Design of Digital Pre-Compensation Filters by H_∞ Optimization

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Abstract—**Recently, several methods have been proposed to digitally compensate for the shortcomings of the analog reconstruction filters in IQ modulators using CPFSK input signals. While these methods have shown to be effective, they result in filters with long coefficients that are computationally demanding to implement on the DSP. In this paper, we present two new techniques for designing the digital compensation filters by means of H_∞ optimization, for which there is ready-made functions in MATLAB. The simulation results show that these techniques are effective and lead to substantial improvement of the output envelope ripples.**

I. Introduction

Consider the DSP (Digital Signal Processor) based *Inphase/Quadrature* (IQ) modulator in Fig. 1. Here, the I and Q channel base-band signals are generated digitally using a DSP and converted into analog signals using digital-to-analog (D/A) converters and analog reconstruction filters before modulation and transmission. However, the performance of such configuration can be limited by the two analog reconstruction filters that are necessary to attenuate digital image components in the base-band signal spectrum before transmission. The transfer characteristics of practical reconstruction filters and errors in their implementation result in the pass-band characteristics departing from constant magnitude and linear phase. Furthermore, implementation errors also result in a mismatch between the I and Q channel reconstruction filter frequency responses.

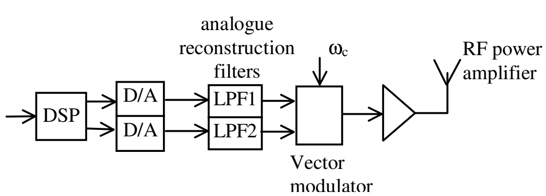

Fig. 1. Radio transmitter architecture incorporating digital IQ modulation.

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In the case of CPFSK (Continuous-Phase-Frequency Shift Keying) signals, these shortcomings cause distortion of the I and Q channel signals resulting in the loss of the constant envelope property of the output CPFSK signal. Ripples in the envelope function cause undesirable spreading of the signal spectrum into adjacent channels when the signal passes through a nonlinear radio-frequency (RF) power amplifier (PA) in the transmitter [1],[2].

Digital compensation for the shortcomings in the analog subsystems of quadrature modulators has received considerable attention in the literature [3]-[7] lately. In [4], a method was proposed to remove the unwanted ripples at the vector modulator's output signal envelope using digital signal pre-shaping filters in the I and Q channels. This is to pre-compensate for both imbalances in the analogue reconstruction filters' frequency responses as well as departures from constant magnitude, linear phase in the pass-band of each reconstruction filter. This method employs a *least-squares* optimization approach where the pre-compensation Finite Impulse Response (FIR) filters are computed by minimizing the H_2 norm of the error transfer function. An alternative solution was given in [5] using state-space approach. However, these methods result in FIR filters that have a large number of coefficients and are computationally demanding to implement on the DSP. Furthermore, the method in [4] requires special attention to numerical issues in order to achieve good results in practical application. Specifically, the solution matrix to a least squares optimization problem must first be regularized by discarding eigenvalues that are smaller than some threshold value before the solution vector is computed. In [6] a technique is presented to reduce the computational load by increasing tap spacing of the FIR filters and some encouraging results were obtained.

Recently [7] proposed a digital compensation scheme using Infinite Impulse Response (IIR) filters since IIR filters are known to be able to produce long impulse responses using only a small number of filter coefficients and are thus will be useful in such application. These IIR filters are designed using an indirect method, where the filters are obtained from FIR filters using model reduction technique. The method involves two steps: First a FIR filter is designed using the optimization technique proposed in

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[4],[5]; next a low-order IIR filter is obtained using model reduction technique of [8]. This approach again requires the special attention to numerical issues. Otherwise, conversion from a FIR to an IIR filter will produce inconsistent results.

In this paper, we present two state-space approaches of obtaining the digital compensation filters; one results in IIR filters while the other FIR filters. Both approaches minimize the H_∞ norm instead of the H_2 of the error transfer function. Design of control system by H_∞ minimization is now a standard technique [12]. The IIR filters are obtained directly from the solution rather than an intermediate FIR solution using a ready-made function (*hinfsys*) in MATLAB. The FIR filters, however, are obtained using the LMI Toolbox in MATLAB.

II. Digital Precompensation

Fig. 2 shows a typical digital pre-compensation structure [4],[5]. The additional components of the digital compensation structure are the two digital filters, i.e. F1 and F2. These filters are designed to pre-compensate for departures from a constant magnitude and phase response (in the pass-bands) of each of the signal reconstruction filters, LPF1 and LPF2, and to achieve gain and phase balance between these two filters. The A/D converters are used to digitize the output signal from the reconstruction filters, so that measurements can be made on the DSP system.

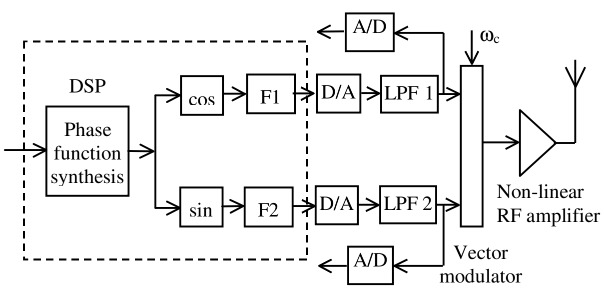

Fig. 2. IQ modulator and digital pre-compensation structure.

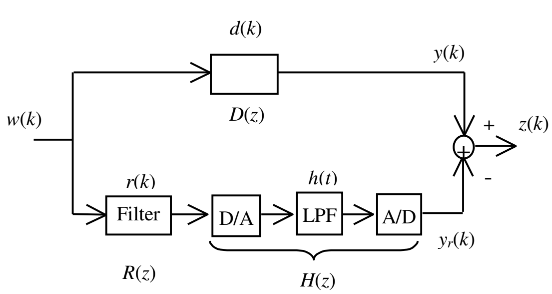

Fig. 3. I and Q channel optimization structure.

The aim is to find F1 and F2 such that for each of the I and Q channel, the overall discrete-time channel (from the input of the digital filter to the output of the A/D converter) has transfer function that closely approximates some desired function [4],[5].This optimization structure is shown in Fig. 3.

In this discrete system, $D(z)$ is the nominal desired response, $H(z)$ is the discrete-time equivalent transfer function of the D/A, analog reconstruction filter and A/D converter while $R(z)$ is the transfer function of the compensation filter. In this paper, the parameters of the analog reconstruction filter are assumed to be known *a priori*. Our objective is to find a stable transfer function $R(z)$ such that the cascaded system of $R(z)H(z)$ closely approximates the desired response, $D(z)$. This is equivalent to minimizing $\|D(z)-R(z)H(z)\|_\infty$, the H_∞ norm of the transfer matrix $D(z)$-$R(z)H(z)$, or in some parts of this paper, we have referred to this as the error transfer function. Thus, we define the cost function as

$$J := \inf_{R(z)} \|D(z)-R(z)H(z)\|_\infty \qquad (1)$$

Our precise *design problem statement* is as follows:

Given stable FIR filters $D(z)$ and $H(z)$, find causal stable digital filter, IIR or FIR, $R(z)$ to minimize J.

This quantity, J is taken to be the performance measure of the digital compensation. A small value of J means that the error $z(k)$ is small uniformly over all inputs $w(k)$. Ideally, we require $J = 0$, so that the I and Q channels are perfectly matched. This optimization is over all matrices $R(z)$ that are analytic and bounded outside the unit disc.

III. IIR Formulation

The MATLAB programs for H_∞ optimization use state-space representations of transfer functions. In this section, we present the state-space formulas relevant for the design program. Let $D(z)$ and $H(z)$ be stable systems with the following minimal realizations:

$$D(z)=\left[\begin{array}{c|c} A_D & B_D \\ \hline C_D & D_D \end{array}\right] \quad \text{and} \quad H(z)=\left[\begin{array}{c|c} A_H & B_H \\ \hline C_H & D_H \end{array}\right]$$

The *model-matching* problem in Fig. 3 can be recast as a standard H_∞ control problem [9] in Fig. 4 by defining (see Appendix for details of derivation)

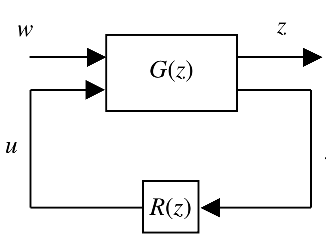

Fig. 4. The standard block diagram.

$$G(z) = \begin{bmatrix} D(z) & -I \\ H(z) & 0 \end{bmatrix} \qquad (2)$$

Then Fig. 3 and Fig. 4 are equivalent.

Lemma 1: *The transfer matrix, G(z) has the realization,*

$$G(z) = \left[\begin{array}{c|c} A_G & B_G \\ \hline C_G & D_G \end{array}\right] = \left[\begin{array}{cc|cc} A_D & 0 & B_D & 0 \\ 0 & A_H & B_H & 0 \\ \hline C_D & 0 & D_D & -I \\ 0 & C_H & D_H & 0 \end{array}\right] \qquad (3)$$

Proof: *Expanding (2) gives*

$$G(z) = \begin{bmatrix} D(z) & -I \\ H(z) & 0 \end{bmatrix} = \begin{bmatrix} C_D(zI - A_D)^{-1}B_D + D_D & -I \\ C_H(zI - A_H)^{-1}B_H + D_H & 0 \end{bmatrix}$$

$$= \begin{bmatrix} C_D(zI - A_D)^{-1}B_D & 0 \\ C_H(zI - A_H)^{-1}B_H & 0 \end{bmatrix} + \begin{bmatrix} D_D & -I \\ D_H & 0 \end{bmatrix}$$

$$= \begin{bmatrix} C_D & 0 \\ 0 & C_H \end{bmatrix} \begin{bmatrix} zI - A_D & 0 \\ 0 & zI - A_H \end{bmatrix}^{-1} \begin{bmatrix} B_D & 0 \\ B_H & 0 \end{bmatrix} + \begin{bmatrix} D_D & -I \\ D_H & 0 \end{bmatrix}$$

$$= C_G\left(zI - A_G\right)^{-1}B_G + D_G$$

where

$$A_G = \begin{bmatrix} A_D & 0 \\ 0 & A_H \end{bmatrix}, \quad B_G = \begin{bmatrix} B_D & 0 \\ B_H & 0 \end{bmatrix}$$

$$C_G = \begin{bmatrix} C_D & 0 \\ 0 & C_H \end{bmatrix}, \quad D_G = \begin{bmatrix} D_D & -I \\ D_H & 0 \end{bmatrix}$$

Therefore, we have

$$G(z) = \left[\begin{array}{c|c} A_G & B_G \\ \hline C_G & D_G \end{array}\right] = \left[\begin{array}{cc|cc} A_D & 0 & B_D & 0 \\ 0 & A_H & B_H & 0 \\ \hline C_D & 0 & D_D & -I \\ 0 & C_H & D_H & 0 \end{array}\right]$$

□

The MATLAB function *hinfsyn* takes in a realization for $G(z)$ as input and outputs a realization of $R(z)$. The resulting filter $R(z)$ is an IIR filter with the same order as $G(z)$, that minimizes the H_∞ norm from w to z in Fig. 4. Since *hinfsyn* works in continuous-time domain, one must perform a bilinear transformation of $G(z)$ to $G_c(s)$, then run *hinfsyn* to obtain $R_c(s)$, and then perform a bilinear transformation back to the discrete-time domain to get $R(z)$.

However, the function *hinfsyn* results in IIR filters of quite high order and are not desirable for practical implementation. It may be possible to get lower order compensation filters if one reduces to minimal realizations at each appropriate stage, that is, if one discards uncontrollable and unobservable states. Another alternative to reduce the order of the resulting IIR filter is through model reduction techniques. There are a number of techniques available for model reduction. Some of the well-known techniques are balanced truncation [8] and optimal Hankel norm approximation [10]. In this paper, we consider only the technique of balanced truncation to obtain a low order IIR filter. The preceding results can now be summarized in the following algorithm.

Algorithm 1: Summary of IIR Algorithm

1. Compute a state-space realization $\{A_G, B_G, C_G, D_G\}$ of the plant $G(z)$ using (3).
2. Use bilinear transformation to transform $G(z)$ into continuous time domain to obtain $G_c(s)$.
3. Compute $R_c(s)$ using *hinfsyn* function in MATLAB.
4. Transform $R_c(s)$ back to discrete time domain using bilinear transformation to get $R(z)$.
5. Use model reduction [8] to eliminate nearly uncontrollable or unobservable states.

IV. FIR FORMULATION

In this section, we derive the necessary formulas for solving the design problem in LMI (Linear Matrix Inequalities) Toolbox in MATLAB. We first introduce the following lemma.

Lemma 2 [11]: *Let*

$$M(z) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$$

be stable, then $\|M(z)\|_\infty < 1$ *if and only if there is a X > 0 such that*

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^T \begin{bmatrix} X & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} - \begin{bmatrix} X & 0 \\ 0 & I \end{bmatrix} < 0$$

Theorem 1: *Let*

$$M(z) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$$

be stable, then $\|M(z)\|_\infty < \mu$ *if and only if there is a X > 0 such that*

$$\begin{bmatrix} A^TXA - X & A^TXB & C^T \\ B^TXA & B^TXB - I & D^T \\ C & D & -\mu^2 I \end{bmatrix} < 0 \qquad (4)$$

The proof of this theorem requires the following lemma.

Lemma 3: *Suppose a Hermitian matrix is partitioned as*

$\begin{bmatrix} A & B \\ B^T & C \end{bmatrix}$, *where* $A = A^T$ *and* $C = C^T < 0$. $\begin{bmatrix} A & B \\ B^T & C \end{bmatrix} < 0$ *if and only if* $A - BC^{-1}B^T < 0$.

Proof of Theorem 1: *Let*

$$\|M(z)\|_\infty < \mu$$

Then

$$\|\mu^{-1}M(z)\|_\infty < 1$$

$$\Leftrightarrow \begin{bmatrix} A & \mu^{-1}B \\ C & \mu^{-1}D \end{bmatrix}^T \begin{bmatrix} X & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A & \mu^{-1}B \\ C & \mu^{-1}D \end{bmatrix} - \begin{bmatrix} X & 0 \\ 0 & I \end{bmatrix} < 0$$

$$\Leftrightarrow \begin{bmatrix} A^T XA - X + \mu^{-2}C^T C & A^T XB + \mu^{-2}C^T D \\ B^T XA + \mu^{-2}D^T C & B^T XB + \mu^{-2}D^T D - I \end{bmatrix} < 0$$

$$\Leftrightarrow \begin{bmatrix} A^T XA - X & A^T XB \\ B^T XA & B^T XB - I \end{bmatrix} - \begin{bmatrix} C^T \\ D^T \end{bmatrix} \left[-\mu^2 I\right]^{-1} \begin{bmatrix} C & D \end{bmatrix} < 0$$

$$\Leftrightarrow \begin{bmatrix} A^T XA - X & A^T XB & C^T \\ B^T XA & B^T XB - I & D^T \\ C & D & -\mu^2 I \end{bmatrix} < 0$$

□

Suppose

$$D(z) = \left[\begin{array}{c|c} A_D & B_D \\ \hline C_D & D_D \end{array}\right], H(z) = \left[\begin{array}{c|c} A_H & B_H \\ \hline C_H & D_H \end{array}\right] \text{and } R(z) = \left[\begin{array}{c|c} A_R & B_R \\ \hline C_R & D_R \end{array}\right]$$

Then the overall transfer function

$$M(z) = D(z) + H(z)R(z)$$

has minimal realization given by

$$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = \left[\begin{array}{ccc|c} A_H & 0 & 0 & B_H \\ B_R C_H & A_R & 0 & B_R D_H \\ 0 & 0 & A_D & B_D \\ \hline D_R C_H & C_R & C_D & D_R D_H + D_D \end{array}\right] \quad (5)$$

Then $R(z)$ has the following realization when expressed in controllability canonical form

$$A_R = \begin{bmatrix} 0 & 0 & \ldots & 0 \\ 1 & 0 & \ldots & 0 \\ \vdots & & & \vdots \\ 0 & & 1 & 0 \end{bmatrix}, \; B_R = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$C_R = \begin{bmatrix} r_2 & r_3 & \ldots & r_N \end{bmatrix}$$

$$D_R = r_1$$

where $r_1, r_2, \ldots, r_N$ are the impulse response of $R(z)$.

It is clear that in transfer function $M(z)=\{A, B, C, D\}$, only matrices C and D are unknown. Therefore, we can use LMIs to minimize $\|D(z)+H(z)R(z)\|_\infty$.

Note that we minimize $\|D(z)+H(z)R(z)\|_\infty$ instead of $\|D(z)-H(z)R(z)\|_\infty$ in order to simplify expression. Once we obtain C_R and D_R that minimize $\|D(z)+H(z)R(z)\|_\infty$, then $-C_R$ and $-D_R$ minimize $\|D(z)-H(z)R(z)\|_\infty$.

Algorithm 2: Summary of FIR Algorithm

1. Compute state-space matrices A and B using (5).
2. Construct the LMIs using (4) and solve using LMI Toolbox in MATLAB.
3. Negate matrices C_R and D_R, i.e. $C_{Rnew} = -C_R$, $D_{Rnew} = -D_R$.
4. Compute $R(z) = C_{Rnew}\left(zI - A_R\right)^{-1} B_R + D_{Rnew}$

V. Simulation Results

In this section, we present some simulation results to show the effectiveness of the proposed methods. The simulation studies are centered on a single channel ERMES (European Radio Message System) modulation format transmitter [4],[5]. For the results presented in Section A, the two low-pass filters, LPF 1 and LPF 2 have a nominal 6th order low-pass characteristic while for the results presented in Section B the analog filters have nominal 4th order low-pass characteristic. In both sections, the analog filters have cut-off frequency of 20kHz, but each response corresponds to particular realization of the filter circuit resulting from the perturbations of the component values about its nominal values. These analog filters are implemented using cascaded *Sallen & Key* 2nd order sections where the circuit component tolerance are assumed to be 5% for resistors and 10% for capacitors. The desired response $D(z)$ is chosen to have the same magnitude characteristics as the nominal response of the reconstruction filters but constrained to have linear phase. The digital system sampling frequency is 200kHz. In the following two sections, we present and compare some results on RMS envelope ripples as a measure of the modulator's performance.

A. IIR Pre-Compensated System

The original IIR filter obtained is 100th order, while the 57th order IIR filter is obtained by discarding the uncontrollable and unobservable states. Further reduction was carried out using model reduction technique of [8] to obtain lower order filters. Table I summarizes the RMS envelope ripples obtained using different filter order. It can be seen from Table I that low-order IIR filters are still able to provide substantial reduction without significantly degrading the performance of the modulator.

Using the same simulation setup, a set of RMS values is recorded in Table II using the IIR pre-compensation filters proposed by [7]. From Table I and II, it can be seen that the new method outlined in Section III gives better results in terms of reducing the output envelope ripples. In addition, the technique in [7] requires special attention to numerical issues in order to get a good approximation of the original FIR filter. Therefore, it is difficult to achieve significant order reduction from FIR to IIR. Fig. 5 shows the output envelope signals for the uncompensated and pre-compensated cases and it is evident that the presence of the 15th order IIR pre-compensation filters greatly reduces the output envelope ripples.

TABLE I
RMS ENVELOPE RIPPLE FOR DIFFERENT
IIR FILTER LENGTHS

IIR filter length	RMS envelope ripple (mV)	Ripple reduction factor
Uncompensated	3.158	-
100th order IIR	0.057	54
57th order IIR	0.057	54
20th order IIR	0.132	24
15th order IIR	0.137	23
11th order IIR	0.149	21
10th order IIR	0.251	13

TABLE II
RMS ENVELOPE RIPPLE FOR DIFFERENT IIR FILTER LENGTHS
USING METHOD OF [6]

IIR filter length	RMS envelope ripple (mV)	Ripple reduction factor
Uncompensated	3.158	-
20th order IIR	0.146	22
18th order IIR	0.242	13
15th order IIR	0.246	12

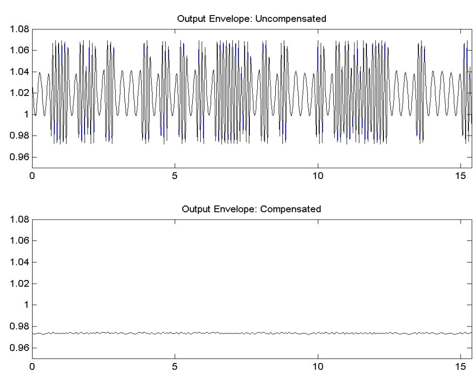

Fig. 5. Vector modulator output envelope for the uncompensated and precompensated systems.
upper plot: Uncompensated system
lower plot: 15th order IIR compensated system

B. FIR Pre-Compensated System

The proposed method yields 13th order FIR pre-compensation filters. Fig. 6 shows the output envelope of the vector modulator with and without compensation. It is clear from the figure that the 13th order FIR filters are still able to provide substantial reduction on the magnitude of the envelope ripples. The reduction factor using such FIR filters is not as good as those in the IIR case. This can be attributed to the limitations of system modeling in LMI optimization. LMIs are computationally demanding to simulate in MATLAB and therefore the impulse responses of the desired and analog filters' responses were chosen as short as possible. In this simulation example, the impulse responses of $D(z)$ and $H(z)$ were truncated to 30 taps.

TABLE III
RMS ENVELOPE RIPPLE FOR DIFFERENT
FIR FILTER LENGTHS

FIR filter length	RMS envelope ripple (mV)	Ripple reduction factor
Uncompensated	1.77	-
13th order FIR	0.28	6.4
10th order FIR	0.45	3.9

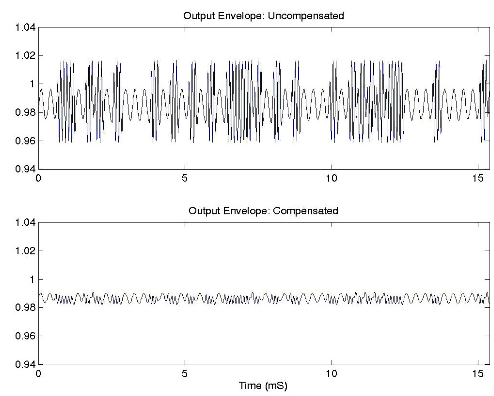

Fig. 6: Vector modulator output envelope functions of uncompensated system and 13th order FIR compensated system.

VI. CONCLUSIONS

In Section III and IV, we presented two state-space solutions for solving the H_∞ optimization problem. The simulation results in Section V shows substantial improvements in RMS envelope ripples can be achieved using the algorithms outlined in Section III and IV. These methods are simple and easy to implement using the readily available functions in MATLAB and furthermore, the stability of the models is guaranteed.

However, there is a disadvantage with using LMI optimization due to the increasing of LMI computational load with increasing filter order and thus, higher order FIR filters could not be tested. Nevertheless, research on LMI optimization is still very active and substantial speed-ups can be expected in the future.

APPENDIX

Conversion from a three-block to one-block standard problem

Consider the standard block diagram shown in Fig. 4. In this figure *w*, *u*, *z*, and *y* are vector-valued signals: *w* is the exogenous input; *u* is the control signal; *z* is the output to be controlled; and *y* is the measured output. The transfer matrices *G* and *R* are, by assumption, real-rational and proper: *G* represents a generalized plant, and R represents a controller.

In the analysis presented in Section III and IV, $D(z)$, $R(z)$ and $H(z)$ are rational functions of *z* (rather than matrices of rational functions of *z*) since $w(k)$ and $z(k)$ in Fig. 3 are one-dimensional signals. Therefore, the order of the cascaded system can be interchanged without affecting the overall output of the system, i.e. $R(z)H(z) = H(z)R(z)$.

We then partition *G* as

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}$$

Then Fig. 4 stands for the algebraic equations

$$z = G_{11}w + G_{12}u \qquad \text{(A1)}$$

$$y = G_{21}w + G_{22}u \qquad \text{(A2)}$$

$$u = Ry \qquad \text{(A3)}$$

Substituting (A2) into (A3), we have

$$u = R\left(G_{21}w + G_{22}u\right) \qquad \text{(A4)}$$

$$\Rightarrow \left(I - RG_{22}\right)u = RG_{21}w$$

$$u = \left(I - RG_{22}\right)^{-1} RG_{21}w \qquad \text{(A5)}$$

Then the transfer matrix from *w* to *z* is a linear-fractional transformation of *R*:

$$z = \left[G_{11} + G_{12}R\left(I - G_{22}R\right)^{-1} G_{21}\right]w \qquad \text{(A6)}$$

By comparing the closed-loop transfer matrix of the optimization problem presented in Section II, $D(z) - R(z)H(z)$ with (A6), it is easy to see that

$$G_{11} = D(z) \ , \ G_{12} = -I$$
$$G_{21} = H(z) \ , \ G_{22} = 0$$

Thus,

$$G(z) = \begin{bmatrix} D(z) & -I \\ H(z) & 0 \end{bmatrix} \qquad \text{(A7)}$$

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