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Optimal impulse response tracking and disturbance rejecting controllers

Anna Soffía Hauksdóttir

***Abstract*— The tuning of PID controllers can essentially be posed as the problem of selecting open-loop zeros such as to obtain a desired system response. In this paper, the general case wherein stable open-loop system zeros can be cancelled is considered, allowing more freedom in placing open-loop zeros, as opposed to just two zeros in the case of a PID controller. Subsequently, optimal open-loop zeros are computed such as to minimize the deviation from a desired reference impulse response, while maintaining the relative degree and the type of the reference system, thus giving the controlled system desired input tracking and disturbance rejection properties. Further, due to the inverse compensation of the plant zeros, the controller is in general causal when the relative degree of the plant and the reference system are similar.**

I. INTRODUCTION

It is well known, that continuous–time as well as discrete–time transfer function responses are strongly affected, not only by the eigenvalues or poles, but the numerator coefficients, or equivalently, the system's zeros, as well. In general, the zeros of a continuous–time system are determined by properties of the plant as well as the location of sensors and actuators. The zeros of discrete–time systems naturally arise as determined by system identification procedures, see, e.g., [1],[2] or as a result of transforming a continuous–time transfer function to a discrete–time one, by different transformations, see, e.g., [3]-[9]. Thus, to some extent, discrete–time zeros and continuous–time zeros have different origins, but strongly affect the systems response, in both cases.

Pole placement has been much discussed in the literature and methods for optimal pole placement using standard state feedback, e.g., the linear quadratic regulator, are well known. Zero placement is also a very relevant design issue, as evident for example in publications on zero placement of linear multivariable systems [10]–[13]. Controllers that affect zeros can be designed, although zeros are not affected by state feedback in SISO (Single Input Single Output) systems. One example of such a controller is the well known PID controller. In a similar manner, stable zeros can be affected by simple inverse compensation. Further, such a controller using dynamic output feedback and dynamic feedforward, can be designed to place the poles as well as to move the (stable) zeros of a system effectively by cancellation, see, e.g., [14].

Much interest has been shown in the general shaping of system responses, as evident, e.g., in the extrema–free problem and closely related problems, such as the non–overshooting problem. In fact, the extrema–free and related problems are extremely important, and there are many practical applications where this is the relevant design criteria, e.g., some chemical processes, machine tool axis control and trajectory–following in robotics[15]. The undershoot problem was studied for scalar systems in [16]. The lack of closed–form expressions of discrete–time transfer function responses led to linear programming approaches in the past, in the solution of overshoot-related design criteria. The problem of designing non-overshooting feedback control systems for step inputs, has been discussed in [17]. There, controllers are designed to track a step optimally, with some predetermined amount of allowable overshoot, leading to an infinite linear programming problem. A technique for choosing zero locations for minimal overshoot is discussed in [18], where an approach to the specification of optimal overshoot controllers for a fixed controller order, is presented for given closed-loop system poles. The solution is obtained by solving an affine minimax optimization problem. Finally, in [19], the problem of minimizing the amplitude of a regulated output due to a specific bounded input, is solved via linear programming.

Transfer function responses for continuous-time as well as discrete–time systems are of considerable interest in the area of control systems and in filter design. Closed–form continuous–time transfer function responses were derived in [20] and extended to the case of complex eigenvalues in [21]. Naturally, the closed form lends itself well to analysis as in [22] and opens up many new interesting applications, e.g., solving for optimal zero locations by minimizing transient responses[20]; tracking a given reference step response in [23]-[26]; and solving the model reduction problem in [27]. The closed–form expressions are further used in the direct computation of coefficients for PID controllers in [28].

Similar to the continuous–time case, closed–form discrete–time transfer function responses derived in [29], and extended to the case of complex eigenvalues in [21], were used to solve for optimal zero locations by minimizing transient responses in [29] and applied to the discrete–time model reduction problem in [30]. The problem of optimal zero locations of discrete–time systems with distinct poles tracking reference step responses is considered in [31]. The discrete–time closed–form expressions are used in the direct computation of coefficients for PID controllers in [32], wherein a hardware-in-the-loop application is also reported.

The direct computation of coefficients for PID controllers

A.S. Hauksdóttir is with the Department of Electrical and Computer Engineering, University of Iceland, Hjardarhaga 2-6, IS-107 Reykjavík, Iceland. ash@hi.is

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in [28], essentially involves computation of optimal open-loop zeros tracking a desired reference impulse response. In this paper, the approach is extended to a more general case wherein stable open-loop zeros are cancelled, allowing more freedom in placing open-loop zeros, as opposed to just two zeros in the case of a PID controller. Subsequently, optimal open-loop zeros are computed such that a desired reference impulse response is tracked, while maintaining the relative degree of the reference system and the type, thus giving the controlled system desired input tracking and disturbance rejection properties. The problem is formulated in Section II, including input tracking and disturbance rejection properties as well as specification of the reference system. The optimal open-loop zeros are computed in Section III, by minimizing the impulse response deviation between the reference and the actual system, including an example. Conclusions and future work are discussed in Section IV.

II. PROBLEM FORMULATION

Consider the closed-loop control system setup shown in Fig. 1. The plant zeros are assumed stable and are given by

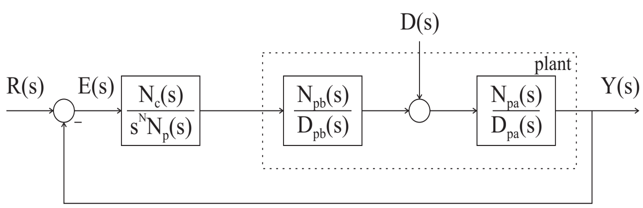

Fig. 1. Closed-loop control system setup.

$$N_p = N_{pb} N_{pa}, \tag{1}$$

where N_p is type zero and the Laplace variable s has been dropped to simplify notation. All N polynomials are of the generic form

$$N = b_0 s^m + b_1 s^{m-1} + ... + b_m. \tag{2}$$

Likewise, the plant poles are assumed stable[1] and are given by

$$D_p = D_{pb} D_{pa}. \tag{3}$$

D_p is also type zero, i.e., the term $\frac{1}{s^N}$ grouped by the controller includes all open–loop pure integrators. All D polynomials are of the generic form

$$D = s^n + a_1 s^{n-1} + ... + a_n = (s+\lambda_1)(s+\lambda_2)\cdots(s+\lambda_n) \tag{4}$$

The plant is affected by the disturbance input D and its output is Y. The closed-loop control system is driven by the input R and the controller, driven by the error E, is of the form

$$\frac{N_c}{s^N N_p} \tag{5}$$

[1]For the case of unstable plant poles, an inner–loop state–feedback type controller can be designed, stabilizing the plant.

The controller cancels the plant zeros by inverse compensation[2], the term $\frac{1}{s^N}$ includes all controller and plant integrators. The controller zeros will be selected such as to optimally track a reference impulse response, maintaining the relative degree and the type of the reference system.

A. Input Tracking

The input tracking for a setup such as in Fig. 1 is easily obtained in a standard manner. The transfer function from the input R to the error E is given by

$$\frac{E}{R} = \frac{1}{1 + \frac{N_c}{s^N N_p}\frac{N_p}{D_p}} = \frac{s^N D_p}{s^N D_p + N_c}. \tag{6}$$

For a unit step input R when $N \geq 1$, the steady state error is given by

$$e_{ss} = \lim_{s\to 0} s \frac{s^N D_p}{s^N D_p + N_c}\frac{1}{s} = 0. \tag{7}$$

Similarly, the steady state error for a unit ramp input R when $N \geq 2$ is given by

$$e_{ss} = \lim_{s\to 0} s \frac{s^N D_p}{s^N D_p + N_c}\frac{1}{s^2} = \lim_{s\to 0} \frac{s^{N-1} D_p}{s^N D_p + N_c} = 0. \tag{8}$$

B. Disturbance Rejection

Likewise, the disturbance rejection for the closed loop is easily obtained in a standard manner. The transfer function from the disturbance input D to the error E is given by

$$\frac{E}{D} = \frac{-\frac{N_{pa}}{D_{pa}}}{1 + \frac{N_c}{s^N N_p}\frac{N_p}{D_p}} = \frac{-s^N N_{pa} D_{pb}}{s^N D_p + N_c}. \tag{9}$$

Then, the steady state error for a unit step disturbance input D when $N \geq 1$ is given by

$$e_{ss} = \lim_{s\to 0} s \frac{-s^N N_{pa} D_{pb}}{s^N D_p + N_c}\frac{1}{s} = 0. \tag{10}$$

The steady state error for a unit ramp disturbance input D when $N \geq 2$ is given by

$$e_{ss} = \lim_{s\to 0} s \frac{-s^N N_{pa} D_{pb}}{s^N D_p + N_c}\frac{1}{s^2} = \lim_{s\to 0} \frac{-s^{N-1} N_{pa} D_{pb}}{s^N D_p + N_c} = 0. \tag{11}$$

C. Reference System Specification

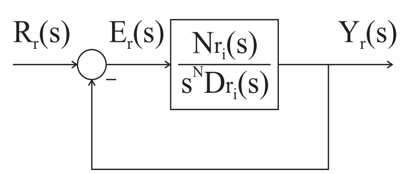

Fig. 2. Closed-loop reference system.

The design criteria is specified as a desired reference closed-loop transfer function and the reference open-loop

[2]For the case of unstable plant zeros, see [28].

transfer function is subsequently solved for, see Fig. 2. The transfer function from the input R_r to Y_r is given by

$$\frac{Y_r}{R_r} = \frac{N_r}{D_r} = \frac{\frac{N_{r_i}}{s^N D_{r_i}}}{1 + \frac{N_{r_i}}{s^N D_{r_i}}} = \frac{N_{r_i}}{s^N D_{r_i} + N_{r_i}}, \qquad (12)$$

where N_{r_i} and D_{r_i} are type zero. Solving for N_{r_i} and D_{r_i} gives

$$N_{r_i} = N_r \qquad (13)$$

and

$$D_{r_i} = s^{-N}(D_r - N_r). \qquad (14)$$

In order to ensure that D_{r_i} is a regular polynomial in s, D_r and N_r must satisfy the following nonrestrictive criteria:

- If $N = 0$, $b_{rm_r} \neq a_{rn_r}$
- If $N = 1$, $b_{rm_r} = a_{rn_r}$ and $b_{rm_r-1} \neq a_{rn_r-1}$
- If $N = 2$, $b_{rm_r} = a_{rn_r}$, $b_{rm_r-1} = a_{rn_r-1}$ and $b_{rm_r-2} \neq a_{rn_r-2}$
- etc.

III. OPTIMAL IMPULSE RESPONSE TRACKING

We now wish to match the open-loop impulse responses of the controlled system, see Fig. 3 for a simplified block diagram, and the reference system, see Fig. 2, as closely as possible. The open–loop impulse response of the controlled system characterized by the causal transfer function N_c/D_p

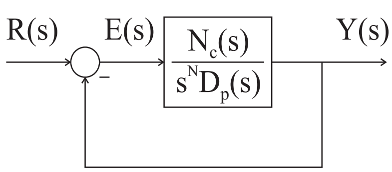

Fig. 3. A simplified block diagram of the controlled system.

with distinct poles, has the impulse response

$$y_{cpi}(t) = \mathcal{B}_c \Lambda_p \mathcal{E}_p(t), \qquad (15)$$

whereas the open–loop impulse response characterized by the causal transfer function N_{r_i}/D_{r_i} with distinct poles, has the impulse response

$$y_{r_i i}(t) = \mathcal{B}_{r_i} \Lambda_{r_i} \mathcal{E}_{r_i}(t). \qquad (16)$$

The impulse responses are of the generic form [20] (for the case of repeated poles see [21]),

$$y_i(t) = \mathcal{B}\Lambda\mathcal{E}(t), \quad t > 0 \qquad (17)$$

where

$$\mathcal{B} = [b_m \;\; -b_{m-1} \;\; b_{m-2} \;\; \cdots \;\; (-1)^m b_0],$$

$$\Lambda = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \lambda_1 & \lambda_2 & \cdots & \lambda_n \\ \lambda_1^2 & \lambda_2^2 & \cdots & \lambda_n^2 \\ \vdots & \vdots & & \vdots \\ \lambda_1^m & \lambda_2^m & \cdots & \lambda_n^m \end{bmatrix} \text{ and}$$

$$\mathcal{E}(t) = \begin{bmatrix} \frac{e^{-\lambda_1 t}}{\prod_{i=2}^{n}(-\lambda_1+\lambda_i)} \\ \frac{e^{-\lambda_2 t}}{\prod_{i=1, i\neq 2}^{n}(-\lambda_2+\lambda_i)} \\ \vdots \\ \frac{e^{-\lambda_k t}}{\prod_{i=1, i\neq k}^{n}(-\lambda_k+\lambda_i)} \\ \vdots \\ \frac{e^{-\lambda_n t}}{\prod_{i=1}^{n-1}(-\lambda_n+\lambda_i)} \end{bmatrix}.$$

We then define a cost function measuring the controlled impulse response deviation from the reference impulse response as

$$\begin{aligned} J &= \int_0^\infty \left(y_{r_i i}(t) - y_{cpi}(t)\right)^2 dt \\ &= \int_0^\infty \left(\mathcal{B}_{r_i}\Lambda_{r_i}\mathcal{E}_{r_i}(t) - \mathcal{B}_c\Lambda_p\mathcal{E}_p(t)\right)^2 dt. \end{aligned} \qquad (18)$$

Assuming b_{cm} is fixed, e.g., such as to obtain a specific DC gain and defining

$$\mathcal{B}_c^{'} = \left[\; -b_{c(m_c-1)} \quad b_{c(m_c-2)} \quad \cdots \quad (-1)^{m_c} b_{c0} \;\right], \qquad (19)$$

differentiating the cost function with respect to $\mathcal{B}_c^{'}$ and setting the result equal to zero gives,

$$\begin{aligned} \frac{\partial J}{\partial \mathcal{B}_c^{'}} &= \frac{\partial}{\partial \mathcal{B}_c^{'}} \int_0^\infty \left(\mathcal{B}_{r_i}\Lambda_{r_i}\mathcal{E}_{r_i}(t) - \mathcal{B}_c\Lambda_p\mathcal{E}_p(t)\right)^2 dt \\ &= \int_0^\infty \frac{\partial}{\partial \mathcal{B}_c^{'}} \left(\mathcal{B}_{r_i}\Lambda_{r_i}\mathcal{E}_{r_i}(t) - \mathcal{B}_c\Lambda_p\mathcal{E}_p(t)\right)^2 dt \\ &= \int_0^\infty \frac{\partial}{\partial \mathcal{B}_c^{'}} \Big(\left(\mathcal{B}_{r_i}\Lambda_{r_i}\mathcal{E}_{r_i}(t)\right)^2 \\ &\quad -2\mathcal{B}_{r_i}\Lambda_{r_i}\mathcal{E}_{r_i}(t)\mathcal{B}_c\Lambda_p\mathcal{E}_p(t) + \left(\mathcal{B}_c\Lambda_p\mathcal{E}_p(t)\right)^2 \Big) dt \\ &= \int_0^\infty \Big(-2\mathcal{B}_{r_i}\Lambda_{r_i}\mathcal{E}_{r_i}(t) \left(\Lambda_{p_{2:(m_c+1),:}} \mathcal{E}_p(t)\right)^T \\ &\quad + 2\left(\mathcal{B}_c\Lambda_p\mathcal{E}_p(t)\right) \left(\Lambda_{p_{2:(m_c+1),:}} \mathcal{E}_p(t)\right)^T \Big) dt \\ &= 2\mathcal{D} + 2\mathcal{B}_c\mathcal{A} = 0. \end{aligned} \qquad (20)$$

Here $\Lambda_{p_{2:(m_c+1),:}}$ denotes all but the first row in Λ_p,

$$\mathcal{D} = -\mathcal{B}_{r_i}\Lambda_{r_i} \int_0^\infty \mathcal{E}_{r_i}(t)\mathcal{E}_p(t)^T dt \left(\Lambda_{p_{2:(m_c+1),:}}\right)^T \qquad (21)$$

is an $1 \times m_c$ dimensional vector and

$$\mathcal{A} = \Lambda_p \int_0^\infty \mathcal{E}_p(t)\mathcal{E}_p(t)^T dt \left(\Lambda_{p_{2:(m_c+1),:}}\right)^T \qquad (22)$$

is an $(m_c+1) \times m_c$ matrix. Assuming stable eigenvalues, and calculating the kj-th element in the matrix

$\int_0^\infty \mathcal{E}_{r_i}(t)\mathcal{E}_p(t)^T dt,\ k = 1,2,\ldots,n_{r_i},\ j = 1,2,\ldots,n_p$ results in

$$\int_0^\infty \mathcal{E}_{r_ik}(t)\mathcal{E}_{pj}(t)^T dt = \frac{1}{(\lambda_{r_ik}+\lambda_{pj})\prod_{i=1,i\neq k}^{n_{r_i}}(-\lambda_{r_ik}+\lambda_{r_ii})\prod_{i=1,i\neq j}^{n_p}(-\lambda_{pj}+\lambda_{pi})}. \tag{23}$$

Similarly, calculating the kj-th element in the matrix $\int_0^\infty \mathcal{E}_p(t)\mathcal{E}_p(t)^T dt,\ k,j = 1,2,\ldots,n_p$, results in

$$\int_0^\infty \mathcal{E}_{pk}(t)\mathcal{E}_{pj}(t)^T dt = \frac{1}{(\lambda_{pk}+\lambda_{pj})\prod_{i=1,i\neq k}^{n_p}(-\lambda_{pk}+\lambda_{pi})\prod_{i=1,i\neq j}^{n_p}(-\lambda_{pj}+\lambda_{pi})}. \tag{24}$$

Now,

$$\begin{aligned}&\mathcal{D}+\mathcal{B}_c\mathcal{A}\\ &\quad = \mathcal{D}+b_{cm_c}\mathcal{A}_{1\cdot} - b_{c(m_c-1)}\mathcal{A}_{2\cdot}\\ &\qquad + b_{c(m_c-2)}\mathcal{A}_{3\cdot}\cdots(-1)^{m_c}b_{c0}\mathcal{A}_{(m_c+1)\cdot}\\ &\quad = 0\end{aligned} \tag{25}$$

or

$$\mathcal{D}+b_{cm_c}\mathcal{A}_{1\cdot} = -\mathcal{B}_c^{'}\mathcal{A}_{\substack{2\cdot\\(m_c+1)\cdot}}. \tag{26}$$

Here, $\mathcal{A}_{k\cdot}$ denotes the k-th row in the $\mathcal{A}$ matrix, whereas $\mathcal{A}_{\substack{2\cdot\\(m_c+1)\cdot}}$ denotes all but the first row in the $\mathcal{A}$ matrix. The coefficient b_{cm_c} is normally chosen, e.g., to give the inner loop the same static gain as the inner loop of the reference system's, i.e.,

$$b_{cm_c} = a_{pn_p}b_{r_im_{r_i}}/a_{r_in_{r_i}}. \tag{27}$$

Then, a system of m_c equations in the m_c unknowns, $b_{c(m_c-1)}, b_{c(m_c-2)}, \cdots, b_{c0}$, results, giving the explicit easily computable result

$$\mathcal{B}^{'} = -\left(\mathcal{D}+b_{cm_c}\mathcal{A}_{1\cdot}\right)\left(\mathcal{A}_{\substack{2\cdot\\(m_c+1)\cdot}}\right)^{-1}. \tag{28}$$

Note that in general for distinct poles, the $m_c \times m_c$ matrix

$$\mathcal{A}_{\substack{2\cdot\\(m_c+1)\cdot}} = \Lambda_{p\,\substack{2\cdot\\(m_c+1)\cdot}}\int_0^\infty \mathcal{E}_p(t)\mathcal{E}_p(t)^T dt\left(\Lambda_{p\,\substack{2\cdot\\(m_c+1)\cdot}}\right)^T \tag{29}$$

will be of rank m_c. Here,

$$\Lambda_{p\,\substack{2\cdot\\(m_c+1)\cdot}} = \begin{bmatrix} 1 & 1 & \cdots & 1\\ \lambda_{p1} & \lambda_{p2} & \cdots & \lambda_{pn_p}\\ \lambda_{p1}^2 & \lambda_{p2}^2 & \cdots & \lambda_{pn_p}^2\\ \vdots & \vdots & & \vdots\\ \lambda_{p1}^{m_c-1} & \lambda_{p2}^{m_c-1} & \cdots & \lambda_{pn_p}^{m_c-1}\end{bmatrix} \tag{30}$$

has rank m_c for distinct poles and the symmetric $n_p \times n_p$ matrix $\int_0^\infty \mathcal{E}_p(t)\mathcal{E}_p(t)^T dt$ is also of full rank for distinct poles. Further, if numerical difficulties arise in the inversion, the number of zeros, m_p, can be adjusted, to avoid such difficulties.

In general, the relative degree of the inner loop of the controlled system should preferably be selected the same as the relative degree of the inner loop of the reference system, to ease the matching of the two systems. Also note, that the above method may still be used even though more numerator coefficients are fixed (see [28]), e.g., in the case of unstable plant zeros.

Example:

Consider a third–order highly underdamped plant with an input disturbance, where the plant transfer function is given by

$$\frac{N_p}{D_p} = \frac{N_{pa}}{D_{pa}} = \frac{s+3}{s^3+7s^2+36s+130} = \frac{s+3}{(s+1\pm 5i)(s+5)}. \tag{31}$$

It is desired to track a well damped type-one closed-loop transfer function given by the transfer function

$$\frac{Y_r}{R_r} = \frac{18}{s^2+6s+18} = \frac{18}{(s+3\pm 3i)}, \tag{32}$$

thus having the inner loop

$$\frac{N_{r_i}}{s^N D_{r_i}} = \frac{18}{s(s+6)}, \tag{33}$$

where $N = 1$. Then, computing the optimal N_c based on Eq. (28) maintaining the same relative degree as the reference system's inner loop, results in

$$N_c = 16.1s^2+35.5s+390 = 16.1(s+1.1\pm 4.8i). \tag{34}$$

The zero-pole locations of the open–loop original plant, the compensated inner loop $\frac{N_c}{N_p}$ and the open–loop reference transfer function $\frac{N_{r_i}}{D_{r_i}}$ are shown in Fig. 4.

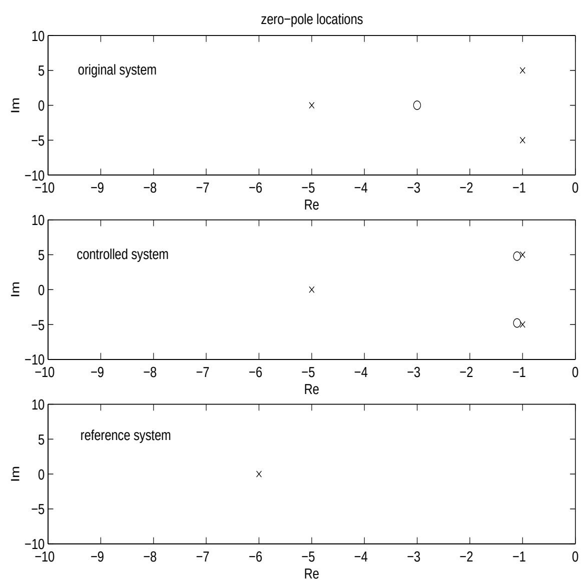

Fig. 4. Pole/zero locations.

The step responses of the open–loop original plant, the open–loop reference transfer function $\frac{N_{r_i}}{D_{r_i}}$ and the compensated inner loop $\frac{N_c}{N_p}$, are shown in Fig. 5, where the compensated inner loop is following the reference step response quite closely. Finally, subjecting the closed-loop as in Fig. 1, to a step input at $time = 1$ and to a unit disturbance at $time = 10$, results in the response shown in Fig. 6. As may be noted, the controlled system follows the reference system very closely during the step input and the disturbance rejection is excellent.

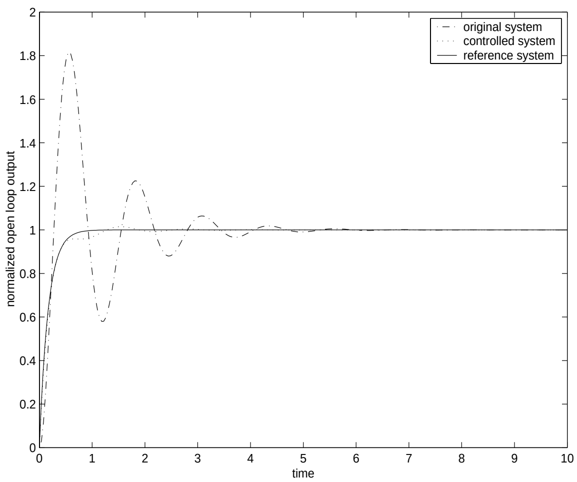

Fig. 5. Normalized open–loop step responses.

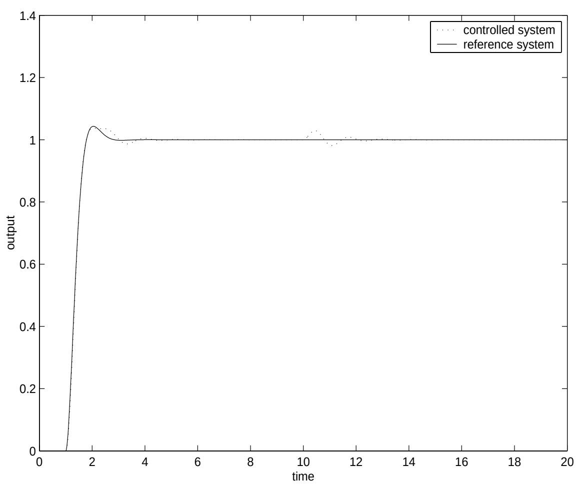

Fig. 6. Closed-loop step responses with the onset of a unit step at time=1 and an onset of a unit disturbance at time=10.

IV. CONCLUSIONS AND FUTURE WORK

A. Conclusions

The tuning of PID controllers can essentially be posed as the problem of selecting open-loop zeros such as to obtain a desired system response. In this paper, the idealogy behind the PID controller was extended to the general case wherein stable open-loop system zeros can be cancelled, thus allowing more freedom in placing open-loop zeros, as opposed to just two zeros in the case of a PID controller. Subsequently, optimal open-loop zeros were computed such as to minimize the deviation from a desired reference impulse response, while maintaining the relative degree and the type of the reference system, thus giving the controlled system desired input tracking and disturbance rejection properties. Due to the inverse compensation of the plant zeros, the controller is in general causal when the relative degree of the plant and the reference system are similar. In cases when the controller is noncausal, which happens if the plant has a high relative degree and the reference system has a low relative degree, the controller can be realized using poles to limit the high frequency response, as is frequently done in a practical setup of a PID controller.

Excellent results were obtained, wherein a highly underdamped system tracked a well behaved reference system response. The controlled system was shown to have excellent input tracking and disturbance rejection properties.

B. Future Work

It is of interest to show that the minimal deviation between the reference and the controlled system does occur when the relative degrees of the two systems are the same. It is further of interest to explore the stability properties of the closed–loop controlled system, in particular to obtain an estimate of the maximum possible deviation between the controlled system and a well-behaved and stable reference closed–loop system, such that stability of the controlled system is guaranteed.

V. ACKNOWLEDGMENTS

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