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Nonlinear Tire Lateral Force versus Slip Angle Curve Identification

Shiang-Lung Koo, Han-Shue Tan and Masayoshi Tomizuka

***Abstract*— Vehicle steering dynamics depends on the tire lateral characteristics. In particular, the relationship between the tire lateral force and the slip angle is a crucial element in the analysis and design of vehicle lateral dynamics and controls. This relationship is generally obtained through laboratory tests, but the laboratory test procedures have two major limitations: (I) the simulated road surface may not represent certain real road conditions; (II) the test environment may not sufficiently mimic the tire characteristics under realistic operating conditions, such as large tire-vehicle interactions under emergency conditions or the changes of suspension and/or steering geometry. It is advantageous to develop an identification approach for tire lateral characteristics using a minimum set of on-board vehicle sensors when the tires are operated in a real configuration and environment. An ideal identification procedure should be insensitive to noise and predict the vehicle lateral behavior accurately. In this paper, a pointwise updating approach with a Kalman filter using a nonlinear vehicle model is designed to address this problem. The effectiveness of the approach is demonstrated using tires with and without snow chains.**

Keyword: Kalman filtering, system identification, slip angle estimation, snow chains.

I. Introduction

Tires follow steering commands and generate forces to control and stabilize the vehicle under various external disturbances. Many researchers divide the tire characteristics into two categories, longitudinal and lateral forces, because of the complexity of tackling both aspects as a whole. This paper focuses on the issue of the tire lateral force with respect to the side-slip angle. This is the essential characteristic, which determines vehicle handling and lateral stability properties. Under emergency conditions, severe maneuvers, or extreme road conditions, this relationship can be highly nonlinear and the handling properties may be significantly different from those generated by the linear tire model. Advanced control systems proposed for maintaining lateral stability under these conditions often require accurate tire models to estimate the vehicle state for feedback. In practice, the tire model is obtained through laboratory tests using either a drum or a flat steel belt with various surface treatments to simulate road surfaces. These laboratory procedures have two major limitations: (I) the simulated road condition does not truly represent the actual road surface; (II) there is no guarantee that the tires in a real vehicle configuration will behave exactly the same as those in the laboratory, especially since the tire facility usually tests one tire at a time. For example, it is difficult to simulate snow-covered road conditions or tires under large steering/suspension geometry changes. It is highly desirable to have a simple identification approach using only a minimum set of on-board vehicle sensors, such as wheel encoders and inertial sensors, to identify the tire characteristics under realistic vehicle and road conditions.

The difficulties of such an approach lie on two issues: noises and nonlinearities. In this study, a modified Kalman Filter is employed to deal with the noise problem under nonlinearities. An iterative procedure is proposed to search for a nonlinear relation between the slip angles and the tire forces under quasi-steady-state conditions. This paper describes the offline identification procedure and compares the experimental results. An example of comparing tires with and without snow chains on a sand-covered test track was used to demonstrate the effectiveness of this technique. It is an interesting example because tires with snow chains under extreme road conditions are rarely discussed in literature and the characteristics are difficult to obtain through laboratory test procedure.

II. Nonlinear Vehicle and Tire Lateral Characteristics

When a vehicle is cornering, tires generate appropriate lateral forces to support the vehicle along a certain path. These forces create the deformation in the tire tread [1]. As a result of the deformation, the traveling direction of the tire differs from the wheel center plane by the slip angle. The relation between lateral forces and slip angles determines the

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vehicle lateral dynamics, which can be very different from those generated by the "geometric model". A typical tire lateral characteristic curve is shown in figure 1. This curve is usually divided into three regions: *linear/elastic*, *transitional*, and *frictional* [2]. Many researchers use a linear tire model to perform analysis on vehicle lateral dynamics and controller design. A linear tire model can be used to predict the properties in the *elastic* region but generally cannot be employed in *transitional* and *frictional* regions. For instance, the vehicle dynamics are unstable when the tires are operated in the *frictional/sliding* region. A linear tire model can not predict the instability.

P215/60R15 Goodyear Eagle GT-S (shaved for racing) 31 psi. Load = 1,800 lb

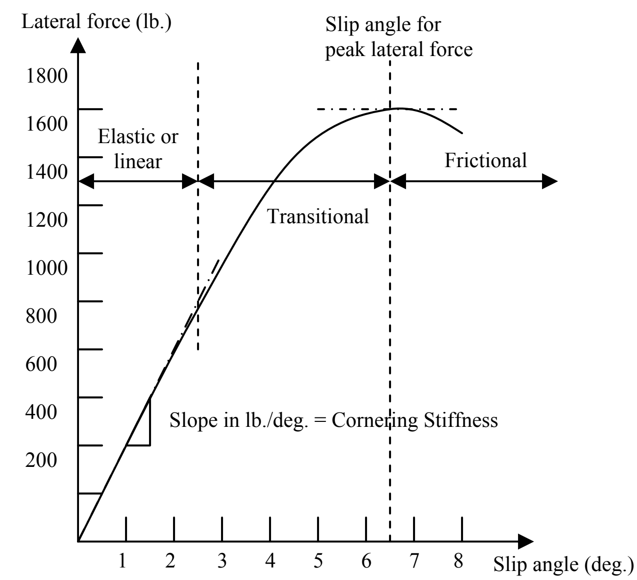

Figure 1: typical lateral force versus slip angle [2]

Since the tread deformation results in side velocity at each tire, the direction of the linear motion of the tire differs from the longitudinal direction of the tire resulting the tire slip angle. The tire slip angle is given by [3]

$$\alpha_f = \frac{\dot{y}_u + l_1\dot{\varepsilon}}{\dot{x}_u} \quad (1)$$

$$\alpha_r = \frac{\dot{y}_u - l_2\dot{\varepsilon}}{\dot{x}_u} \quad (2)$$

where $\alpha_{f,r}$: slip angles of front/rear axles

$l_{1,2}$:length between front/rear axles & vehicle C.G.

$\dot{x}_u$: longitudinal velocity in unsprung mass coordinates

$\dot{y}_u$:lateral velocity in unsprung mass coordinates

$\dot{\varepsilon}$: yaw rate.

The two equations show that the slip angles can be calculated as long as the vehicle speed, yaw rate and lateral velocity at C.G. can be measured or estimated. When inertia sensors are used to sense yaw rate, Eq. (3) can be employed to estimate lateral velocity at the vehicle C.G.

$$a_x = \ddot{x}_u - \dot{\varepsilon}\dot{y}_u \quad (3)$$

where a_x : longitudinal acceleration in inertia coordinates

$\ddot{x}_u$: longitudinal acceleration in unsprung mass coordinates.

The longitudinal acceleration with respect to the inertia frame can be directly measured by a longitudinal accelerometer and the acceleration with respect to the unsprung mass frame can be numerically differentiated from wheel speed sensors. The lateral velocity at C.G. can then be estimated as in Eq. (3). A simple method to predict the lateral force is to use vehicle's onboard sensors and a lateral model given by Eqs. (4) and (5) below. However, the static nonlinear relationship represented by the resulting force-slip-angle pairs is generally extremely noisy.

$$ma_y = F_f \cos\delta + F_r \quad (4)$$

$$I_{zz}\ddot{\varepsilon} = F_f \cos\delta l_1 - F_r l_2 \quad (5)$$

where $F_{f,r}$: tire lateral forces at front/rear axles

a_y : lateral acceleration in inertia coordinates

δ : steering angle

m : vehicle mass

I_{zz} : inertia moment of yaw motion.

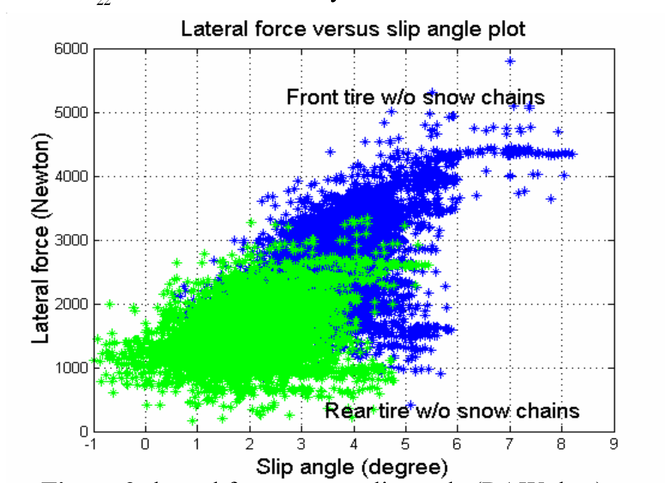

Figure 2: lateral force versus slip angle (RAW data)

Figure 2 is an example of a dispersed plot of such characteristics, which results from feeding the raw test data from a typical vehicle cornering experiment (details in section IV) into the equations of force and slip angle estimations. Determining a static nonlinear relationship by using general curve fitting techniques can be difficult. The difficulties lie in: (I) the inherited large noises contained in the vehicle environments and the resulting noise amplification from a nonlinear relationship, (II) a very low signal to noise ratio in the slip angle estimate, (III) the nonlinear relationship between the slip angle and lateral force, and (IV) nonlinearities in the vehicle model under large-angle operating conditions. As a result, the effects of uncertainties and noises are amplified during the calculation of slip angles.

III. POINTWISE UPDATING APPROACH WITH KALMAN FILTERING

Figure 3 shows a typical identification procedure based on a linear model. The adaptation block is designed to minimize the prediction errors by adjusting the system parameters in the linear model [4]. However, this approach cannot be applied to the tire identification due to the nonlinearities and system input noise problems as presented in section II.

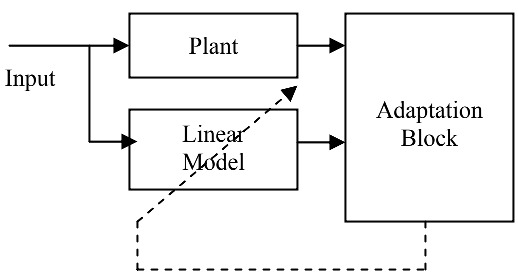

Figure 3: typical linear system identification

Frequency-weighted filters can attenuate the measurement noise but fail to account for the nonlinear dynamics. In this study, a Kalman filter is used to reduce the noise effect in the measurements, while a nonlinear vehicle model is incorporated to the filter to account for the nonlinearities. The model-based filtering is found to be effective in addressing both the nonlinearities in the system and the noise in the measurement. The structure of this identification approach is shown in Figure 4.

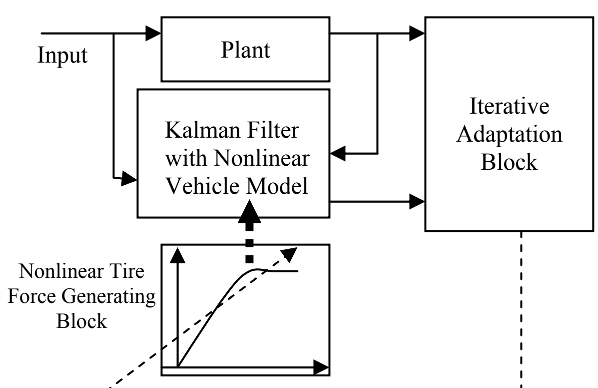

Figure 4: proposed identification procedure

The nonlinear vehicle model [5] used in the modified Kalman filter is expressed as

$$m(\ddot{y}_u + \dot{x}_u \dot{\varepsilon}) = F_f(\alpha_f)\cos\delta + F_r(\alpha_r) \tag{6}$$

$$I_{zz}\ddot{\varepsilon} = F_f(\alpha_f)\cos\delta l_1 - F_r(\alpha_r)l_2 \tag{7}$$

Currently, no simple parametric models can effectively describe the tire force generating functions, $F_f(\alpha_f)$ and $F_r(\alpha_r)$, in Eqs. (6) and (7). "Non-parametric" tire functions are proposed in this paper to represent the nonlinear characteristic curves. In practice, the functions $F_f(\alpha_f)$ and $F_r(\alpha_r)$ are parameterized as a set of points (force vs. slip) in a look-up table (see Figure 5). Two look-up tables are used to represent the front and rear tire characteristics, respectively. By inserting the two look-up tables into the model in the Kalman Filter, significantly better state estimates can be obtained despite the input/output noise and corresponding nonlinear amplification in the slip angle computation. The errors between the estimates and the measurements will be employed to update the two look-up tables. The "points" on the two "non-parametric" curves will be adjusted iteratively until they converge. The final look-up tables in the iteration are the identified non-parametric curves.

Three key issues in this identification approach will be discussed in detail: (I) modified Kalman filtering, (II) quasi-steady-state operation, and (III) parameter updating algorithm.

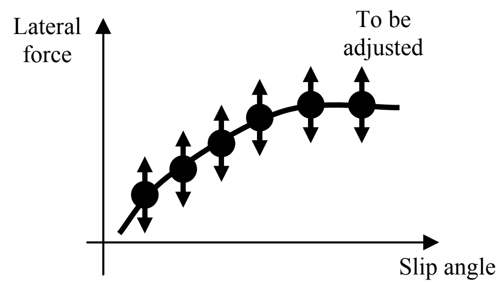

Figure 5: non-parametric approach using look-up tables

(I) A bicycle vehicle model [6] is used in the modified Kalman filter under the following assumptions:

1. roll and pitch motions are neglected;
2. front/rear two tires are lumped;
3. large steering angle effects are considered;
4. system parameters, such as the inertia and geometric locations are known;
5. time-invariant static nonlinear relations are assumed.

Two system states, yaw rate and lateral velocity at C.G. with respect to the unsprung mass frame, describe the vehicle lateral dynamics. The filter uses yaw rate and the steering angle to generate smooth system states, where the steering angle is the system input and yaw rate is the system output. The filter gain is determined by solving the standard Kalman

Filter problem with the linearized mathematical model around each equilibrium point. Gain scheduling is used to ensure the local stability of the filter.

(II) Various types of input can be selected for system identification. One simple choice is to apply step input till the system reaches steady state. The advantages of using step input in the identification of tire lateral properties are: (a) the test can be performed within a small site; (b) noise attenuation can be easily accomplished through "averaging" under steady state conditions. At steady state, a step input usually results in different slip angles on the front and rear tires. If the experiment can be designed to allow both front and rear tires to experience the *elastic*, *transitional*, and *frictional* regions, the parameters on these two characteristic curves can then be fully determined. In the first study, only the characteristic curves in *elastic* and *transitional* regions will be evaluated, since manually steering and stabilizing a vehicle with tires operated in the *frictional* region is difficult. Two basic approaches can be used to create the preferred quasi-steady-state experimental situations:

1. keeping the longitudinal velocity constant and gradually varying the steering angle;
2. keeping steering angle constant and slowly changing the longitudinal velocity.

In this paper, the second approach was applied because it allows for a smaller test track using large steering angles.

(III) The two characteristic curves in the look-up tables are the unknowns and need to be initialized before iteration. Iteration can begin with an appropriate guess for each curve. Wrong parameters used in the vehicle model during iteration result in biased estimates. Bias in the estimated states can be computed by averaging the measured and estimated states with a similar estimated slip angle. One widely used approach is to formulate the minimization of the bias as a least-square problem. Consider the following least-square problem. The cost function, $J(\theta) = \|\varepsilon\|^2$ is minimized over θ, where θ represents the unknown system parameters and ε is the vector of the state bias. The gradient descent technique can be employed to search for the minimizer of the cost function. This formulation is similar to the LMS algorithm in the adaptive filtering as can be seen in Eq. (9) [7].

$$\hat{\theta}_{k+1} = \hat{\theta}_k - \mu \left.\frac{\partial J}{\partial \theta}\right|_{\hat{\theta}_k} = \hat{\theta}_k + K\varepsilon \tag{9}$$

The gradient of J calculated with the current estimated parameters, $\hat{\theta}_k$, is the steepest descent direction. μ is used for tuning the convergence rate. After some algebra, the term, $-\mu \left.\frac{\partial J}{\partial \theta}\right|_{\hat{\theta}_k}$, can be expressed as the linear combination of the state bias, $K\varepsilon$, where K is a matrix gain. As a result, the bias is used to update the parameters in the look-up tables. As shown in Fig. 6, for any given slip angle, $\hat{\alpha}_i$, a corresponding estimated force, $\hat{F}_i(\hat{\alpha}_i)$, will need to be determined, where the subscript i can be f (front) or r (rear). Two consecutive points are connected by splines. The local slope at each point on the characteristic curve can be approximated by differentiating the splines. The local slope is then used for the Kalman filter design. This identification problem can then be regarded as a set of least square problems in solving the forces, F_i's, at the corresponding slip angle, α_i's.

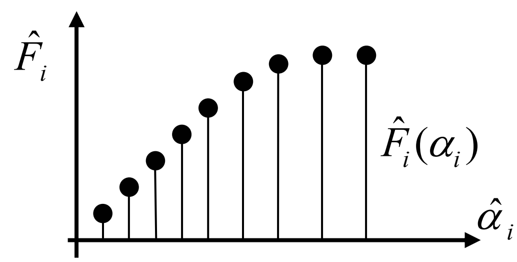

Figure 6: estimated force at each slip angle of the nonlinear relation

The identification procedure is summarized in Fig. 7. Test data is obtained under the following quasi-steady-state vehicle experiment: slowly changing longitudinal velocities under approximately the same steering input. The data is then fed into the modified Kalman filter. The nonlinear curves of the front and the rear tires in the Kalman filter are initialized by two guessed curves. Estimated and measured states with the similar estimated slip angle are sorted in the same group. The biases are computed by averaging the measured and the estimated states in each group. The curves in the two look-up tables were adjusted by using the linear combination of the state bias. Iteration starts again with newly obtained curves.

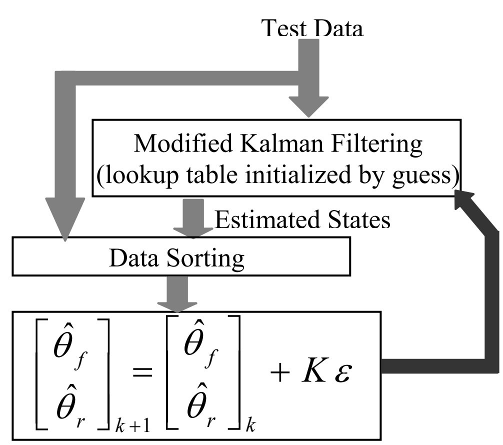

Figure 7: flow diagram of the identification procedure

IV. Experimental Results

The experiments were conducted on a sand-covered test track. The sand was used to simulate a snow-covered road with reduced road-holding capability. The road was a 2.5-meter-wide circular track with an inner radius of 7.5 meters. The steering angle was fixed at approximately 19 degrees on the front tires and the longitudinal speed was slowly changed from 2 m/sec to 6 m/sec until the wheels began to slide. Three independent configurations were tested in the experiments as shown in table 1.

	Front tire	Rear tire
Configuration 1	w/o snow chains	w/o snow chains
Configuration 2	with snow chains	w/o snow chains
Configuration 3	w/o snow chains	with snow chains

Table 1: three different configurations using tires with or without snow chains

Using the proposed identification approach, figure 8(a), 8(b), and 8(c) show the final tire lateral force versus slip angle curves of configurations 1, 2, and 3, respectively. Each dot represents the force estimate directly calculated from the measurements and the corresponding slip angle estimate produced from Kalman Filter. The solid line is the non-parametric characteristic curve based on the final iteration. As can be seen in Figs. 8(d) and 8(e), blue lines, green lines, and red lines represent the identified curves for the three configurations, respectively. As expected, the curve of the front tire in configuration 1 is the same as that of the front tire in configuration 3 since neither has snow chains installed. Similarly, the curves for the rear tires in configuration 1 and 2 are the same. The slope of the *elastic* region is almost the same for both tires with and without snow chains. The *elastic* region for tires with snow chains is slightly narrower than that for tires without snow chains. In the *transitional* region, the curves differ much more. With snow chains installed, the *transitional* region of tires with snow chains is able to extend to higher slip angles. On the other hand, the forces in this region will rise up more slowly than those without snow chains. Tires generally slide should they go beyond the *transitional* region. The solid line in the *frictional/sliding* region is not shown in Fig. 8 because it either violated the assumption of the quasi-steady-state operation or the vehicle was temporarily driven outside the sand-covered test track and onto the pavement. The length of the solid line before the *frictional* region is determined based on the statistics of collected data. Some of the points in the two look-up tables were neglected if the probability was less than a preset threshold. An example is shown in Fig. 9.

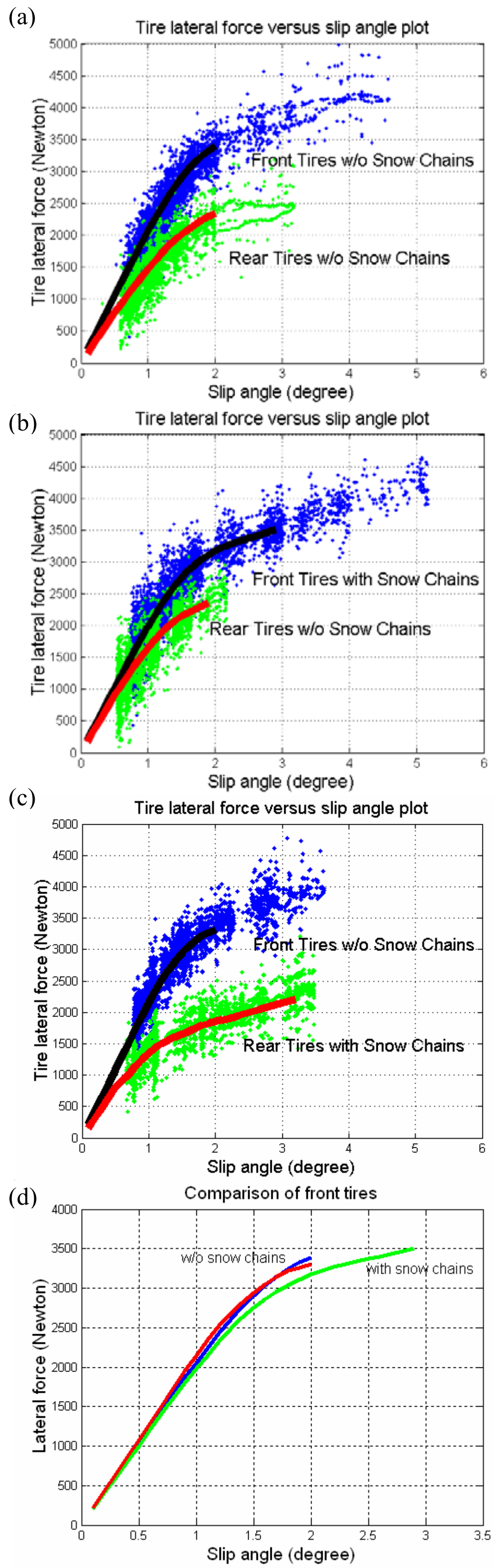

Figure 8: experimental results on sand-covered road

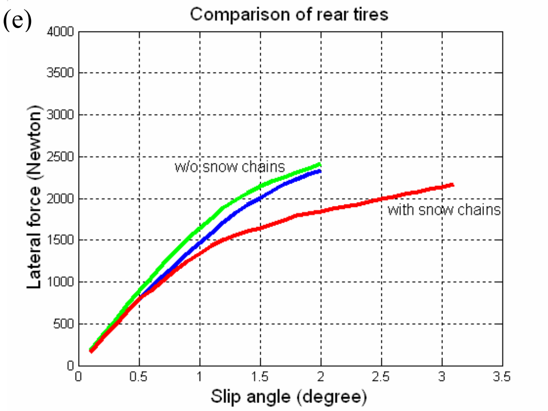

Figure 8: experimental results on sand-covered road (continued)

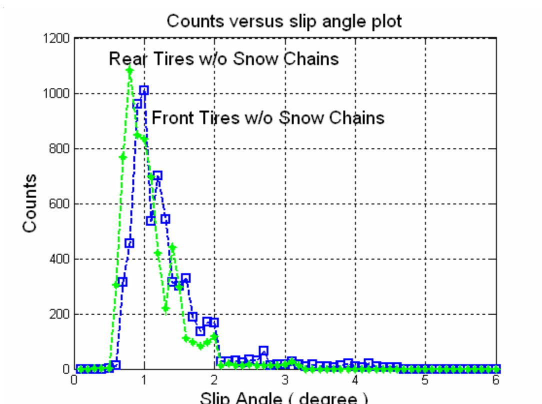

Figure 9: # of points at each slip angle in configuration 1

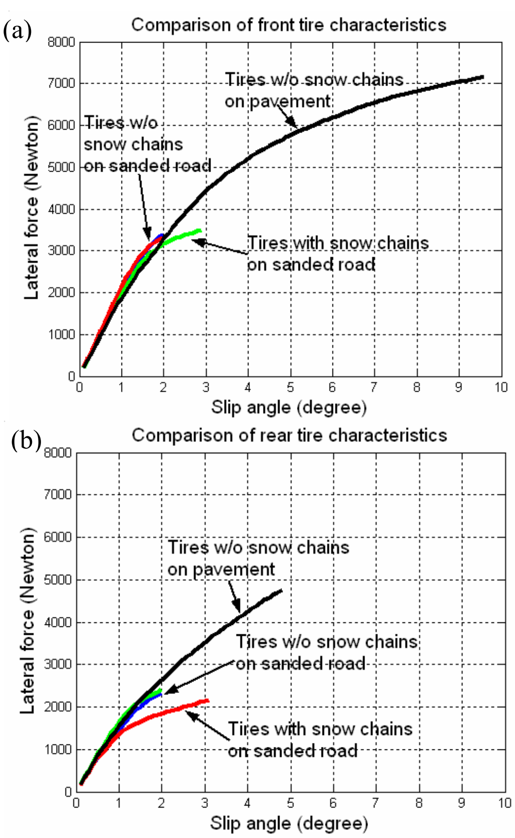

Figure 10: experimental results on dry pavement

The experiments on dry pavement without snow chains were also performed (configuration 4). The comparison among the identified curves of the four configurations is presented in figure 10(a) and 10(b). It can be observed that the curves for either the front or the rear tires in all configurations have approximately the same slope in the *elastic* region. The differences lie in *transitional* regions. The front tire forces saturated at 9.5 degrees for tires without snow chains on pavement, at 2 degrees for tires without snow chains on a sand-covered road, and at 3 degrees for tires with snow chains on a sand-covered road. The rear tire forces can be extended as high as 5 degrees for tires without chains on the pavement, 2 degrees for tires without chains on a sand-covered road, and 3 degrees for tires with snow chains on a sanded road. The results show that this identification approach can be applied to various road conditions.

V. Conclusion

This paper demonstrated that a point-wise updating approach with Kalman filtering using a nonlinear vehicle model can effectively determine the nonlinear tire lateral characteristics by using simple on-vehicle sensors. This offline method successfully addressed the nonlinearities and the noise problems in a real vehicle environment. The possibility of real-time identification is under investigation. In the example of tires with snow chains, the curve is extended to higher slip angles while the cornering stiffness in the *transitional* region is reduced, which follows our intuition.

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