\theta_{7i+2}x_{1,i} + \theta_{7i+3}) \\ &\quad +\tau_i(k)\theta_{7i+4}\,\sin(\theta_{7i+5}x_{1,i} + \theta_{7i+6}). \end{cases} \tag{32}$$

Figures 1 and 2 show the obtained results in the presence of the emulated actuator fault. Namely, it can be seen

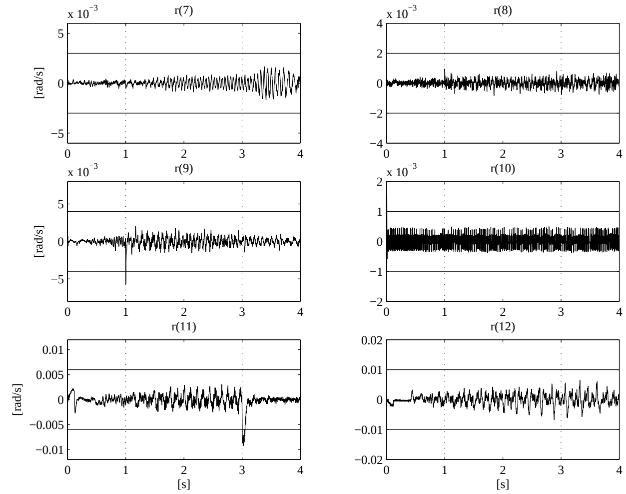

Fig. 1. Residuals for the emulated actuator fault with adaptive estimation of uncertainties.

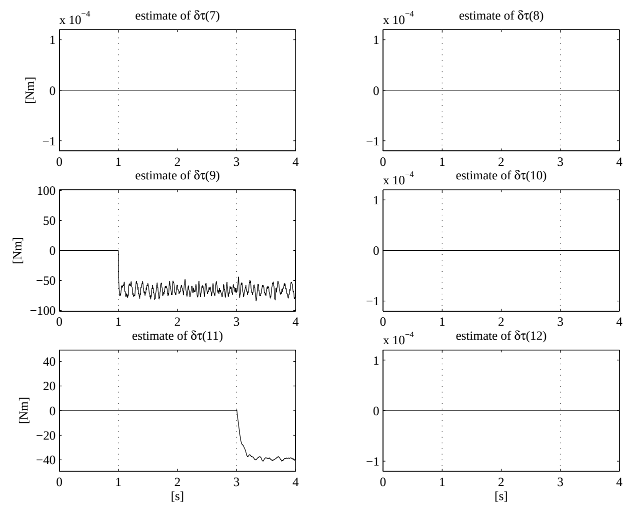

Fig. 2. Estimate of the fault time evolution with adaptive estimation of uncertainties.

that a good trade-off between robustness to uncertainties (i.e., low residuals) and sensitivity to faults is achieved by adopting the proposed adaptive estimation technique; this result is due to the accurate choice of the uncertainties model. Once a fault is detected, it can be reliably isolated, since the sole residuals corresponding to the faulty actuators become larger than the corresponding thresholds. Also, a fairly accurate fault identification is achieved (see Figure 2).

VIII. Conclusion

In this paper an adaptive discrete-time fault diagnosis approach for a class of nonlinear systems has been analyzed. The approach combines the use of a diagnostic observer with an adaptive uncertainties estimation technique, which makes use of a parametric model of the uncertainties. Convergence analysis of the resulting adaptive estimation scheme has been carried out by using Lyapunov techniques in discrete-time. Finally, the proposed approach has been experimentally tested on a six-degree-of-freedom industrial robot.

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