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A Design of Model Matching Systems for Fat Plants

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***Abstract*— In this paper, a design of model matching systems for a plant having more inputs than the outputs (e.g., fat plant) will be presented. For this aim, an inverted interactorizing system by state feedback will be investigated. It will be shown that the highest frequency gain matrix of given fat plant cannot be assigned arbitrarily. This means that the unstable pole-zero cancellation may occur by applying the control law for the previous reported method, even if the plant has no unstable zeros. A special selection of generalized inverse of the highest frequency gain matrix will be proposed to achieve stable model matching systems.**

Key words : linear multivariable systems, model matching systems, interactor, inverted interactorizing, fat systems.

I. Introduction

The aim of model matching control is to combine the transfer function matrix of the plant and controllers to match the reference model, and one of the ideal design of control systems. The basic design of model matching was presented in [1] and [2], and developed in [3] relating with the parameter adaptive control. In the parameterization by [3], the denominator polynomial matrix of the plant is assigned to its numerator polynomial matrix and interactor matrix [4]. Therefore, the stability of the numerator polynomial matrix is necessary and sufficient condition for an internally stable model matching systems. Unfortunately, the methods mentioned above is only used for the plant having same number of inputs and outputs.

In the case of the plant having more inputs than the outputs, which have "fat" transfer function matrix, it is considered that the problem can be reduced to the square case by eliminating some inputs. But it is not clear how to select the eliminating inputs. That is, by eliminating some columns in the transfer function matrix, some unstable zeros may be appeared even if the original plant has no unstable invariant zeros. Thus, the elimination method is not adequate.

In this paper, it will be proposed a design of model matching system for fat plants. For this development, it will be investigated that the derivation method of an interactor and inverted interactorizing using the state feedback [5]. The inverted interactorizing gives the basic structure of model matching control systems and, of course, an interactor plays an important role there. The authors proposed a simple derivation of the interactor for square transfer matrices [7]. In this paper, it will be extended to the fat case. Then, it will be presented an effective proof of the inverted interactorizing to make clear its structure. By these developments, the pole-zero cancellation mechanism will be clear in the inverted interactorizing systems. A necessary and sufficient condition for an internally stable inverted interactorizing systems will be presented.

The paper organized as follows. In the next section, the definition of the interactor and a simple derivation of it for fat plants will be given. In section 3, a simple proof of the inverted interactorizing will be presented and a necessary and sufficient condition for the internally stability will be given. The extension for model matching system will be discussed in section 4. The proposed method is easy to apply for a plant with measurement noise and numerical example will be shown in section 5. Concluding remarks will be given in section 6.

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II. Interactor

For a given $m \times p$ ($m \leq p$) strictly proper, full rank transfer function matrix $G(z)$, there exists $m \times m$ polynomial matrix $L(z)$ such that

$$\lim_{z \to \infty} L(z)G(z) = K \ (full\ rank). \tag{1}$$

The above $L(z)$ is called an interactor matrix for $G(z)$ [4], and if $p = m$ and $K = I_m$ (m-th dimensional identity matrix), then $L(z)$ is called an identity interactor. At first, the derivation of the identity interactor $\xi(z) := K^{-1}L(z)$ will be discussed.

Set $\xi(z)$ as follows:

$$\xi(z) = \xi_0 + z\xi_1 + z^2\xi_2 + \cdots + z^w\xi_w \tag{2}$$

where ξ_i ($i = 0, 1, \ldots, w$) is an $m \times m$ matrix to be determined, and w will be defined later. Let (A, B, C) denote a realization of $G(z)$. Then, the following relation holds [6]:

$$\boldsymbol{\xi} \boldsymbol{T}_{w-1} = \boldsymbol{J}_{w-1} \tag{3}$$

where

$$\boldsymbol{\xi} = \begin{bmatrix} \xi_1 & \cdots & \xi_w \end{bmatrix}, \quad \boldsymbol{J}_{w-1} = \begin{bmatrix} I_m & 0_{m \times m(w-1)} \end{bmatrix},$$

$$\boldsymbol{T}_{w-1} = \begin{bmatrix} CB & 0 & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{w-1}B & CA^{w-2}B & \cdots & CB \end{bmatrix},$$

$$S_{I_m}^w(z) = \begin{bmatrix} I_m & zI_m & \cdots & z^w I_m \end{bmatrix}^T.$$

Now, w is defined by the least integer which satisfies the following equation:

$$\text{rank}\begin{bmatrix} \boldsymbol{T}_{w-1} \\ \boldsymbol{J}_{w-1} \end{bmatrix} = \text{rank}\boldsymbol{T}_{w-1}. \tag{4}$$

From the above, eqn.(3) is solvable, and using the pseudoinverse $\boldsymbol{T}_{w-1}^{\dagger}$ of $\boldsymbol{T}_{w-1}$, the solution is given by

$$\boldsymbol{\xi} = \boldsymbol{J}_{w-1}\boldsymbol{T}_{w-1}^{\dagger}. \tag{5}$$

The above solution has the following properties.

Theorem 1 If the solution of eqn.(3) is given by eqn.(5), then the following properties hold:

$$\textbf{P1} \quad \xi(z)\xi^{\sim}(z) = \boldsymbol{\xi}\boldsymbol{\xi}^T, \tag{6}$$
$$\textbf{P2} \quad \mathcal{O}_{w-1}(C, A_F)B = \boldsymbol{\xi}^{\dagger}, \tag{7}$$
$$\textbf{P3} \quad CA_F^w = 0 \tag{8}$$

where $\boldsymbol{\xi}^{\dagger}$ is the pseudoinverse of $\boldsymbol{\xi}$, and

$$\xi^{\sim}(z) = \xi^T(z^{-1}) = \xi_0^T + z^{-1}\xi_1^T + \cdots + z^{-w}\xi_w^T,$$
$$F = \boldsymbol{\xi}\mathcal{O}_{w-1}(C,A)A, \quad \mathcal{O}_{w-1}(C,A) = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{w-1} \end{bmatrix},$$
$$A_F = A - BF.$$

See [7] for the proof.

P1 means that $\xi(z)$ has all-pass property in discrete-time [8], [9]. P2 and P3 show that non-zero Markov parameters of the inverted interactorizing system are given by the pseudoinverse of $\boldsymbol{\xi}$, since F is a feedback gain of inverted interactorizing [5].

Next, the derivation for fat plants will be discussed. In this case, the relation corresponding to eqn.(3) is given by

$$\boldsymbol{L}\boldsymbol{T}_{w-1} = \bar{\boldsymbol{J}}_{w-1} \tag{9}$$

where

$$L(z) = L_0 + zL_1 + \cdots + z^w L_w,$$
$$\boldsymbol{L} = \begin{bmatrix} L_1 & \cdots & L_w \end{bmatrix},$$
$$\bar{\boldsymbol{J}}_{w-1} = \begin{bmatrix} K & 0_{m\times p(w-1)} \end{bmatrix}.$$

Since any special form of K cannot be assumed, find it by the following iteration.

Set $w = 1$. From eqn.(9), $\boldsymbol{L}$ can be represented by

$$\boldsymbol{L} = \bar{\boldsymbol{J}}_{w-1}\boldsymbol{T}_{w-1} = K\boldsymbol{W}_{w-1} \tag{10}$$

where $\boldsymbol{W}_{w-1}$ represents the matrix which consists of first p-th rows in $\boldsymbol{T}_{w-1}^{\dagger}$. Substituting the above equation to eqn.(9),

$$K\left\{\boldsymbol{W}_{w-1}\boldsymbol{T}_{w-1} - \begin{bmatrix} I_p & 0_{p\times p(w-1)} \end{bmatrix}\right\} = 0. \tag{11}$$

Thus, K can be obtained by calculating left null space of $\left\{\boldsymbol{W}_{w-1}\boldsymbol{T}_{w-1} - \begin{bmatrix} I_p & 0_{p\times p(w-1)} \end{bmatrix}\right\}$. If rank$K < m$, repeat the above procedure setting $w = w+1$. If not, using K, $\boldsymbol{L}$ is given by eqn.(10).

Note that the above method is essentially same as in square case and all zeros of the interactor lie at origin. The following Lemma is trivial but important.

Lemma 1 For a given fat transfer function matrix $G(z)$, let $L_1(z)$ and $L_2(z)$ denote interactor matrices, and K_1 and K_2 denote the highest frequency gains satisfying eqn.(1), respectively. Then, there exists nonsingular matrix M such that

$$K_2 = MK_1. \tag{12}$$

(Proof). As shown in eqn.(9), K_1 and K_2 are given by linearly combinations of rows in $\mathcal{O}_{w-1}(C,A)B$. Thus, eqn.(12) holds.

Example 1 Consider the following transfer function matrix $G(z)$.

$$G(z) = \begin{bmatrix} \frac{1}{z+0.1} & \frac{1}{z+0.2} & \frac{1}{z+0.3} \\ \frac{1}{z+1} & \frac{1}{z+1.1} & \frac{1}{z+1.2} \end{bmatrix}.$$

A state space realization (A, B, C) of $G(z)$ is given by

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -0.1 & -1.1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -0.22 & -1.3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -0.36 & -1.5 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 & 1.1 & 1 & 1.2 & 1 \\ 0.1 & 1 & 0.2 & 1 & 0.3 & 1 \end{bmatrix}.$$

In this case, $w = 3$ and $\boldsymbol{W}_3\boldsymbol{T}_3 - \begin{bmatrix} I_3 & 0_{3\times 6} \end{bmatrix}$ is given by

$$\boldsymbol{W}_3\boldsymbol{T}_3 - \begin{bmatrix} I_3 & 0_{3\times 6} \end{bmatrix} = \begin{bmatrix} -0.1667 & 0.3333 & -0.1667 & 0.0001 & 0 & -0.0001 & 0.0011 & 0.0011 & 0.0011 \\ 0.3333 & -0.1667 & 0.3333 & -0.0002 & 0 & 0.0002 & -0.0022 & -0.0022 & -0.0022 \\ -0.1667 & 0.3333 & -0.1667 & 0.0001 & 0 & -0.0001 & 0.0011 & 0.0011 & 0.0011 \end{bmatrix}.$$

Thus, K is given by

$$K = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$$

and the interactor is given by

$$\boldsymbol{L} = \begin{bmatrix} 0.4158 & 0.4158 & 0.1871 & -0.1871 & 0 & 0 \\ 2.7027 & 2.7027 & -3.7838 & -6.2162 & 11.1111 & -11.1111 \end{bmatrix},$$

$$L(z) = z^3\begin{bmatrix} 0 & 0 \\ 11.1111 & -11.1111 \end{bmatrix} + z^2\begin{bmatrix} 0.1871 & -0.1871 \\ -3.7838 & -6.2162 \end{bmatrix} + z\begin{bmatrix} 0.4158 & 0.4158 \\ 2.7207 & 2.7207 \end{bmatrix}.$$

On the other hand, by the method in [4], K_2 and $L_2(z)$ are given by

$$K_2 = \begin{bmatrix} 1 & 1 & 1 \\ 12 & 15 & 18 \end{bmatrix}, \quad L_2(z) = \begin{bmatrix} z & 0 \\ -z^3+3z^2 & z^3 \end{bmatrix}.$$

It is clear that K and K_2 are linearly dependent as shown in Lemma 1.

III. Inverted Interactorizing by State Feedback

Although the inverted interactorizing was proposed and proved in [5], the proof was complex. A simple method will be proposed for the following discussions.

Lemma 2 For a given $m \times p$ fat transfer function matrix $G(z)$, let (A, B, C) denote a realization of $G(z)$. Define the feedback gain by

$$\begin{aligned} \bar{F} &= \begin{bmatrix} L_0 & \boldsymbol{L} \end{bmatrix} \mathcal{O}_{w-1}(C, A) \\ &= \begin{bmatrix} L_0 & L_1 & \cdots & L_w \end{bmatrix} \begin{bmatrix} C \\ CA \\ \vdots \\ CA^w \end{bmatrix}. \end{aligned} \tag{13}$$

Then, by the control law

$$u(t) = -K^-(\bar{F}x(t) - r(t)), \tag{14}$$

the inverted interactorizing is achieved, where K^- is a generalized inverse of K, and $r(t) \in \mathbf{R}^m$ is a command input.

(Proof). In the state space equation

$$zx(t) = Ax(t) + Bu(t) \tag{15}$$

$$y(t) = Cx(t), \tag{16}$$

Multiplying eqn.(16) by z, and substituting it to eqn.(15) recursively, it follows [10]

$$S^w_{I_m}(z)y(t) = \mathcal{O}_w(C, A)x(t) + \begin{bmatrix} 0_{m \times pw} \\ \boldsymbol{T}_{w-1} \end{bmatrix} S^{w-1}_{I_p}(z)u(t). \tag{17}$$

Leftmultiplying the above equation by $\begin{bmatrix} L_0 & \boldsymbol{L} \end{bmatrix}$, and using eqns.(9) and (13),

$$\begin{aligned} L(z)y(t) &= \bar{F}x(t) + \bar{\boldsymbol{J}}_{w-1}S^{w-1}_{I_p}(z)u(t) \\ &= \bar{F}x(t) + Ku(t). \end{aligned} \tag{18}$$

A solution of the above equation for $u(t)$ is given by

$$u(t) = -K^-(\bar{F}x(t) - L(z)y(t)), \tag{19}$$

and substituting the above to the state equation (15) and (16),

$$y(t) = C(zI - A + BK^-\bar{F})^{-1}BK^-L(z)y(t).$$

Therefore,

$$L^{-1}(z) = C(zI - A + BK^-\bar{F})^{-1}BK^-$$

is obtained.

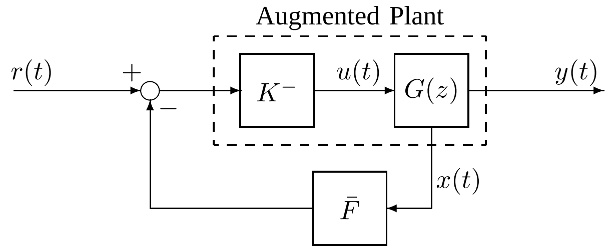

Fig. 1. Inverted Interactorizing Systems by State Feedback

In the control law (14), the generalized inverse K^- can be interpreted as the squalizing pre-compensator (see Fig.1). Then, $\bar{F}$ is the conventional inverted interactorizing feedback gain for the square plant $G(z)K^-$. Therefore, the following Theorem about the stability of the closed-loop system holds.

Theorem 2 Internally stable inverted interactorizing is achieved if and only if $G(z)K^-$ is stably invertible.

Note that it is not adequate to adopt the pseudoinverse $K^\dagger = K^T(KK^T)^{-1}$ of K as a generalized inverse. Since there is no degree of freedom in K from Lemma 1, so as in K^T and $K^\dagger$. Thus, from Theorem 2, the zeros of $G(z)K^T$ are fixed and the inverted interactorizing system may not be stable even if there are no unstable invariant zeros in $G(z)$.

Example 2 Consider the same plant as in Example 1. Note that $G(z)$ has no invariant zeros outside the unit circle. Since a left coprime factorization of $G(z)$ is given by

$$G(z) = \tilde{D}^{-1}(z)\tilde{N}(z),$$

$$\tilde{D}(z) = \begin{bmatrix} \tilde{d}_{11}(z) & 0 \\ 0 & \tilde{D}_{22}(z) \end{bmatrix},$$

$$\tilde{N}(z) = \begin{bmatrix} \tilde{n}_{11}(z) & \tilde{n}_{12}(z) & \tilde{n}_{13}(z) \\ \tilde{n}_{21}(z) & \tilde{n}_{22}(z) & \tilde{n}_{23}(z) \end{bmatrix},$$

$$\tilde{d}_{11}(z) = (z+0.1)(z+0.2)(z+0.3)$$

$$\tilde{d}_{22}(z) = (z+1)(z+1.1)(z+1.2)$$

$$\tilde{n}_{11}(z) = (z+0.2)(z+0.3) \quad \tilde{n}_{21}(z) = (z+1.1)(z+1.2)$$

$$\tilde{n}_{12}(z) = (z+0.1)(z+0.3) \quad \tilde{n}_{22}(z) = (z+1)(z+1.2)$$

$$\tilde{n}_{13}(z) = (z+0.1)(z+0.2) \quad \tilde{n}_{23}(z) = (z+1)(z+1.1)$$

the zeros of $G(z)K^T$ are given by that of

$$\det \tilde{N}(z)K^T = -0.54z^2 - 0.702z - 0.1206$$

that is,

$$z = -1.0963, \ -0.2037$$

which is unstable and an internally stable inverted interactorizing will not be achieved if $K^\dagger$ is used as a generalized inverse. In fact, feedback gain $\bar{F}$ is given by

$$\begin{aligned} \bar{F} &= \boldsymbol{L}\mathcal{O}_2(C, A)A \\ &= \begin{bmatrix} -0.1 & -0.6426 & -0.22 & -0.7595 & -0.36 & -0.8763 \\ -0.1 & 1.8730 & 0 & 3.5135 & 0.36 & 5.5541 \end{bmatrix} \end{aligned}$$

and the eigenvalues of $A - BK^\dagger\bar{F}$ are given by

$$0, \ 0, \ 0, \ 0, \ -0.2037, \ -1.0963.$$

Therefore, an internally stable inverted interactorizing system is not obtained.

On the other hand, define the generalized inverse K^- of K by

$$K^- = K^\#(KK^\#)^{-1}$$

where

$$K^\# = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \alpha & \beta \end{bmatrix}, \quad \det KK^\# = \alpha - 2\beta - 1 \neq 0$$

and the constant α and β are to be determined so that the zeros of $\tilde{N}(z)K^\#$ lie inside the unit circle. Since

$$\begin{aligned}\tilde{N}(z)K^\# = {}& 0.09\{(1-\alpha+2\beta)z^2 + (2.9 - 2.5\alpha + 5.4\beta)z \\ & +(2.08 - 1.54\alpha + 3.6\beta)\},\end{aligned}$$

and $\alpha - 2\beta - 1 \neq 0$, set the 0-th and first order coefficients of the above equation to be zero. Then,

$$\alpha = -1.2321, \quad \beta = -1.0982.$$

Although the feedback gain $\bar{F}$ is same as in the pseudoinverse case, all eigenvalues of $A - BK^-\bar{F}$ are zeros, and an internally stable inverted interactorizing is achieved.

From the above Example, a necessary condition for a design of K^- is obtained.

Theorem 3 Assume that any m columns in $G(z)$ are linearly independent normally. Then, there exists K^- such that the zeros of det$G(z)K^-$ can be assigned arbitrarily if the following inequality holds

$$m(p-m) \geq \dim A - \det L(z). \tag{20}$$

However, it is hard to find a sufficient condition. Since $L(z)$ is stably invertible, the problem is reduced to find K^- such that $L(z)G(z)K^-$. Let $(A,\ B,\ \tilde{C},\ K)$ denote a realization of $L(z)G(z)$. Then, A-matrix of the inverse system $\{L(z)G(z)K^-\}^{-1}$ is $A - BK^-C$. This means that the problem is same as the stabilizability problem by static output feedback. Even though some important facts are reported [11], [12], the problem is still open.

IV. Model Matching for Fat Plants

Consider a right coprime factorization of $G(z)$ as follows:

$$\begin{aligned} & G(z) = N(z)D^{-1}(z), \\ & D(z) \in \mathbf{R}^{p\times p}[z], \quad N(z) \in \mathbf{R}^{m\times p}[z]. \end{aligned} \tag{21}$$

Let $L(z)$ and K denote an interactor and corresponding highest frequency gain satisfying eqn.(1), respectively. Define the control law by

$$u(t) = K^-H^{-1}(z)\{X(z)u(t)+Y(z)y(t)\}+K^-L(z)y_m(t) \tag{22}$$

where $y_m(t) \in \mathbf{R}^m$ is the reference output, $H(z) \in \mathbf{R}^{m\times m}[z]$ is a Hurwitz polynomial matrix, and $X(z) \in \mathbf{R}^{m\times p}[z]$ and $Y(z) \in \mathbf{R}^{m\times m}[z]$ are polynomial matrices which satisfy the following Diophantine equation

$$X(z)D(z) + Y(z)N(z) = H(z)\{KD(z) - L(z)N(z)\}. \tag{23}$$

Theorem 4 Let ν denote the maximum value of observability indices. For an integer $d \geq \nu$, if $H(z)$ is a Hurwitz polynomial matrix with row degree $d-1$ and $G(z)K^-$ is stably invertible, then by the control law (22), $y(t) \to y_m(t)$.

(Proof). Substituting eqn.(23) to eqn.(21) yields

$$\begin{aligned} y(t) = {}& N(z)D^{-1}(z)[I_p - K^-\{K - L(z)N(z)D^{-1}(z)\}]^{-1} \\ & \cdot K^-L(z)y_m(t). \end{aligned} \tag{24}$$

Since $KK^- = I_m$ in this case, the above equation yields

$$\begin{aligned} y(t) &= N(z)D^{-1}(z)K^- \\ &\quad \cdot[I_m - \{K - L(z)N(z)D^{-1}(z)\}K^-]^{-1}L(z)y_m(t) \\ &= N(z)D^{-1}(z)K^-\{L(z)N(z)D^{-1}(z)K^-\}^{-1} \\ &\quad \cdot L(z)y_m(t) \\ &= y_m(t) \end{aligned}$$

and the Theorem is proved.

V. Numerical Example

The proposed method is easy to apply for a plant with measurement noise [13]. Consider the plant in Examples 1 and 2. For the plant, it is supposed that the measured output signal $\bar{y}(t)$ is the sum of the real output signal $y(t)$ and the measurement noise,

$$\bar{y}(t) = y(t) + w(t). \tag{25}$$

The purpose of the design is to construct a model matching system which also minimizes the effect by noise on $y(t)$.

The measurement noise is assumed to be the white noise with covariance $\begin{bmatrix} 0.010 & 0.0083 \\ 0.0083 & 0.0070 \end{bmatrix}$. A pulse train function is used as a reference input. Fig.2 shows the plant output responses when $d = 3$ and the measurement noises are added. Fig.3 shows the output responses when $d = 7$. As is expected [13], the performance is better when increasing the value of d.

VI. Conclusions

In this paper, it was discussed the model matching problem for fat transfer function matrices. In the development, the structure of the inverted interactorizing system, which gives the basic structure of model matching systems, made clear. A necessary and sufficient condition for an internally stable inverted interactorizing systems was given. If the problem of selecting the inputs is solved completely, a procedure is available which completely avoids the introduction of unstable zeros not originally present in the plant.

The effectiveness of the proposed design was confirmed through the numerical example with measurement noise. The extension to the adaptive control is under studying.

REFERENCES

[1] L. M. Silverman, Inversion of multivariable linear systems, *IEEE Trans.*, vol.**AC-14**, no.3, pp.270-276, 1969.

[2] W. A. Wolovich, *Linear multivariable systems*, Springer-Verlag, 1974.

[3] H. Elliott and W. A. Wolovich, Parameterization issues in multivariable adaptive control, *Automatica*, vol.**20**, no.5, pp.533-545, 1984.

[4] W. A. Wolovich and P. L. Falb, Invariants and canonical forms under dynamic compensation, *SIAM J. Contr., and Optmiz.*, **14**-6, pp.996-1008, 1976.

[5] Y. Mutoh and P.N. Nikiforuk, Inversed interactorizing and triangularizing with an arbitrary pole assignment using the state feedback, *IEEE Trans. Automat. Contr.*, vol.**37**, no.5, pp.630-633, 1992.

[6] Y. Mutoh and R. Ortega, Interactor structure estimation for adaptive control of discrete-time multivariable non-decouplable systems, *Automatica, 29*, pp.635-647, 1993.

[7] W. Kase, Y. Mutoh and M. Teranishi, A simple derivation of interactor matrix and its applications, *Proc. of the 38th IEEE Conf. on Dicision and Contr.,* Phoenix, AZ, pp.493-498, 1999.

[8] Y. Peng and M. Kinnaert, Explicit solution to the singular LQ regulation problem, *IEEE Trans.*, vol.**AC-37**, no.5, pp.633-636, 1992.

[9] S. Bittanti, P. Colaneri and M. F. Mongiovi, Singular filtering via spectral interactor matrix, *IEEE Trans.*, vol.**AC-40**, no.8, pp.1492-1497, 1995.

[10] W. Kase, A solution of polynomial matrix equations using extended division algorithm and another description of all-stabilizing controllers, *Int. J. Systems Science, 30*, pp.95-104, 1999.

[11] K. H. Kiritsis, A Necessary condition for pole assignment by constant output feedback", *Systems and Control Letters*, **45**, pp.317-320, 2002.

[12] L. Carotenuto and G. Franze, A general formula for eigenvalue assignment by static output feedback with application to robust design", *Systems and Control Letters*, **49**, pp.175-190, 2003.

[13] W. Kase and Y. Mutoh, Suboptimal exact model matching for multivariable systems with measurement noise, *IEEE Trans. Automat. Contr., 45*, pp.1170-1175 , 2000.

[14] G. Marro and E. Zattoni, A novel geometric insight into the model matching problem with stability, *Proc. of 41st IEEE Conf. on Dicision and Contr.,* Las Vegas, NV, pp.2137-2142, 2002.

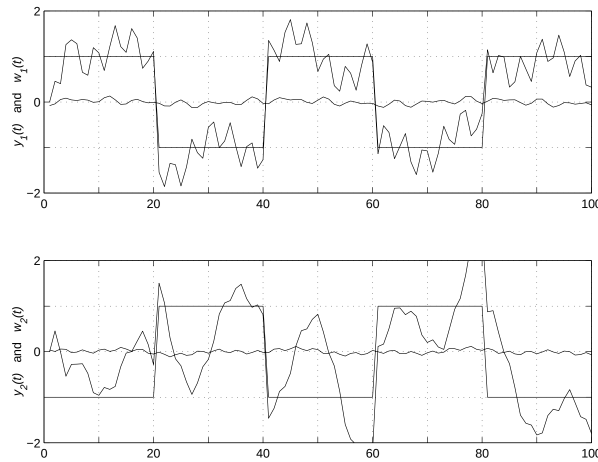

Fig. 2. Simulation result of proposed method ($d = 3$).

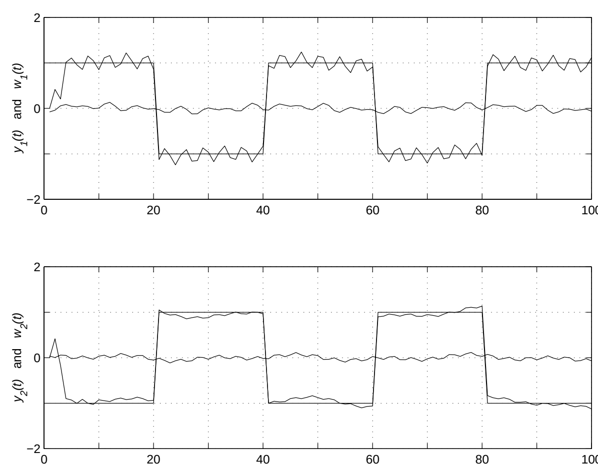

Fig. 3. Simulation result of proposed method ($d = 7$).