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Robust Fault Detection and Isolation Using Robust ℓ_1 Estimation

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Abstract

This paper considers the application of robust ℓ_1 estimation to fault robust fault detection and isolation. This is accomplished by developing a series, or bank, of robust estimators (full-order observers), each of which is designed such that the residual will be sensitive to a certain fault (or faults) while insensitive to the remaining faults. Robustness is incorporated by assuring that the residual remains insensitive to exogenous disturbances as well as modeling uncertainty. Mixed structured singular value and ℓ_1 theories are used to develop the appropriate threshold logic to evaluate the outputs of the estimators used for determining the occurrence and location of a fault. A real-coded genetic algorithm is used to obtain the optimal estimator gain matrices. This approach to FDI is successfully demonstrated using a linearized model of a jet engine.

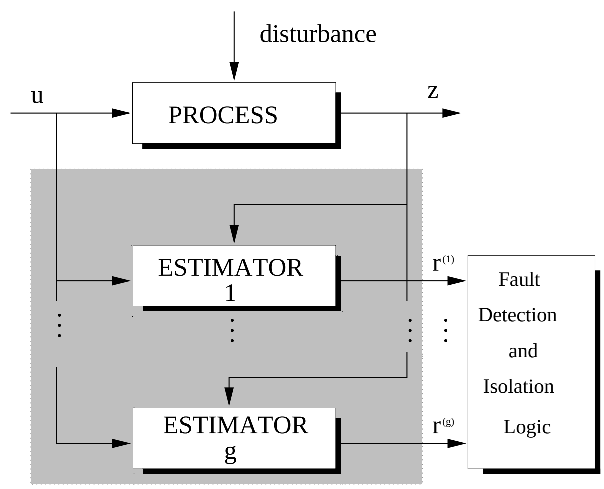

Figure 1: Estimation Based Fault Detection and Isolation

1 Introduction

In modern systems such as aircraft and spacecraft, there is an increasing demand for reliability and safety. For example, a jet engine is very critical for an aircraft and if faults occur, the consequences can be extremely serious [5]. Many dynamic systems are complex technical systems that involve extensive use of multiple sensors, actuators and other system components, any one of which could fail or deteriorate. Hence, health monitoring and supervision of these systems is essential for the improvement of reliability, safety and dependability of operations. This entails continuously checking a physical system for faults and taking appropriate actions to maintain the operation in such situations. In particular, the objective is to detect and isolate failures or anomalies in the sensors, actuators and components.

One of the primary approaches to model-based, fault detection and isolation uses state or output estimators. Detection of a fault is achieved by comparing the actual behavior of the plant to that expected on the basis of the model; deviations are indications of a fault (or disturbances, noise or modeling errors) [4]. Fault isolation can be achieved by dedicating an estimator such that the residual is sensitive to only one particular fault. In particular, referring to Figure 1, a bank of estimators is used to generate residuals $r^{(i)}(t)$. These residuals are then analyzed by some appropriate logic (e.g., logic based on thresholds or fuzzy logic) which infers whether faults have occurred (fault detection) and where they have occurred (fault isolation).

In many approaches to the FDI problem the robustness aspect is commonly introduced in relation to the fault detection. The estimators shown in Figure 1 may be designed in a variety of ways, for example by using Kalman filter theory (i.e., H_2 optimal estimation) [6], H_∞ theory [1], or ℓ_1 theory [2]. Whichever method is used for designing the estimator, it will use an idealized mathematical description of the underlying plant. In practice this model of the plant is never totally accurate, which can degrade the quality of the residuals produced by the estimators. The errors in the plant model may be either parametric or unstructured (e.g., unmodeled dynamics). To reduce the degradation in

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the quality of the residuals upon which the FDI process is based, and hence to reduce the false alarm rate, it is imperative that the plant uncertainty be explicitly taken into account in the design of the estimators.

Until recent work [1, 2, 3, 6], the relatively nonconservative mixed structured singular value techniques had not been applied to robust estimation, although more conservative techniques, based on the small gain theorem or fixed quadratic Lyapunov functions, had been used. The reduced conservatism of mixed structured singular value theory allows the estimators to be used for more accurate fault detection. Specifically, the fixed thresholds are smaller, allowing the detection of smaller faults. With more conservative theories the thresholds are larger, causing some smaller faults to go undetected. Although this paper focuses exclusively on sensor faults the theory is easily applied to actuator faults as well.

The organization of this paper is as follows. Section 2 presents the formulation of the closed-loop uncertain system to which estimation will be applied. The application of robust ℓ_1 estimation to robust fault detection and isolation is presented in Section 3. Section 4 discusses results of an illustrative example of a jet engine, while Section 5 gives concluding remarks.

2 Formulation of Closed-Loop Uncertain System

Consider a discrete-time, linear uncertain dynamic system

$$\begin{aligned} x_p(k+1) &= (A_p+\Delta A_p)x_p(k) + (B_p+\Delta B_p)u_p(k) \\ &\quad + D_{\infty,1}w_\infty(k), \quad k\in\mathcal{Z}^+ \qquad (1)\\ y_p(k) &= (C_p+\Delta C_p)x_p(k) + D_{\infty,2}w_\infty(k) \\ &\quad + R_s f(k), \qquad (2)\end{aligned}$$

where $x_p \in \mathcal{R}^{n_p}$ is the state vector, $u_p \in \mathcal{R}^{d_p}$ is the control input, $y_p \in \mathcal{R}^{p_p}$ denotes the plant measurements, $w_\infty \in \mathcal{R}^{d_\infty}$ denotes an ℓ_∞ disturbance signal satisfying $\|w_\infty(\cdot)\|_{\infty,2} \le 1$, $f \in \mathcal{R}^{r_s}$ is the sensor fault vector. The fault distribution matrices R_s is assumed to be known. The uncertainties ΔA_p, ΔB_p and ΔC_p satisfy

$$\begin{aligned} \Delta A_p \in \mathcal{U}_{A_p} &\triangleq \{\Delta A_p \in \mathcal{R}^{n_p\times n_p} : \Delta A_p = \\ &\quad - H_{A_p}F_{A_p}G_{A_p},\ \ F_{A_p}\in\mathcal{F}_{A_p}\}, \qquad (3)\\ \Delta B_p \in \mathcal{U}_{B_p} &\triangleq \{\Delta B_p \in \mathcal{R}^{n_p\times d_p} : \Delta B_p = \\ &\quad - H_{B_p}F_{B_p}G_{B_p},\ \ F_{B_p}\in\mathcal{F}_{B_p}\}, \qquad (4)\\ \Delta C_p \in \mathcal{U}_{C_p} &\triangleq \{\Delta C_p \in \mathcal{R}^{p_p\times n_p} : \Delta C_p = \\ &\quad - H_{C_p}F_{C_p}G_{C_p},\ \ F_{C_p}\in\mathcal{F}_{C_p}\}, \qquad (5)\end{aligned}$$

where

$$\mathcal{F}_{A_p} \triangleq \{F_{A_p}\in\mathcal{D}^r : M_{A_p,1} \le F_{A_p} \le M_{A_p,2}\}, \qquad (6)$$

$$\mathcal{F}_{B_p} \triangleq \{F_{B_p}\in\mathcal{D}^s : M_{B_p,1} \le F_{B_p} \le M_{B_p,2}\}, \qquad (7)$$

$$\mathcal{F}_{C_p} \triangleq \{F_{C_p}\in\mathcal{D}^s : M_{C_p,1} \le F_{C_p} \le M_{C_p,2}\}, \qquad (8)$$

with $M_{A_{p,1}}$, $M_{A_{p,2}} \in \mathcal{D}^r$, $M_{B_{p,1}}$, $M_{B_{p,2}} \in \mathcal{D}^r$, $M_{C_{p,1}}$, $M_{C_{p,2}} \in \mathcal{D}^s$, $M_{A_{p,2}} - M_{A_{p,1}} \ge 0$, $M_{B_{p,2}} - M_{B_{p,1}} \ge 0$, and $M_{C_{p,2}} - M_{C_{p,1}} \ge 0$. Note that the system in (1) and (2) has uncertainty in the input matrix B_p. This is significant in that current mixed structure singular value (MSSV) theory does not consider this uncertainty.

Assume that the dynamic system in (1) and (2) is controlled by a linear controller,

$$x_c(k+1) = A_c x_c(k) + B_c y_p(k), \qquad (9)$$

$$u_p(k) = C_c x_c(k). \qquad (10)$$

Then, the closed-loop system is described by,

$$x(k+1) = (A+\Delta A)x(k) + D_{w,1}w_\infty(k) + R_1 f(k), \qquad (11)$$

$$y(k) = Cx(k) + D_{w,2}w_\infty(k) + R_2 f(k), \qquad (12)$$

where

$$\begin{aligned} x(k) &= \begin{bmatrix} x_p(k) \\ x_c(k)\end{bmatrix}, \quad A = \begin{bmatrix} A_p & B_pC_c \\ B_cC_p & A_c\end{bmatrix}, \\ D_{w,1} &= \begin{bmatrix} D_{\infty,1} \\ B_cD_{\infty,2}\end{bmatrix}, \quad C = \begin{bmatrix} C_p & 0\end{bmatrix}, \\ R_1 &= \begin{bmatrix} 0 \\ B_cR_s\end{bmatrix}, \quad \Delta A = \begin{bmatrix} \Delta A_p & \Delta B_pC_c \\ B_c\Delta C_p & 0\end{bmatrix}, \\ D_{w,2} &= D_{\infty,2}, \quad R_2 = R_s. \qquad (13)\end{aligned}$$

Furthermore, ΔA and ΔC satisfy

$$\begin{aligned} \Delta A \in \mathcal{U}_A &\triangleq \{\Delta A \in \mathcal{R}^{2n_p\times 2n_p} : \Delta A = \\ &\quad - H_AF_AG_A,\ \ F_A\in\mathcal{F}_A\}, \qquad (14)\\ \Delta C \in \mathcal{U}_C &\triangleq \{\Delta C \in \mathcal{R}^{p_p\times 2n_p} : \Delta C = \\ &\quad - H_CF_CG_C,\ \ F_C\in\mathcal{F}_C\}, \qquad (15)\end{aligned}$$

where

$$\mathcal{F}_A \triangleq \{F_A\in\mathcal{D}^{r+s} : M_{A,1} \le F_A \le M_{A,2}\}, \qquad (16)$$

$$\mathcal{F}_C \triangleq \{F_C\in\mathcal{D}^{2s} : M_{C,1} \le F_C \le M_{C,2}\}, \qquad (17)$$

and

$$F_A = \begin{bmatrix} F_{A_p} & 0 & 0 \\ 0 & F_{C_p} & 0 \\ 0 & 0 & F_{B_p} \end{bmatrix},$$
$$H_A = \begin{bmatrix} H_{A_p} & 0 & H_{B_p} \\ 0 & B_c H_{C_p} & 0 \end{bmatrix},$$
$$G_A = \begin{bmatrix} G_{A_p} & 0 \\ G_{C_p} & 0 \\ 0 & G_{B_p} C_c \end{bmatrix},$$
$$F_C = F_{C_p}, \;\; H_C = H_{C_p}, \;\; G_C = \begin{bmatrix} G_{C_p} & 0 \end{bmatrix}, \tag{18}$$

with

$$\begin{aligned} M_{A,1} &= diag(M_{A_{p,1}}, M_{C_{p,1}}, M_{B_{p,1}}), \\ M_{A,2} &= diag(M_{A_{p,2}}, M_{C_{p,2}}, M_{B_{p,2}}), \\ M_{C,1} &= M_{C_{p,1}}, \qquad M_{C,2} = M_{C_{p,2}}. \end{aligned} \tag{19}$$

Notice that in the closed-loop system (11)-(12) all of the uncertainty appears in the ΔA and ΔC matrices. Hence, the robust estimation and fault detection results of [2] may be applied.

3 Robust FDI Using Robust ℓ_1 Estimation

The closed-loop system in (11)-(12) can be rewritten in the modified form,

$$\begin{aligned} x(k+1) = (A + \Delta A)x(k) + D_{w,1} w_\infty(k) \\ + R_{1,1} f_1(k) + \ldots + R_{1,r_s} f_{r_s}(k), \end{aligned} \tag{20}$$

$$\begin{aligned} y(k) = (C + \Delta C)x(k) + D_{w,2} w_\infty(k) \\ + R_{2,1} f_1(k) + \ldots + R_{2,r_s} f_{r_s}(k), \end{aligned} \tag{21}$$

where $R_{1,i}$ (respectively, $R_{2,i}$) denotes the i^{th} column of the matrix R_1 (respectively, R_2). The term $f_i(k)$, $i \in \{1, 2, \ldots, r_s\}$, represents the i^{th} individual sensor fault of $f(k)$ and $R_{1,i}$ (respectively, $R_{2,i}$) represents its directional characteristics. Assume that $f_i(k)$ is the "target fault," i.e., the fault that it is desired to detect. Without loss of generality, the vector of "nuisance faults", representing the faults that are *not* to be detected (by the robust fault detection filter), is given by $\bar{f}_i \triangleq [f_1(k) \cdots f_{i-1}(k) \;\; f_{i+1}(k) \cdots f_{r_s}(k)]$. Hence, (20) and (21) can be replaced by

$$\begin{aligned} x(k+1) &= (A + \Delta A)x(k) + D_{w,1} w_\infty(k) \\ &\quad + R_{1,i} f_i(k) + \bar{R}_{1,i} \bar{f}_i(k), \end{aligned} \tag{22}$$
$$\begin{aligned} y(k) &= (C + \Delta C)x(k) + D_{w,2} w_\infty(k) \\ &\quad + R_{2,i} f_i(k) + \bar{R}_{2,i} \bar{f}_i(k). \end{aligned} \tag{23}$$

Let $w^{(i)}$ be defined as $w^{(i)} \triangleq [w_\infty^T \; \bar{f}_i^T]^T$. Then, (22) and (23) can be written as a set $\Sigma^{(i)}$ of system equations

$$x(k+1) = (A + \Delta A)x(k) + D_1^{(i)} w^{(i)}(k) + R_{1,i} f_i(k), \tag{24}$$

$$y(k) = (C + \Delta C)x(k) + D_2^{(i)} w^{(i)}(k) + R_{2,i} f_i(k), \tag{25}$$

where

$$D_1^{(i)} = \begin{bmatrix} D_{w,1} & \bar{R}_{1,i} \end{bmatrix}, \;\; D_2^{(i)} = \begin{bmatrix} D_{w,2} & \bar{R}_{2,i} \end{bmatrix}. \tag{26}$$

It is desired to design a bank of full-order observers of the form

$$\begin{aligned} x_e^{(i)}(k+1|k) = &A_e^{(i)} x_e^{(i)}(k|k-1) + W^{(i)}[y(k) \\ &- C x_e^{(i)}(k|k-1)] \end{aligned} \tag{27}$$

to estimate the state vector x, where $A_e^{(i)} \in \mathcal{R}^{2n_p \times 2n_p}$ and $W^{(i)} \in \mathcal{R}^{2n_p \times p_p}$ are the parameters to be determined.

The estimation error is defined as

$$e^{(i)}(k) \triangleq x(k) - x_e^{(i)}(k|k-1), \tag{28}$$

which using (24), (25), and (27) can be shown to obey the evolution equation

$$\begin{aligned} e^{(i)}(k+1) = (A_e^{(i)} - W^{(i)} C) e^{(i)}(k) + (A + \Delta A \\ - W^{(i)} \Delta C - A_e^{(i)}) x(k) + (D_1^{(i)} - W^{(i)} D_2^{(i)}) \cdot \\ w^{(i)}(k) + (R_{1,i} - W^{(i)} R_{2,i}) f_i(k). \end{aligned} \tag{29}$$

Now define the error output $z^{(i)} \in \mathcal{R}^{q_p}$ as $z^{(i)}(k) \triangleq E_\infty^{(i)} e^{(i)}(k)$. Then augmenting (24) with (29) yields

$$\begin{aligned} \tilde{x}^{(i)}(k+1) = (\tilde{A}^{(i)} + \Delta\tilde{A}^{(i)}) \tilde{x}^{(i)}(k) + \tilde{D}_1^{(i)} w^{(i)}(k) \\ + \tilde{R}_1^{(i)} f_i(k), \end{aligned} \tag{30}$$
$$z^{(i)}(k) = \tilde{E} \tilde{x}^{(i)}(k), \tag{31}$$

where

$$\tilde{x}^{(i)}(k) = \begin{bmatrix} x(k) \\ e^{(i)}(k) \end{bmatrix},$$
$$\tilde{A}^{(i)} = \begin{bmatrix} A & 0 \\ A - A_e^{(i)} & A_e^{(i)} - W^{(i)} C \end{bmatrix},$$
$$\tilde{D}_1^{(i)} = \begin{bmatrix} D_1^{(i)} \\ D_1^{(i)} - W^{(i)} D_2^{(i)} \end{bmatrix},$$
$$\tilde{R}_1^{(i)} = \begin{bmatrix} R_{1,i} \\ R_{1,i} - W^{(i)} R_{2,i} \end{bmatrix}, \; \tilde{E} = \begin{bmatrix} 0 & E_\infty^{(i)} \end{bmatrix}. \tag{32}$$

Furthermore, $\Delta\tilde{A}$ satisfies

$$\begin{aligned} \Delta\tilde{A}^{(i)} \in \mathcal{U}_{\tilde{A}} \triangleq \{\Delta\tilde{A}^{(i)} \in \mathcal{R}^{4n_p \times 4n_p} : \Delta\tilde{A}^{(i)} = \\ - H_0^{(i)} F_{\tilde{A}} G_0, \;\; F_{\tilde{A}} \in \mathcal{F}_{\tilde{A}}\}, \end{aligned} \tag{33}$$

where

$$\mathcal{F}_{\tilde{A}} \triangleq \{F_{\tilde{A}} \in \mathcal{D}^{2r+2s} : M_1 \leq F_{\tilde{A}} \leq M_2\}, \tag{34}$$

with

$$F_{\tilde{A}} = \begin{bmatrix} F_A & 0 \\ 0 & F_C \end{bmatrix}, \ H_0^{(i)} = \begin{bmatrix} H_A & 0 \\ H_A & W^{(i)}H_C \end{bmatrix},$$
$$G_0 = \begin{bmatrix} G_A & 0 \\ -G_C & 0 \end{bmatrix}, \tag{35}$$

and

$$M_1 = diag(M_{A,1}, M_{C,1}),$$
$$M_2 = diag(M_{A,2}, M_{C,2}). \tag{36}$$

Let the residual error be defined as

$$r^{(i)}(k) \triangleq P^{(i)}[y(k) - C\hat{x}^{(i)}(k|k-1))], \tag{37}$$

where the gain matrices $P^{(i)}$ are $r_s \times p_p$ chosen such that $r^{(i)}$ have a fixed direction when responding to the target fault.

Let $J_{rw}^{(i)}$ represent the ℓ_1 norm of the system operator from the disturbance vector $w^{(i)}$ to the residual $r^{(i)}$, and let $J_{rf}^{(i)}$ represent the ℓ_1 norm of the system operator from the target fault f_i to the residual $r^{(i)}$. Then, using mixed structured singular value theory, it is possible to characterize upperbounds $\hat{J}_{rw}^{(i)}$ and $\hat{J}_{rf}^{(i)}$ such that

$$\hat{J}_{rw}^{(i)} \geq \max_{\Delta\tilde{A}^{(i)} \in \mathcal{U}} J_{rw}^{(i)}, \tag{38}$$

$$\hat{J}_{rf}^{(i)} \geq \max_{\Delta\tilde{A}^{(i)} \in \mathcal{U}} J_{rf}^{(i)}. \tag{39}$$

Robust fault detection filter design may be approached by choosing $A_e^{(i)}$, $W^{(i)}$ and $P^{(i)}$ so that $\hat{J}_{rw}^{(i)}$ is small and $\hat{J}_{rf}^{(i)}$ is large.[1] A minimization problem that expresses this objective is

$$\min_{A_e^{(i)}, W^{(i)}, P^{(i)}} \mathcal{J}^{()} = \beta \hat{J}_{rw}^{(i)} + (1-\beta)\frac{1}{\hat{J}_{rf}^{(i)}} + \gamma \frac{\hat{J}_{rw}^{(i)}}{\hat{J}_{rf}^{(i)}}, \tag{40}$$

where $\beta \in [0,1]$ and $\gamma > 0$ are arbitrarily chosen weighting scalars.

The robust ℓ_1 estimation problem is to find the estimator parameters $A_e^{(i)}$, $W^{(i)}$ and $P^{(i)}$ such that the combined system (27), (30)-(31) is asymptotically stable over the uncertainty set $\mathcal{U}$, and the ℓ_1 performance function $\mathcal{J}^{()}(A_e^{(i)}, W^{(i)}, P^{(i)})$ is minimized. A real-coded genetic algorithm [7] is used as an appropriate optimization method to select the estimation and projection parameters.

[1] It would be more desirable to make a *lower* bound on $\hat{J}_{rf}^{(i)}$ large. Unfortunately, lower bounds are usually much more difficult to work with computationally than upper bounds.

Now consider the set $\Sigma^{(i)}$ of uncertain discrete-time systems

$$x(k+1) = (A + \Delta A)x(k) + D_1^{(i)}w^{(i)}(k) + R_{1,i}f_i(k), \tag{41}$$

$$y(k) = Cx(k) + D_2^{(i)}w^{(i)}(k) + R_{2,i}f_i(k), \tag{42}$$

where x, y, $w^{(i)}$, and f_i are as previously discussed.

The robust fault detection problem is to generate a robust residual signals $r^{(i)}(k)$ that satisfies

$$\|r^{(i)}(k)\|_p \leq J_{th}^{(i)} \ \text{ if } \ f_i(k) = 0, \tag{43}$$

$$\|r^{(i)}(k)\|_p > J_{th}^{(i)} \ \text{ if } \ f_i(k) \neq 0, \tag{44}$$

where $\|\cdot\|_p$ denotes the p norm of a Lebesgue signal and $J_{th}^{(i)}$ is the i^{th} threshold value. If the estimators (27) are applied to (41)-(42) and $E_\infty^{(i)}$ is chosen as C (37) can be written as

$$r^{(i)}(k) = P^{(i)}z^{(i)}(k) + P^{(i)}D_2^{(i)}w^{(i)}(k) + P^{(i)}R_{2,i}f_i(k). \tag{45}$$

As derived in [2], if $f_i(k) = 0$ (45) satisfies the norm inequality

$$\begin{aligned}\|r^{(i)}\|^2_{(\infty,2),[N_0,N]} \leq \{&[\mathrm{tr}(P^{(i)}E_\infty^{(i)}Q_\infty^{(i)}E_\infty^{(i)T}P^{(i)T})^q]^{\frac{1}{q}} \\ &+ 2\sigma_{\max}(P^{(i)}D_2^{(i)})[\mathrm{tr}(P^{(i)}E_\infty^{(i)}Q_\infty^{(i)} \cdot \\ &E_\infty^{(i)T}P^{(i)T})^q]^{\frac{1}{2q}} + \sigma^2_{\max}(P^{(i)}D_2^{(i)})\} \cdot \\ &\|w^{(i)}\|^2_{(\infty,2),[N_0,N]},\end{aligned} \tag{46}$$

where $Q_\infty^{(i)}$ is a positive-definite matrix satisfying

$$Q_\infty^{(i)} = \alpha(A - W^{(i)}C)Q_\infty^{(i)}(A - W^{(i)}C)^T + \frac{\alpha}{\alpha - 1}V_\infty^{(i)}, \tag{47}$$

with $V_\infty^{(i)} \triangleq (D_1^{(i)} - W^{(i)}D_2^{(i)})(D_1^{(i)} - W^{(i)}D_2^{(i)})^T$ and $\alpha > 1$. The threshold can be chosen as

$$\begin{aligned}J_{th}^{(i)} \triangleq \{&[\mathrm{tr}(P^{(i)}E_\infty^{(i)}Q_\infty^{(i)}E_\infty^{(i)T}P^{(i)T})^q]^{\frac{1}{q}} \\ &+ 2\sigma_{\max}(P^{(i)}D_2^{(i)})[\mathrm{tr}(P^{(i)}E_\infty^{(i)}Q_\infty^{(i)}E_\infty^{(i)T}P^{(i)T})^q]^{\frac{1}{2q}} \\ &+ \sigma^2_{\max}(P^{(i)}D_2^{(i)})\}\|w^{(i)}\|^2_{(\infty,2),[N_0,N]}.\end{aligned} \tag{48}$$

Robust fault detection can be accomplished by comparing $\|r^{(i)}\|_{(\infty,2),[N_0,N]}$ with $J_{th}^{(i)}$. A fault occurs if $\|r^{(i)}\|_{(\infty,2),[N_0,N]} > J_{th}^{(i)}$, i.e.,

$$\|r^{(i)}\|_{(\infty,2),[N_0,N]} > J_{th}^{(i)} \Rightarrow \text{a fault occurred.} \tag{49}$$

4 Illustrative Example of FDI for a Jet Engine

A numerical example is presented in this section to illustrate robust ℓ_1 estimator design using the Popov-Tsypkin multiplier and the application of the robust ℓ_1 estimator to robust fault detection of dynamic systems. The model used was supplied by NASA Glenn Research Center and is given as

$$x_p(k+1) = (A_p + \Delta A_p)x_p(k) + (B_p + \Delta B_p)u_p(k) + D_{\infty,1}w(k), \quad k \geq 0 \tag{50}$$

$$y_p(k) = (C_p + \Delta C_p)x_p(k) + D_{\infty,2}w(k) \tag{51}$$

where the sampling period is 0.01. The state vector $x_p \triangleq [\mathrm{XNH}, \mathrm{XNL}, \mathrm{TMPC}]^T$, where

$$\begin{aligned} \mathrm{XNH} &\triangleq \text{High Pressure Spool Speed (rpm)} \\ \mathrm{XNL} &\triangleq \text{Low Pressure Spool Speed (rpm)} \\ \mathrm{TMPC} &\triangleq \text{High Pressure Compressor Inlet Temperature } (^\circ C). \end{aligned}$$

The control input vector $u_p \triangleq [\mathrm{WF36}, \mathrm{A8}, \mathrm{A16}]^T$, where

$$\begin{aligned} \mathrm{WF36} &\triangleq \text{Main Burner Fuel Flow (kg/hr)} \\ \mathrm{A8} &\triangleq \text{Exhaust Nozzle Throat Area } (m^2) \\ \mathrm{A16} &\triangleq \text{Bypass Duct Area } (m^2). \end{aligned}$$

The output vector y_p measures the states and w denotes a vector of disturbance signals.

The uncertainty matrices, ΔA_p, ΔB_p and ΔC_p, are representative of some engine degradation over time. Thus, it is assumed that a newly constructed engine can be modeled with the nominal matrices A_p, B_p and C_p and with use, the parameters of the degraded engine are encompassed in the uncertainty. The system parameter matrices are

$$A_p = \begin{bmatrix} 0.8938 & 0.0034 & 0.0020 \\ -0.0001 & 0.8940 & 0.0014 \\ 0 & -0.0001 & 0.89960 \end{bmatrix},$$

$$B_p = \begin{bmatrix} 0.0059 & 0.1119 & -0.0160 \\ 0.0042 & 0.0720 & 0.0083 \\ 0.0001 & -0.0003 & 0 \end{bmatrix},$$

$$C_p = \begin{bmatrix} 1.0000 & 0.0077 & 0.0044 \\ -0.0003 & 1.0000 & 0.0031 \\ 0.0299 & -0.0208 & 0.0010 \end{bmatrix},$$

$$D_{p,1} = \mathrm{diag}\{0.1,\ 0.1,\ 0.01\},\ D_{p,2} = 0.1 \times I_{3\times3}. \tag{52}$$

The uncertainty matrices $\Delta A_p = -H_{A_p}F_{A_p}G_{A_p}$, $\Delta B_p = -H_{B_p}F_{B_p}G_{B_p}$, $\Delta C_p = -H_{C_p}F_{C_p}G_{C_p}$, where

$$H_{A_p} = -\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$H_{B_p} = H_{C_p} = -\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$G_{A_p} = \begin{bmatrix} I_{3\times3} \\ I_{3\times3} \end{bmatrix}, \quad G_{B_p} = G_{C_p} = \begin{bmatrix} I_{3\times3} \end{bmatrix},$$

$$\begin{aligned} F_{A_p} &= \mathrm{diag}\{\delta_{A_1},\ \delta_{A_2},\ \delta_{A_3},\ \delta_{A_4},\ \delta_{A_5},\ \delta_{A_6}\} \\ F_{B_p} &= \mathrm{diag}\{\delta_{B_1},\ \delta_{B_2},\ \delta_{B_3}\}\ \ F_{C_p} = \mathrm{diag}\{\delta_{C_1},\ \delta_{C_2},\ \delta_{C_3}\}, \end{aligned} \tag{53}$$

with

$$\begin{aligned} &0 \leq \delta_{A_1} \leq 0.0011, \quad -0.0008 \leq \delta_{A_2} \leq 0, \\ &0 \leq \delta_{A_3} \leq 0.0002, \quad -0.0001 \leq \delta_{A_4} \leq 0, \\ &0 \leq \delta_{A_5} \leq 0.0005, \quad 0 \leq \delta_{A_6} \leq 0.0001, \end{aligned} \tag{54}$$

$$\begin{aligned} &0 \leq \delta_{B_1} \leq 0.0008, \quad 0 \leq \delta_{B_2} \leq 0.0115, \\ &-0.0005 \leq \delta_{B_3} \leq 0, \end{aligned} \tag{55}$$

$$\begin{aligned} &-0.0001 \leq \delta_{C_1} \leq 0, \quad -0.0015 \leq \delta_{C_2} \leq 0, \\ &0 \leq \delta_{C_3} \leq 0.0004. \end{aligned} \tag{56}$$

Note that the uncertain parameters $\delta_{A_1} \ldots \delta_{A_6}$ correspond to parameter fluctuations in the first two rows of matrix A_p, $\delta_{B_1} \ldots \delta_{B_3}$ and $\delta_{C_1} \ldots \delta_{C_3}$ to the first row of B_p and C_p, respectively. By using the objective function (40) with stability constraints in a real-coded genetic algorithm the respective gain and projection matrices are obtained for a bank of estimators.

In order to illustrate the application of the robust ℓ_1 estimator to robust fault detection, FDI of the closed-loop system in (30) and (31) subject to plant disturbances was performed. A bank of estimators (as described in Section 3) was designed for the set of y_{p_i}, $i \in \{1, 2, 3\}$, sensor outputs, i.e., the i^{th} estimator is designed to detect a fault in the y_{p_i} sensor while neglecting faults in the remaining sensors. Here the robust case (estimation with uncertainty) is considered for the FDI process. In order to show the extent of robustness, uncertainty for all system matrices was considered. The uncertain parameters are assigned the nonzero values of their upper or lower bounds. For example δ_{A_1}, whose lower and upper bounds are 0 and 0.0011, respectively, was assigned its upper bound 0.0011, whereas, δ_{A_2}, whose lower and upper bounds are -0.0008 and 0, respectively, was assigned its lower bound -0.0008.

This paper only considered the occurrence of sensor faults within the system. A typical sensor fault in the jet engine is a drift in the sensor reading. Thus, a slow drifting (or ramping) sensor fault was added to a sensor reading at a particular instant in time. Specifically,

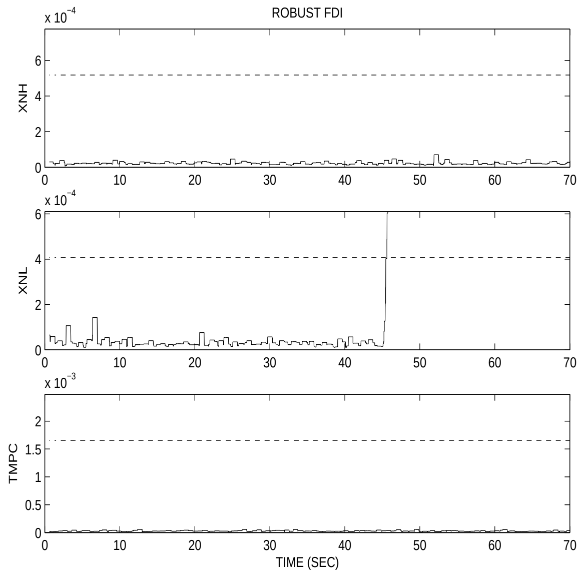

Figure 2: Robust ℓ_1 FDI: fault in XNL sensor at $t = 45$sec.

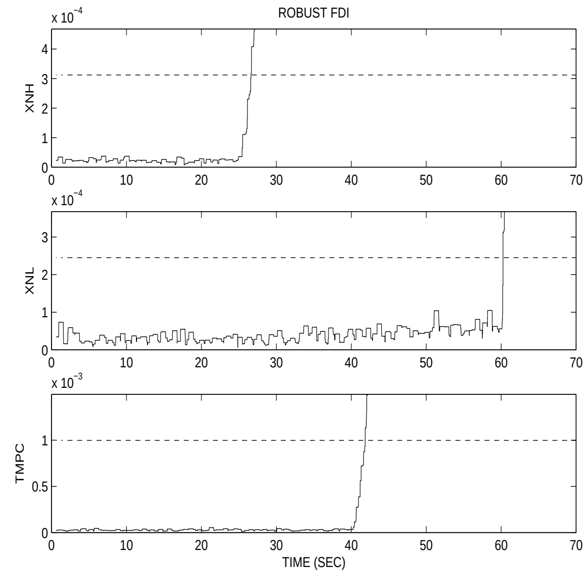

Figure 3: Robust ℓ_1 FDI: fault in XNH sensor at $t =$ 25 sec., XNL sensor at $t = 60$ sec., and TMPC sensor at $t = 40$ sec.

the error between the actual value and the value of the *faulty* sensor's reading became increasingly larger over time. Due to the disturbance the finite-horizon infinity norm (46) of the residual with $N - N_0 = 60$ (corresponding to a time interval of 0.6 sec.), was nonzero even in the absence of faults.

In Figure 2 a single sensor fault was introduced in the system. It can be seen that each fault was successfully detected and isolated in the respective residual. In Figure 3 multiple sensor faults were introduced in the system at particular times. It can be observed that in this instant the estimators were able to isolate each target fault from the other nuisance faults. As such, the ℓ_1 estimators demonstrate a degree of the robustness in the presence of plant disturbance.

5 Conclusions

This paper considered the application of robust ℓ_1 estimation for uncertain, linear discrete-time systems to the robust fault detection and isolation of dynamic systems. Mixed structured singular value theory of [2] was used to design a bank of robust ℓ_1 estimators and the resulting fixed threshold logic. By considering a discrete, linear model of a jet engine with real parametric uncertainties, an LQG controller was implemented to form a closed-loop system. Using this closed-loop system and introducing drifting sensor faults, it was shown that the robust FDI methodology based on fixed thresholds was capable of detecting and isolating failures in each of the particular sensors.

References

[1] Collins, E. G. Jr. and T. Song, "Multiplier-Based Robust H_∞ Estimation with Application to Robust Fault Detection," *Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 5, pp. 857-864, 2000.

[2] Collins, E. G. Jr. and T. Song, "Robust ℓ_1 Estimation Using the Popov-Tsypkin Multiplier with Application to Robust Fault Detection," *International Journal of Control*, Vol. 74, No. 3, pp. 303-313, 2001.

[3] Curry, T., E. G. Collins, Jr., and M. Selekwa, "Robust Fault Detection Using Robust ℓ_1 Estimation and Fuzzy Logic," *Proceedings of the American Control Conference*, pp. 1753-1758, June 2001.

[4] Gertler, J., "Fault Detection and Isolation Using Parity Relations," *Control Engineering Practice*, Vol. 5, No. 5, pp. 653-661, 1997.

[5] Patton, R. J. and J. Chen, "A Robustness Study of Model-Based Fault Detection for Jet Engine Systems," *Proceedings of the IEEE Conference on Control Applications*, pp. 871-876, 1992.

[6] Song, T. and E. G. Collins, Jr., "Robust H_2 Estimation with Application to Robust Fault Detection," *Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 6, pp. 1067-1071, 2001.

[7] Wright, A. H., "Genetic Algorithms for Real Parameter Optimization, in *Foundations of Genetic Algorithms*," Rawlins, G., Ed., Morgan Kaufmann Publishers, Los Altos, pp. 205-218, 1991.