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Output Feedback Sampled-Data Stabilization of Nonlinear Systems*

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Abstract

This paper studies sampled-data output feedback control of a class of nonlinear systems. It is shown that the performance of a stabilizing continuous-time state feedback controller can be recovered by a sampled-data output feedback controller when the sampling period is sufficiently small. The output feedback controller uses a deadbeat discrete-time observer to estimate the unmeasured states. Two schemes are proposed to overcome large initial transients when the controller is switched on.

1 Introduction

While nonlinear control theory is developed mainly for continuous-time systems, it is important to study sampled-data control because most controllers are implemented using digital computers. Over the past several years, there has been a number of studies, which can be broadly classified into two categories. In the first category, an approximate discrete-time model of the nonlinear plant is derived and used to design a discrete-time controller. In the second category, a continuous-time controller is designed using a continuous-time model; then it is discretized and implemented using sample/hold devices. Papers in the first category include [11], [14], [15], and [18], while those in the second category include [2], [4], [5], [6], [8], [10], [12], [16], [17], and [19], Unlike linear systems [1, 7], in the nonlinear case we cannot, in general, have an exact discrete-time model of the plant. Therefore, results in nonlinear sampled-data control are, usually, asymptotic results that establish certain properties of the system when the sampling period tends to zero. The current paper belongs to the second category and deals specifically with the stabilization of a class of nonlinear systems under output feedback.

Output feedback sampled-data control of nonlinear systems has been studied in [6], [9], and [10]. In [9], the nonlinear system is linearized about the origin and a linear digital controller is designed using the exact discrete-time model of the linearization. It is shown that the linear controller will stabilize the origin of the nonlinear system. The result of course is local, but does not require the sampling period to be small. In [10], a continuous-time (static state feedback or dynamic output feedback) controller is designed such that a certain dissipation inequality is satisfied globally in the presence of disturbance inputs. The controller is discretized (satisfying certain consistency conditions) and implemented using zero-order hold. It is shown that, for sufficiently small sampling period, the sampled-data controller recovers the dissipation inequality in a semiglobal practical sense. When specialized to the stabilization problem, the results of [10] show semiglobal practical stabilization of the exact discrete-time model. They do not show the inter-sampling behavior nor the property $\lim_{t\to\infty} x(t) = 0$. Finally, [6] considers a class of nonlinear systems that can be represented in coordinates where some of the state variables are derivatives of a measured output. A continuous-time static partial state feedback controller is deigned to stabilize the origin. The controller uses all measured signals together with those unmeasured states that are derivatives of the output. Using the separation principle of [3], a continuous-time high-gain observer is designed to estimate the unmeasured states. Such a continuous-time output feedback controller recovers the performance of the state feedback controller when an observer parameter ε is sufficiently small. In sampled-data control, the observer is discretized. Since reducing ε increases the bandwidth of the high-gain observer, it is clear that for the sampled-data controller to capture the properties of the continuous-time controller, it is necessary that the sampling frequency be high enough to cover the observer bandwidth. This fact is recognized in [6] by taking the sampling period T to be of the order of ε; i.e., $T = \alpha\varepsilon$. It is shown that the sampled-data output feedback controller recovers the performance of the continuous-time state feedback controller as $\varepsilon \to 0$. Performance recovery includes recovery of the trajectories and the property $\lim_{t\to\infty} x(t) = 0$.

In this paper, we consider the same class of nonlinear systems treated in [6] and start with the same continuous-time static partial state feedback controller that stabilizes the origin. However, we do not design observers in the continuous-time domain. Instead, we derive an exact discrete-time model of the sampled-data

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system and pursue the observer design in discrete-time by following the ideas of [13]; namely, the observer problem is studied in the context of solving sets of simultaneous nonlinear equations. Towards that end, we derive the nonlinear map from the state and a set of consecutive controls to a set of consecutive outputs and exploit the special structure of the system to solve the set of nonlinear equations to obtain the partial state (the vector whose components are unmeasured derivatives of the outputs) in terms of vectors of consecutive controls and outputs. This solution depends on the exact nonlinear model, which cannot be computed. However, we show that an $O(T)$ approximation of the solution is given by a linear map that depends only on the linear structure of the system; more specifically, on the number of integrators in each output channel. This linear map is equivalent to a deadbeat observer for a linear discrete-time system, which is the exact discretization of a chain of integrators. In initializing such an observer, the state estimates could experience a peaking phenomenon in the initial transient due to division by positive powers of T. We describe two schemes to overcome peaking during initialization. The first scheme, which is motivated by our prior experience with high-gain observers, allows the observer to start with arbitrary initial conditions and relies on global boundedness of the control law to overcome peaking. The second scheme, which represents the state of the art in the digital control literature, keeps the control signal fixed for an initial period until enough past measurements of the output have become available to calculate the estimated states without peaking. We prove that the sampled-data output feedback controller, with either scheme, recovers the performance of the continuous-time state feedback controller as the sampling period becomes arbitrarily small. Performance recovery is established by showing four different properties: First, boundedness of all states when the initial states belong to any compact subset of the region of attraction; second, ultimate boundedness of the states with an ultimate bound that can be made arbitrarily small by choosing the sampling period small enough; third, closeness of the trajectories under continuous-time state and sampled-data output feedback for sufficiently small sampling period. Finally, we show that if the closed-loop system under continuous-time state feedback is exponentially stable, then the trajectories under sampled-data output feedback converge to zero as time tends to infinity.

2 Controller design

We consider a multi-input–multi-output nonlinear system represented by

$$\dot{x} = Ax + B\phi(x,z,u) \quad (1)$$

$$\dot{z} = \psi(x,z,u) \quad (2)$$

$$y = Cx \quad (3)$$

$$\zeta = q(x,z) \quad (4)$$

where $u \in R^p$ is the control input, $y \in R^m$ and $\zeta \in R^s$ are measured outputs, and $x \in R^\rho$ and $z \in R^\ell$ constitute the state vector. The $\rho \times \rho$ matrix A, the $\rho \times m$ matrix B, and the $m \times \rho$ matrix C represent m chains of integrators. They are block diagonal matrices, with the diagonal blocks

$$A_i = \begin{bmatrix} 0 & 1 & \cdots & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & & \vdots \\ 0 & \cdots & \cdots & 0 & 1 \\ 0 & \cdots & \cdots & \cdots & 0 \end{bmatrix}_{\rho_i \times \rho_i}, \quad B_i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{\rho_i \times 1}$$

$$C_i = \begin{bmatrix} 1 & 0 & \cdots & \cdots & 0 \end{bmatrix}_{1 \times \rho_i}$$

where $1 \le i \le m$ and $\rho = \rho_1 + \cdots + \rho_m$. The functions ϕ, ψ, and q are locally Lipschitz in their arguments for $(x,z,u) \in D_x \times D_z \times R^p$, where $D_x \subset R^\rho$ and $D_z \subset R^s$ are domains that contain their respective origins. Moreover, $\phi(0,0,0) = 0$, $\psi(0,0,0) = 0$, and $q(0,0) = 0$.

Various sources of the model (1)–(4), including the normal form of input-output linearizable systems and mechanical/elctromechanical systems, are discussed in [3].

The task of the sampled-data output feedback controller is to stabilize the origin of the closed loop system using only the measured outputs y and ζ. We follow a two-step approach to this problem. First, we design a continuous-time partial state feedback controller that uses measurements of x and ζ. Then, we implement the same controller in sampled-data, with x replaced by an estimate $\hat{x}$ that is calculated from the output y. The state feedback controller takes the form

$$u = \gamma(x,\zeta) \quad (5)$$

where γ is a locally Lipschitz function in its arguments over the domain of interest and $\gamma(0,0) = 0$. For convenience, we write the continuous-time closed-loop system under state feedback control as

$$\dot{\mathcal{X}} = f(\mathcal{X}) \quad (6)$$

where

$$\mathcal{X} = \begin{bmatrix} x \\ z \end{bmatrix}, \quad f(\mathcal{X}) = \begin{bmatrix} Ax + B\phi(x,z,\gamma(x,q(x,z))) \\ \psi(x,z,\gamma(x,q(x,z))) \end{bmatrix}$$

We consider a zero-order-hold system where u is held constant in between the (uniformly spaced) sampling points. Let T be the sampling period and denote the signals at the kth sampling point by $x(k)$, $u(k)$, etc. To discretize the plant dynamics, we rewrite (1)–(2) as

$$\dot{\mathcal{X}} = F(\mathcal{X},u) \quad (7)$$

where $F(\mathcal{X},u) = \begin{bmatrix} Ax + B\phi(x,z,u) \\ \psi(x,z,u) \end{bmatrix}$. The solution of (7) over the sampling period $[kT, kT+T]$ is given by

$$\begin{aligned}\mathcal{X}(t) &= \mathcal{X}(k) + (t-kT)F(\mathcal{X}(k),u(k)) \\ &\quad + \int_{kT}^{t} [F(\mathcal{X}(\tau),u(k)) - F(\mathcal{X}(k),u(k))]\, d\tau\end{aligned}$$

On compact sets of $\mathcal{X}$ and u, we can use the Lipschitz property of F and the Gronwall-Bellman inequality, to show that

$$\|\mathcal{X}(t) - \mathcal{X}(k)\| \leq \frac{1}{L_1}\left[e^{(t-kT)L_1} - 1\right]\|F(\mathcal{X}(k),u(k))\| \tag{8}$$

$\forall\ t \in [kT, kT+T]$, where L_1 is a Lipschitz constant of F with respect to $\mathcal{X}$. Hence,

$$\mathcal{X}(k+1) = \mathcal{X}(k) + TF(\mathcal{X}(k),u(k)) + T^2\Phi(\mathcal{X}(k),u(k),T) \tag{9}$$

where Φ is locally Lipschitz in $(\mathcal{X},u)$ and uniformly bounded in T, for sufficiently small T. This model and inequality (8) are sufficient to characterize the plant dynamics. However, to estimate x from y, we need a more detailed model of the state x that makes use of the special structure of (1) and the properties of the matrices A, B, and C. Towards that end, let

$$D_i = \text{diag}\ [1,\ T,\ \ldots, T^{\rho_i - 1}] \tag{10}$$

and verify the following expressions for every $1 \leq i \leq m$:

$$\left.\begin{aligned} D_iB_i &= T^{\rho_i-1}B_i \\ C_iD_i &= C_i \\ A_i^{\rho_i} &= 0 \\ T^kD_iA_i^k &= A_i^kD_i, \quad \text{for } k \geq 0 \\ D_i(e^{A_iT})^k &= (e^{A_i})^kD_i, \quad \text{for } k \geq 0 \\ C_i(e^{A_iT})^k &= C_i(e^{A_i})^kD_i, \quad \text{for } k \geq 0 \end{aligned}\right\} \tag{11}$$

Following the derivations of [6], it can be shown that

$$x^i = [y_i,\ \dot{y}_i,\ y_i^{(2)},\ \ldots, y_i^{(\rho_i-1)}]^T$$

satisfies the equations

$$x^i(k+1) = e^{A_iT}x^i(k) + T^{\rho_i}D_i^{-1}h_i \tag{12}$$

$$y_i(k) = C_ix^i(k) \tag{13}$$

where $h_i = h_i(\mathcal{X}(k),u(k),T)$ is locally Lipschitz in $(\mathcal{X},u)$ and uniformly bounded in T, on compact sets of $(\mathcal{X},u)$, for sufficiently small T. By recursive application of (12) and (13), we arrive at

$$Y^i_{[k,k+\rho_i-1]} = M_iD_ix^i(k) + T^{\rho_i}g_i(\mathcal{X}(k), U_{[k,k+\rho_i-2]}, T) \tag{14}$$

for some locally Lipschitz function g_i that is uniformly bounded in T, where

$$Y^i_{[k,k+\rho_i-1]} = [y_i(k),\ y_i(k+1),\ \ldots,\ y_i(k+\rho_i-1)]^T$$

$$U_{[k,k+\rho_i-2]} = [u(k),\ u(k+1),\ \ldots,\ u(k+\rho_i-2)]^T$$

$$M_i = \begin{bmatrix} C_i \\ C_ie^{A_i} \\ \vdots \\ C_i(e^{A_i})^{\rho_i-1} \end{bmatrix}$$

Writing (14) with k replaced by $k-\rho_i+1$, solving for $x^i(k-\rho_i+1)$, and then using (12) to calculate $x^i(k)$, we end up with

$$\begin{aligned} x^i(k) &= D_i^{-1}\left(e^{A_i}\right)^{\rho_i-1}M_i^{-1}Y^i_{[k-\rho_i+1,k]} \\ &\quad + TH_i(\mathcal{X}(k-\rho_i+1), U_{[k-\rho_i+1,k-1]}, T) \end{aligned} \tag{15}$$

for $k \geq \rho_i - 1$, where H_i is locally Lipschitz in $(\mathcal{X},U)$ and uniformly bounded in T, for sufficiently small T. From (15) we see that, for $k \geq \rho_i - 1$, an estimate $\hat{x}^i$ of x^i, with an order $O(T)$ error, can be taken as

$$\hat{x}^i(k) = D_i^{-1}\left(e^{A_i}\right)^{\rho_i-1}M_i^{-1}Y^i_{[k-\rho_i+1,k]}, \quad 1 \leq i \leq m \tag{16}$$

Then, the output feedback sampled-data controller is taken as

$$u(k) = \gamma(\hat{x}(k), \zeta(k)) \tag{17}$$

for $k \geq k_0$, where $k_0 = \max_i\{\rho_i\} - 1$. The right-hand side of (16) has negative powers of T in D_i^{-1}. However, it can be verified that for all $k \geq \rho_i - 1$, $\hat{x}^i(k)$ will be bounded, uniformly in T, as long as $\mathcal{X}(k-\rho_i+1)$ and $U_{[k-\rho_i+1,k-1]}$ remain bounded.

It remains to determine how to initialize the controller for $0 \leq k < k_0$. There are two possible schemes:

- Arbitrary initialization of $Y^i_{k-\rho_i+1}$
- Arbitrary initialization of the control $u(k)$

In the first scheme, we replace the unavailable data $y_i(-\rho_i+1), \cdots, y_i(-1)$ with arbitrary numbers $\bar{y}_i(-\rho_i+1), \cdots, \bar{y}_i(-1)$. At $k=0$, we use these numbers together with $y(0)$ to form the vector $Y^i_{-\rho_i+1}$ in (16). As we move from $k=0$ to $k=1$, we use $\bar{y}_i(-\rho_i+2), \cdots, \bar{y}_i(-1)$, together with $y^i(0)$ and $y^i(1)$. By the time we reach $k = \rho_i - 1$, we recover the formula (16). The sampled-data controller of this scheme can be described by

$$\left.\begin{aligned} u(k) &= \gamma(\hat{x}(k), \zeta(k)), \quad \text{for } k \geq 0 \\ \hat{x}^i(k) &= D_i^{-1}\left(e^{A_i}\right)^{\rho_i-1}M_i^{-1}\Omega_i \end{aligned}\right\} \tag{18}$$

where

$$\Omega_i = A_i^{k+1}\begin{bmatrix} 0 \\ \bar{y}_i(-\rho_i+1) \\ \vdots \\ \bar{y}_i(-1) \end{bmatrix} + E^i_{k+1}Y^i_{[k-\rho_i+1,k]}$$

$$E^i_k = \begin{cases} \begin{bmatrix} 0_{(\rho_i-k)\times(\rho_i-k)} & 0 \\ 0 & I_{k\times k} \end{bmatrix} & \text{for } 1 \leq k < \rho_i - 1 \\ I_{\rho_i} & \text{for } k \geq \rho_i \end{cases}$$

in which I_k is the $k \times k$ identity matrix and 0_k is the $k \times k$ zero matrix. The scheme is equivalent to a deadbeat observer in which the transient response due to initial conditions converges to zero in a finite number of steps [13]. For $0 \leq k < \rho_i - 1$, the estimate $\hat{x}^i(k)$ may experience peaking for arbitrary choices of the initial conditions $\bar{y}_i(-\rho_i + 1), \ldots, \bar{y}_i(-1)$, due to the negative powers of T in D_i^{-1}. During this peaking, $\hat{x}^i(k)$ may take values of the order of negative powers of T, which gets worse with faster sampling. This peaking phenomenon is similar to the one experienced in high-gain observers and can be overcome by designing $\gamma(x, \zeta)$ to be a globally bounded function in x. During the peaking period, u saturates and protects the plant from peaking in $\hat{x}$, while the state of the plant $\mathcal{X}(k)$ cannot change by more than $O(T)$ from its initial value. Hence, for sufficiently small T, $\mathcal{X}(k_0)$ will be arbitrarily close to $\mathcal{X}(0)$. The global boundedness requirement is typically achieved by saturation of γ, or its x-input, outside a compact region of interest. It may also follow from design constraints. We will refer to this case as the *saturation scheme.*

In the second scheme, the controller is given by

$$\left.\begin{array}{rcl} u(k) & = & \bar{u}(k), \text{ for } 0 \leq k < k_0 \\ u(k) & = & \gamma(\hat{x}(k), \zeta(k)), \text{ for } k \geq k_0 \\ \hat{x}^i(k) & = & D_i^{-1}\left(e^{A_i}\right)^{\rho_i - 1} M_i^{-1} Y^i_{[k-\rho_i+1,k]} \end{array}\right\} \quad (19)$$

In this scheme, the formula (16) for estimating x^i is used only for $k \geq \rho_i - 1$; hence, there is no peaking in the calculation of $\hat{x}^i$. The control $u(k) = \gamma(\hat{x}(k), \zeta(k))$ can be calculated only starting from $k_0 = \max_i \rho_i - 1$. For $k < k_0$, we set the control at the arbitrary numbers $\bar{u}(0), \cdots, \bar{u}(k_0 - 1)$. Over the time period $[0, k_0 T]$, the state of the plant $\mathcal{X}(k)$ cannot change by more than $O(T)$ from its initial value. Hence, for sufficiently small T, $\mathcal{X}(k_0)$ will be arbitrarily close to $\mathcal{X}(0)$. We will refer to this case as the *initialization scheme.* Note that the controllers (18) and (19) coincide for $k \geq k_0$.

3 Performance Recovery

The output feedback sampled-data controller (18), where $\gamma(x, \zeta)$ is a globally bounded function in x, or (19) recovers the performance of the state feedback continuous-time controller $u(t) = \gamma(x(t), \zeta(t))$ for sufficiently small T. The performance recovery is summarized in the following theorem. The theorem shows that the solution of the closed-loop system under sampled-data output feedback control, which starts in a compact subset of the region of attraction of (6), stays bounded for all future time, comes arbitrarily close to the origin as time progresses, and converges to the solution of (6) as T tends to zero. When the origin of (6) is exponentially stable, the theorem shows that, for sufficiently small T, the solution of the closed-loop system converges to zero at time tends to infinity.

Theorem 1 *Consider the closed-loop system of the plant* (1)–(4) *and the sampled-data output feedback controller* (18), *where* $\gamma(x, \zeta)$ *is a globally bounded function in* x, *or* (19). *Suppose the origin of* (6) *is asymptotically stable and* $\mathcal{R}$ *is its region of attraction. Let* $\mathcal{S}$ *be any compact set in the interior of* $\mathcal{R}$, $\mathcal{X}(t)$ *be a solution of the closed-loop sampled-data system that starts in* $\mathcal{S}$, *and* $\mathcal{X}_c(t)$ *be the solution of closed-loop continuous-time system* (6) *that starts at* $\mathcal{X}_c(0) = \mathcal{X}(0)$. *Then,*

- *there exists* $T_1^* > 0$ *such that, for every* $0 < T \leq T_1^*$, $\mathcal{X}(t)$ *is bounded for all* $t \geq 0$ *and* $\hat{x}^i(k)$ *is bounded for* $k \geq \rho_i - 1$, *uniformly in* T.
- *given any* $\mu > 0$, *there exist* $T_2^* > 0$ *and* $T_a > 0$, *both dependent on* μ *and* $\mathcal{S}$, *such that, for every* $0 < T \leq T_2^*$, $\mathcal{X}(t)$ *satisfies*

$$\|\mathcal{X}(t)\| \leq \mu, \quad \forall\ t \geq T_a \qquad (20)$$

- *given any* $\mu > 0$, *there exists* $T_3^* > 0$, *dependent on* μ *and* $\mathcal{S}$, *such that, for every* $0 < T \leq T_3^*$, $\mathcal{X}(t)$ *satisfies*

$$\|\mathcal{X}(t) - \mathcal{X}_c(t)\| \leq \mu, \quad \forall\ t \geq 0 \qquad (21)$$

- *if the origin of* (6) *is exponentially stable and* $f(\mathcal{X})$ *is twice continuously differentiable in some neighborhood of the origin, then there exists* $T_4^* > 0$ *such that, for every* $0 < T \leq T_4^*$, $\mathcal{X}(t)$ *decays to zero, exponentially fast, as* $t \to \infty$.

The proof is omitted due to space limitations.

Example 1 Consider the system

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = x_3, \quad \dot{x}_3 = x_4, \quad \dot{x}_4 = \theta x_2^3 + u, \quad y = x_1$$

where θ is an unknown parameter that satisfies $|\theta| \leq 3$, together with the state feedback control law

$$u = -\beta(x)\ \mathrm{sat}\left(\frac{s}{0.015}\right)$$

where

$$\beta(x) = |x_2 + 2x_3 + x_4| + 3|x_2|^3 + 1, \ s = x_1 + 2x_2 + x_3 + x_4$$

Using sliding mode control analysis, it can be verified that the state feedback control globally stabilizes the origin $x = 0$. For initial conditions in the set $\{|x_i| \leq 0.5\}$, we determined via simulation that $|u| \leq 5$. In sampled-data output feedback, we estimate the state using (16) and take the control as

$$u(k) = -\beta(\hat{x}(k))\ \mathrm{sat}\left(\frac{\hat{s}(k)}{0.015}\right)$$

We consider two schemes for determining $u(k)$ for $k = 0$, 1, and 2. In the saturation scheme (18), we use the foregoing equations to calculate $u(k)$ for all k starting with $y(-3) = y(-2) = y(-1) = 0$ and saturate the control at ± 5. In the initialization scheme (19), we take $u(0) = u(1) = u(2) = 0$ and use the foregoing equations to calculate $u(k)$ starting from $k = 3$. Simulation is performed using Simulink with ode45 (Runge-Kutta). The fixed step size is automatically chosen by Simulink. In the sampled-data simulation, zero-order-hold blocks, with sampling period T, are used at the input and output of the plant. Figure 1 shows simulation results for the continuous-time state feedback and the two sampled-data output feedback schemes when $\theta = 2$. The initial conditions are $x_1(0) = x_2(0) = x_3(0) = x_4(0) = 0.5$ and the sampling period is $T = 0.001$. The trajectories are almost indistinguishable between the three cases. Figure 2 gives us a better picture of the closeness of trajectories by plotting the error in x_1 and x_4 between the state feedback and each of the two output feedback schemes. The figure demonstrates the trajectory convergence property of Theorem 1 by showing that the error decreases with T. The two figures illustrate how the two schemes overcome the peaking phenomenon, either by saturating the control or by initializing it at fixed values. There is an important difference though between the two schemes in that initializing the control requires us to detect any instant where the state suddenly deviates from zero and program the controller to reinitialize itself starting at that moment. Failing to do so could lead to peaking. This situation is illustrated in Figure 3, where at time $t = 20$ we inserted a pulse at the input of $\dot{x}_1$ of amplitude 500 and duration 0.001, which was good enough to increase the state of x_1 by about 0.5. The saturation scheme handles the situation gracefully and takes care of peaking through saturation. The initialization scheme is not reinitialized and consequently the system goes unstable.

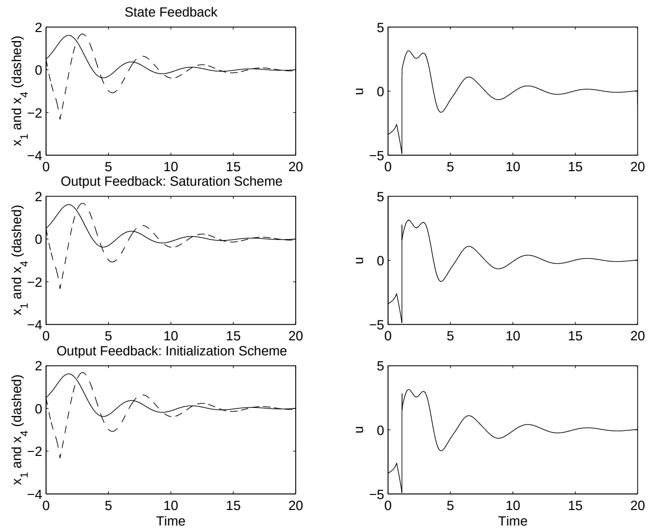

Figure 1: Simulation results for Example 1, showing state and control trajectories for the continuous-time state feedback control and the two sampled-data output feedback control schemes: saturation (18) and initialization (19). The sampling period $T = 0.001$.

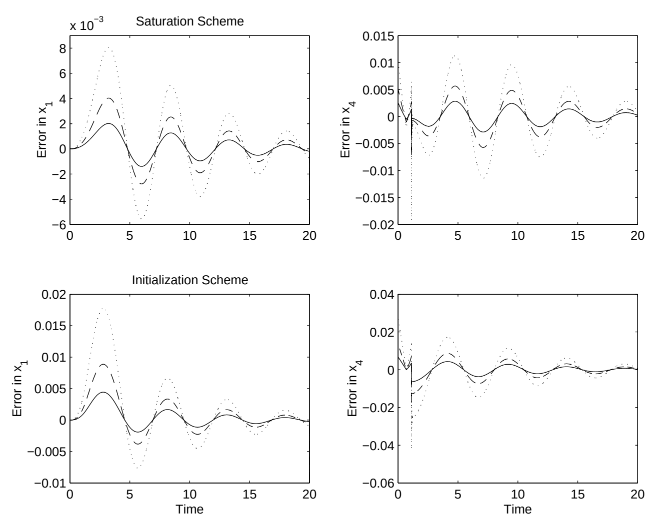

Figure 2: Simulation results for Example 1, showing the error in the state trajectories between the continuous-time state feedback control and each of the two sampled-data output feedback control schemes for $T = 0.0005$ (solid), 0.001 (dashed), and 0.002 (dotted).

4 Conclusions

We have presented a method for sampled-data stabilization of a class of nonlinear systems. The method designs a stabilizing state feedback controller in the continuous-time domain and recovers its performance in the sampled-data domain, for sufficiently small sampling period, by using a simple algorithm (16) to estimate the derivatives of the output. To estimate the derivatives of the output y_i at time k, the algorithm (16) uses the past values $y_i(k - \rho_i + 1)$ to $y_i(k-1)$. Using arbitrary past values at the initial time could cause peaking, where the transient behavior would of the order of negative powers of T. We presented two schemes to overcome this peaking phenomenon.

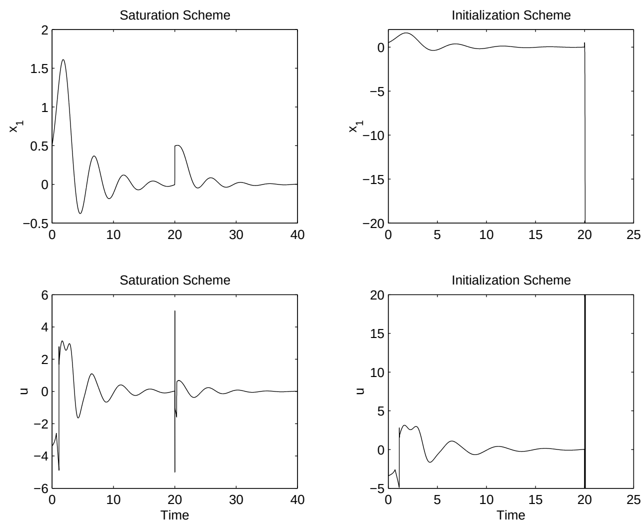

Figure 3: Simulation results for Example 1 with an impulsive-like disturbance inserted at $t = 20$. The saturation schemes (18) handles the disturbance gracefully. The initialization scheme (19) is not reinitialized following the disturbance and, consequently, the system goes unstable. The sampling period $T = 0.001$.

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