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# Sliding-Mode Controller Design with Internal Model Principle for Systems Subject to Periodic Signals

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***Abstract*—This paper proposes a sliding-mode control (SMC) scheme based on the internal model principle (IMP) for robust reference tracking and disturbance rejection. The linear IMP controller is known for the capability of perfect tracking and disturbance rejection with an internal model of exogenous signals, while the SMC controller is robust to system perturbations and exogenous signals with unknown dynamics. In this paper, an SMC design based on IMP is proposed to combine the best feature of these two fundamentally different but effective methods. Furthermore, with the help of the SMC, an initial state of the internal model is determined independently of system perturbations in order that transient performance is greatly improved as compared with that of the linear IMP controller. In addition, by properly assigning the initial state of the internal model, a sliding control law is derived to ensure the existence of a sliding mode during an entire response. This global sliding motion yields excellent robustness of the entire system at the beginning of system response and afterwards. Simulation results show the feasibility of the proposed scheme.**

***Index Terms* — Sliding-Mode Control, Internal Model Principle.**

## I. Introduction

SLIDING-mode control (SMC) [1] is a robust nonlinear control scheme, in which system state is directed towards some predefined switching plane and maintained on it through switching control effort. During the sliding motion, system response is completely insensitive to system perturbations satisfying the so-called matching condition. However, due to finite switching frequencies in physical implementation, this invariance property can not be thoroughly preserved, and perfect tracking performance cannot be achieved. An alternative approach ensuring robust tracking in linear control theory is based on the internal model principle (IMP) [2] which states that a model of the non-decaying exogenous signal in the loop transfer function ensures perfect asymptotic tracking and disturbance rejection. With the internal model of a reference signal, robust tracking performance can be ensured even when system parameters are perturbed away from their nominal values to a certain extent. However, with the extra dynamics in the control loop, the system tends to have large overshoot or to oscillate significantly before settling down [3]. Besides, system perturbation is apt to have notable influences on system performance by this linear control technique.

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To improve the transient performance of a linear control system based on the IMP, Wu [3] proposed a strategy that first applied a sliding controller for an improved transient response and then switched to a linear IMP controller in the steady state. A switching mechanism was devised to yield a smooth transition between a sliding controller and an IMP controller by incorporating an observer-like IMP controller to track the equivalent control effort [1] of the sliding controller in the transient phase. In the transient phase, however, the IMP controller made no contribution to control activities, and thus the transient performance would be deteriorated by exogenous disturbances even with known dynamics. On the other hand, in the steady phase the sliding controller is inactive, which weakened system robustness to parameter variations and unexpected disturbances. Moreover, since the active controller might jump back and forth between two controllers, the stability of the overall system is not ensured. In [4], a linear IMP controller was augmented by an integral SMC [1] to enhance the robustness of a linear IMP control system. Basically, the integral SMC design procedure formulated in [1] allows for incorporating any linear control with the integral SMC, where the linear control is designed for the nominal system and then the sliding control is applied to enhancing system robustness. Therefore, the nominal linear design based on IMP in [4] represented the desired system dynamics, in which the problem of large overshoot or significant oscillations associated with the linear IMP control system remains unsolved. The experimental results represented in [4] also showed great overshoot in step responses even with disturbance compensation. Moreover, system responses tended to be oscillatory after adopting the continuous approximation of the discontinuous sign function, and thus the information on time derivative of the switching function was incorporated into the control law. The measurement of this derivative signal however reduces the noise-immune capability of the whole system.

SMC design belongs to time-domain approaches. The essential feature of this nonlinear state-space method is that feedback gains are locally high when system state is close to some predefined switching hyperplane. When system state moves away from the switching hyperplane, the equivalent linear feedback gain is reduced. On the other hand, the linear control based on IMP features locally-high feedback gains in the frequency domain. The internal models in loop transfer functions usually have gains of infinite magnitudes at the frequencies of exogenous signals. In this paper, a systematic design approach is proposed to combine the best features of these two fundamentally different control schemes. A state-space model including the model of exogenous signals is first formulated, and then an SMC design approach is introduced to the joined system. The initial state of the internal model is assigned not only to make the initial value of a switching function zero for the existence of a global sliding mode [5] but also to yield non-overshooting output responses. In this manner, excellent transient performance is guaranteed while robust tracking is also achieved in the steady phase. Simulation validation shows the effectiveness of the proposed scheme.

## II. Sliding-Mode Control Based on Internal Model Principle

### A. Combined Model of Plant and Exo-system

Consider the following uncertain system of $n$-th order:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}(u+d) \qquad y = \boldsymbol{C}\boldsymbol{x} \tag{1}$$

where $y$ is the scalar output of interest, $u$ is the scalar plant input, $\boldsymbol{x} = [x_1\ x_2\ x_3 \cdots x_n]^T \in \Re^n$ being the state vector of the plant, $\boldsymbol{A} = [a_{ij}] \in \Re^{n\times n}$, $\boldsymbol{B} = [0 \ \cdots \ 0 \ \ b]^T \in \Re^{n\times 1}$, $\boldsymbol{C} = [c_i] \in \Re^{1\times n}$, and $d$ denotes the external disturbance. We assume that the plant is completely controllable and has no zeros at the roots of the exogenous signal's characteristic equation, and that its uncertainties satisfy the so-called matching condition. Define

$$\beta = b^{-1},\ \alpha_j = b^{-1} a_{nj} \ \text{ for } \ j = 1, 2, \cdots, n. \tag{2}$$

Bounds on parameter uncertainties and external disturbance are assumed to be known, i.e.

$$|d| < \Delta d,\ \left|\beta - \hat{\beta}\right| < \Delta\beta, \qquad \left|\alpha_j - \hat{\alpha}_j\right| = \Delta\alpha_j \ \text{ for } \ j = 1, 2, \cdots, n \tag{3}$$

where $\hat{\beta}$ and $\hat{\alpha}_j$ are the estimates of $\beta$ and $\alpha_j$, respectively, and $\Delta d$, $\Delta\beta$ and $\Delta\alpha_j$ are uncertainty bounds assumed to be known.

The control objective is to have the output $y$ track a reference input $r$ in the presence of external disturbance signal $d$. Assume that the exogenous signal, either reference or disturbance, is a pure tonal signal; i.e. a sinusoid of a single frequency described by

$$\ddot{r} + \omega^2 r = 0, \quad \ddot{d} + \omega^2 d = 0. \tag{4}$$

The limitation on the dynamics of reference/disturbance signals is for the convenience of elaborations, while the following design can be extended in principle to the case of exogenous signals with high-order dynamics. The structure of the proposed closed-loop system is shown in Fig. 1, where the tracking error of the system is defined as $e = y - r$. Notice that the controller contains an internal model whose input is the tracking error. The state equations of the overall system are then

$$\frac{d}{dt}\begin{bmatrix} z_1 \\ z_2 \\ \boldsymbol{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -\omega^2 & 0 & \boldsymbol{C} \\ 0 & 0 & \boldsymbol{A} \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \\ \boldsymbol{x} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \boldsymbol{B} \end{bmatrix}(u+d) - \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} r \tag{5}$$

where $z_1$ and $z_2$ are state variables of the internal model. Based on this combined model of the plant and the internal model, an SMC is developed to enhance system robustness to parameter variations and unexpected disturbances, while the internal model having infinite gain at the frequency $\omega$ forces the tracking error to converge asymptotically.

### B. Switching Function

In designing SMC, first determine a desired switching function, and then find a sliding control law that is able to constraint system state on the switching hyperplane, that is, to force the predefined switching function to zero. Rewrite (5) as

$$\frac{d}{dt}\begin{bmatrix} \bar{\boldsymbol{x}} \\ x_n \end{bmatrix} = \left[\begin{array}{ccc:c} 0 & 1 & \boldsymbol{0} & 0 \\ -\omega^2 & 0 & \boldsymbol{C}_1 & \boldsymbol{C}_2 \\ 0 & 0 & \boldsymbol{A}_{11} & \boldsymbol{A}_{12} \\ \hdashline 0 & 0 & \boldsymbol{A}_{21} & \boldsymbol{A}_{22} \end{array}\right]\begin{bmatrix} \bar{\boldsymbol{x}} \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \boldsymbol{0} \\ b \end{bmatrix}(u+d) - \begin{bmatrix} 0 \\ 1 \\ \boldsymbol{0}_{(n-1)} \\ 0 \end{bmatrix} r \tag{6}$$

where

$$\bar{\boldsymbol{x}} = [z_1 \quad z_2 \quad x_1 \quad \cdots \quad x_{n-1}]^T \in \Re^{n+1},\ \boldsymbol{A} = \left[\begin{array}{c:c} \boldsymbol{A}_{11} & \boldsymbol{A}_{12} \\ \hdashline \boldsymbol{A}_{21} & \boldsymbol{A}_{22} \end{array}\right],$$

$\boldsymbol{C}_1 = [c_1 \quad c_2 \quad \cdots \quad c_{n-1}]$, and $\boldsymbol{C}_2 = [c_n]$.

With this state partition, the joined model (6) can be divided into two parts as follows:

$$\dot{\bar{\boldsymbol{x}}} = \bar{\boldsymbol{A}}\bar{\boldsymbol{x}} + \bar{\boldsymbol{B}}x_n + \bar{\boldsymbol{H}}r \tag{7}$$

$$\dot{x}_n = [0 \quad 0 \quad \boldsymbol{A}_{21}]\bar{\boldsymbol{x}} + \boldsymbol{A}_{22}x_n + b(u+d) \tag{8}$$

where

$$\bar{\boldsymbol{A}} = \begin{bmatrix} 0 & 1 & \boldsymbol{0} \\ -\omega^2 & 0 & \boldsymbol{C}_1 \\ 0 & 0 & \boldsymbol{A}_{11} \end{bmatrix},\ \bar{\boldsymbol{B}} = \begin{bmatrix} 0 \\ \boldsymbol{C}_2 \\ \boldsymbol{A}_{12} \end{bmatrix}, \text{ and}$$

$$\bar{\boldsymbol{H}} = -[0 \quad 1 \quad \boldsymbol{0}_{(n-1)\times 1}]^T \in \Re^{n+1}.$$

Equation (7) describes the null space dynamics while equation (8) represents the range space dynamics. In designing an SMC, the variable $x_n$ in (7) is regarded as a control input to the null space dynamics and should be so assigned that the null space dynamics is shaped to the desired sliding dynamics. Let

$$x_n = -\boldsymbol{\lambda}\overline{\boldsymbol{x}} + \upsilon r \tag{9}$$

where $\boldsymbol{\lambda} = [\lambda_1 \quad \lambda_2 \quad \cdots \quad \lambda_{n+1}]$ being a constant row vector, and $\upsilon$ is a constant feedforward gain to be determined. The state feedback with feedback gains $\boldsymbol{\lambda}$ can place the poles of the null space dynamics to any desired locations only when the null space dynamics is completely controllable. It turns out that it is possible to relate the controllability of the null space dynamics with that of the original system.

*Lemma 1:* If $(\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C})$ is completely controllable and has no invariant zeros at $\pm i\omega$, then the matrix pair $(\overline{\boldsymbol{A}}, \overline{\boldsymbol{B}})$ is completely controllable.

The proof is omitted. Since the plant is assumed to be completely controllable and have no zeros at $\pm i\omega$, the null space dynamics is completely controllable, and any linear state feedback method can be utilized to determine $\boldsymbol{\lambda}$. Several approaches have been developed, including linear quadratic minimization [6] and direct eigenvalue assignment [7].

In this paper, the reference signal *r* is assumed to be a pure tonal signal. In case the reference signal contains nonzero direct-current component, the internal model shown in Fig. 1 does not fit in with the dynamics of the exogenous signal, and an integrator should be added to the internal model. Instead of incorporating an integrator into the internal model, a constant feedforward gain $\upsilon$ is applied to tracking the direct-current component of the reference signal. To determine the feedforward gain $\upsilon$, substituting (9) into (7) and considering only the direct-current component of the state vector gives

$$\overline{\boldsymbol{x}} = -(\overline{\boldsymbol{A}} - \overline{\boldsymbol{B}}\boldsymbol{\lambda})^{-1}(\overline{\boldsymbol{B}}\upsilon + \overline{\boldsymbol{H}})r \,. \tag{10}$$

Rewrite the output equation of the system as

$$y = \overline{\boldsymbol{C}}\overline{\boldsymbol{x}} + \overline{\boldsymbol{D}}x_n \tag{11}$$

where

$$\overline{\boldsymbol{C}} = [0 \quad 0 \quad c_1 \quad c_2 \quad \cdots \quad c_{n-1}] \text{ and } \overline{\boldsymbol{D}} = [c_n].$$

Substituting (9) and (10) into (11), we obtain the direct-current component of the output

$$y = \left[-(\overline{\boldsymbol{C}} - \overline{\boldsymbol{D}}\boldsymbol{\lambda})(\overline{\boldsymbol{A}} - \overline{\boldsymbol{B}}\boldsymbol{\lambda})^{-1}(\overline{\boldsymbol{B}}\upsilon + \overline{\boldsymbol{H}}) + \overline{\boldsymbol{D}}\upsilon\right] r \,. \tag{12}$$

Equating it with *r* yields

$$\upsilon = \left[-(\overline{\boldsymbol{C}} - \overline{\boldsymbol{D}}\boldsymbol{\lambda})(\overline{\boldsymbol{A}} - \overline{\boldsymbol{B}}\boldsymbol{\lambda})^{-1}\overline{\boldsymbol{B}} + \overline{\boldsymbol{D}}\right]^{-1} \left[1 + (\overline{\boldsymbol{C}} - \overline{\boldsymbol{D}}\boldsymbol{\lambda})(\overline{\boldsymbol{A}} - \overline{\boldsymbol{B}}\boldsymbol{\lambda})^{-1}\overline{\boldsymbol{H}}\right] \tag{13}$$

when $-(\overline{\boldsymbol{C}} - \overline{\boldsymbol{D}}\boldsymbol{\lambda})(\overline{\boldsymbol{A}} - \overline{\boldsymbol{B}}\boldsymbol{\lambda})^{-1}\overline{\boldsymbol{B}} + \overline{\boldsymbol{D}} \neq 0$ that is then the condition for the existence of a solution for the constant feedforward gain.

*Lemma 2:* If $(\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C})$ has no invariant zeros at the origin of the complex plane and the frequency $\omega$ is nonzero, then (13) gives a finite solution for the feedforward gain $\upsilon$.

Due to limited space, the proof is omitted here. Notice that the loop transfer function of the system itself contains the model of a direct-current signal when the frequency $\omega$ is equal to zero and the plant has less than two invariant zeros at the origin. In this case, there is no need to introduce the feedforward compensation $\upsilon r$, and the feedforward gain $\upsilon$ should be set to zero. From (13), it is clear that the determination of the feedforward gain $\upsilon$ is independent of matched uncertainties. When there exists a finite solution for $\upsilon$, the feedforward compensation in (9) ensures the exact tracking of a reference signal's direct-current component. This together with the internal model guarantees the precise tracking of a reference signal that consists of a direct-current part and a sinusoid of a single frequency.

To shape the null space dynamics, the state variable $x_n$ should be constrained to maintain the equation (9) valid. According to (9), define the switching function

$$\sigma(t) = s(t) + k\int_0^t s(\tau)d\tau \tag{14}$$

where $s = x_n + \boldsymbol{\lambda}\overline{\boldsymbol{x}} - \upsilon r$ and *k* is a constant parameter. The introduction of an integral action in the switching function is to suppress an offset error in the plant's output caused by a constant disturbance. As mentioned previously, the internal model shown in Fig. 1 can be modified to one with an integrator for restraining the offset error due to a constant disturbance. Here, the integral action is carried out in the switching function for its simplicity and convenience.

### *C. Determination of Initial Conditions of the Internal Model*

The output response of a control system containing an internal model of exogenous signals tends to have significant overshoot and/or serious oscillations before settling down even if the closed-loop poles are well placed to look for highly-damped system behavior. In fact, the response of a system is primarily determined not only by its transfer function but also by its initial state. Assigning the initial state of a controller properly would have an essential improvement in system's transient responses. On the other hand, for a certain initial state, the system response is apt to be influenced by parameter variations and external disturbances, which makes it difficult to assign a controller's initial state properly. This problem, however, does not exist in the proposed formulation as the sliding dynamics that determines the system output is the shaped null space dynamics and is free from matched system perturbations. Matched system perturbations have an effect on the range space dynamics, but an SMC can suppress their effect efficiently.

In this paper, the initial state of an internal model is assigned to make the initial value of the switching function (14) zero as well as to have a smooth start-up. When the switching function (14) is forced to be initially zero and a sliding control law is so designed that the sliding condition [8] is valid, a global sliding mode control (GSMC) [5] is achieved and robust performance is thus ensured. To have $\sigma(0)=0$, setting $s(0)=0$ gives

$$\lambda_2 z_2(0)+\lambda_1 z_1(0)=-\left(x_n(0)+\sum_{i=1}^{n-1}\lambda_{i+2}x_i(0)\right)+\upsilon r(0) \quad (15)$$

which is the necessary condition for global sliding behavior. Since in a global sliding mode $\sigma(t)=0$ during an entire response, we have $\dot{\sigma}(t)=0$. This together with the requirement of a smooth start-up demands $\dot{\sigma}(0)=0$ and $\dot{x}_n(0)=0$. Taking the time derivative of (14) and noting that $s(0)=0$, we have

$$\boldsymbol{\lambda}\left(\overline{\boldsymbol{A}}\overline{\boldsymbol{x}}(0)+\overline{\boldsymbol{B}}x_n(0)+\overline{\boldsymbol{H}}r(0)\right)-\upsilon\dot{r}(0)=0\,. \quad (16)$$

Solve the simultaneous algebraic equations, (15) and (16), for the required initial values of the internal model, $z_1(0)$ and $z_2(0)$, which is independent of matched system perturbations. Since the internal model is implemented inside a controller, its initial state can be arbitrarily assigned. When determined by (15) and (16), the initial state of the internal model leads the overall system state initially on the predefined sliding hyperplane and guarantees a smooth start-up behavior. The arrangement for a smooth start-up reduces the necessary starting torque, lowers mechanical/electrical stress on the plant, and improves the transient response as well.

### D. Sliding Control Law

The objective of a sliding control law is to attract system state onto the switching hyperplane so that system state reaches the switching hyperplane and stays on it thereafter. This can be achieved by designing a control law that satisfies the sliding condition, $\sigma(t)\dot{\sigma}(t)<0$. Taking the derivative of (14) with respect to time and substituting (7) yields

$$\dot{\sigma}=\dot{x}_n+\boldsymbol{\lambda}\dot{\overline{\boldsymbol{x}}}-\upsilon\dot{r}+ks=\dot{x}_n+\eta \quad (17)$$

where $\eta=\boldsymbol{\lambda}\left(\overline{\boldsymbol{A}}\overline{\boldsymbol{x}}+\overline{\boldsymbol{B}}x_n+\overline{\boldsymbol{H}}r\right)-\upsilon\dot{r}+ks$. Dividing both sides of (17) by $b$ and substituting (8) gives

$$\beta\dot{\sigma}=\sum_{i=1}^{n}\alpha_i x_i+u+d+\beta\eta \quad (18)$$

which leads to the sliding control law

$$u=-\left(\hat{\beta}\eta+\sum_{i=1}^{n}\hat{\alpha}_i x_i\right)-\left(\Delta\beta|\eta|+\Delta d+\sum_{i=1}^{n}\Delta\alpha_i|x_i|\right)\operatorname{sgn}(\sigma) \quad (19)$$

where $\operatorname{sgn}(\cdot)$ denotes the discontinuous sign function. Since $\beta=b^{-1}$ is assumed to be positive, it can be easily verified that the sliding control law (19) ensures the satisfaction of the sliding condition $\sigma(t)\dot{\sigma}(t)<0$ for $\sigma(t)\neq 0$ and $t\geq 0$.

According to (15), the initial value of the switching function is set to zero. This together with the satisfaction of the sliding condition implies that the sliding mode exists throughout an entire response, i.e.

$$\sigma=0 \quad \text{for all } t\geq 0 \quad (20)$$

Therefore, an initial period of time is not required to reach the sliding regime $\sigma=0$, and the reaching phase is eliminated in this design. As the sliding mode exits throughout an entire response, robust performance is thus guaranteed.

## III. Simulation Validation

### A. System Description and Controller Design

Consider the second-order model of a voice-coil motor described by

$$\frac{X(s)}{U(s)}=\frac{b}{s^2+a_2 s+a_1} \quad (\mu\text{m/volt}) \quad (21)$$

where $x$ is the output of interest, $a_2=100\times(1\pm 0.5)$, $a_1=(1.0\text{e}5)\times(1\pm 0.5)$, and $b=(7.0\text{e}8)\times(1\pm 0.5)$. Let

$$d(t)=0.1\sin(\omega t)-0.07\left(\mathit{1}(t-0.5)-\mathit{1}(t-0.55)\right) \quad (22)$$

where $\mathit{1}(\cdot)$ denotes the unit-step function, and $\omega=1200$ (rad/s).

From the extreme values of uncertain parameters, we get $\hat{\alpha}_1=-2.3810\text{e}-4$, $\hat{\alpha}_2=-2.3810\text{e}-7$, $\hat{\beta}=1.9048\text{e}-9$, $\Delta\alpha_1=1.9048\text{e}-4$, $\Delta\alpha_2=1.9048\text{e}-7$, $\Delta\beta=9.5238\text{e}-10$. Moreover, the bounds on external disturbance $\Delta d=0.1$. For performance comparisons, we designed three kinds of controllers, i.e. the conventional SMC, the linear IMP controller, and the proposed controller. All three controllers were based on the same parameter values listed above, and all poles were assigned at -600 in the nominal case. To alleviate chattering phenomenon, the boundary layer method is adopted in all sliding controllers. For the regulation problem $r$=0, the conventional SMC is designed as

$$u=-\left[\left(600\hat{\beta}+\hat{\alpha}_2\right)\dot{x}+\hat{\alpha}_1 x\right] \\ -\left[\left(600\Delta\beta+\Delta\alpha_2\right)|\dot{x}|+\Delta\alpha_1|x|+\Delta d\right]\ \operatorname{sat}(\sigma/\varepsilon) \quad (23)$$

where $\sigma=\dot{x}+600x$, $\varepsilon=5.0\text{e}2$, and $\operatorname{sat}(\cdot)$ denotes the saturation function defined as

$$\text{sat}(x)=\begin{cases}\text{sgn}(x) & \text{for } |x|>1\\ x & \text{for } |x|\le 1\end{cases} \quad (24)$$

A detailed derivation of a linear IMP controller can be found in [9], and the controller structure is shown in Fig. 2, where the state feedback gains $\boldsymbol{K}=[K_3 \quad K_4]$ contribute a control component $(K_3x+K_4\dot{x})$ to $u$ in our case. To have all closed-loop poles at -600, we have $K_1=1880064$, $K_2=1.5799\text{e}3$, $K_3=0.4084$, $K_4=4.5476\text{e}-5$. According to (19), the proposed controller is designed as

$$\begin{aligned}u=&-\left(\hat{\beta}\eta+\hat{\alpha}_2\dot{x}+\hat{\alpha}_1x\right)\\&-\left(\Delta\beta|\eta|+\Delta d+\Delta\alpha_2|\dot{x}|+\Delta\alpha_1|x|\right)\text{sat}(\sigma/\varepsilon)\end{aligned} \quad (25)$$

where

$$\sigma=s+600\int_0^t s(\tau)d\tau,\ \varepsilon=1.0\text{e}3,$$

$$s=\dot{x}+18000x+106560000z_2+1.9008\text{e}11z_1-18000r,$$

$$\eta=18000\dot{x}+106560000\left(e-\omega^2z_1\right)+1.9008\text{e}11z_2-18000\dot{r}+600s.$$

*B. Dynamic Response*

To test the effectiveness of the controllers, different parameter values and an external disturbance are applied to the plant in simulations, where there exists an aperiodic disturbance component during the period between 0.5s and 0.55s. Assume that $x(0)=5$ and $\dot{x}(0)=0$. Consider the following three cases of plant model

*Case 0)* $a_2=-\hat{\alpha}_2/\hat{\beta}$, $a_1=-\hat{\alpha}_1/\hat{\beta}$, $b=1/\hat{\beta}$

*Case 1)* $a_2=100\times(1+0.5)$, $a_1=(1.0\text{e}5)\times(1+0.5)$, $b=(7.0\text{e}8)\times(1+0.5)$

*Case 2)* $a_2=100\times(1-0.5)$, $a_1=(1.0\text{e}5)\times(1-0.5)$, $b=(7.0\text{e}8)\times(1-0.5)$

Figure 3 shows the regulation performance with the conventional SMC. Due to the existence of a reaching phase in the conventional SMC the transient performance is not robust. Moreover, the steady-state performance is deteriorated by the external disturbance. Reducing the width of the boundary layer can improve steady-state responses. However, this would increase the switching level in the control and lead to more severe chattering phenomenon. The dynamical responses with the linear IMP controller are shown in Fig. 4. It is seen that the periodic disturbance component is suppressed by this approach, but significant undershoot appears in the output response. Figure 5 shows the regulation performance using the proposed scheme. It is clear that the disturbance is effectively restrained by this approach without undershoot in the output response. Moreover, the transient responses are robust to parameter uncertainties and external disturbances without causing significant chatter in the control. It is seen that the conventional SMC is not effective in dealing with periodic disturbances while the linear IMP controller is incapable of eliminating unexpected disturbances well. On the other hand, the proposed approach rejects both periodic and sudden disturbances efficiently. Figure 6 shows the tracking performance by the proposed scheme, where the reference signal $r(t)=10(1+\sin(\omega t))$. Output performance is excellent in tracking both constant and sinusoidal reference signals.

## IV. CONCLUSIONS

This paper has presented the design of integrating two essentially different approaches. In principle, the SMC possesses the property of locally high feedback gains in time domain while the IMP design makes use of locally high feedback gains in frequency domain. To obtain the best features of these two schemes, the proposed scheme was designed based on a combined model that consists of the plant and the internal model. With the help of SMC, the IMP scheme became robust to unexpected system perturbations. On the other hand, the IMP method enhanced the capabilities of SMC for tracking reference signals and rejecting external disturbances with known dynamics. Furthermore, through assigning the initial state of an internal model properly the problem of excessive overshoot or oscillating response caused by the conventional IMP scheme was alleviated greatly. The determination of this initial state can be performed precisely since it is free from the influence of matched system perturbations with the aid of the SMC approach. At the same time, the assignment of this initial state was so determined that a sliding control law ensured a global sliding motion, implying that system robustness is maintained during an entire response. Simulation results demonstrated the effectiveness of the proposed scheme.

## V. ACKNOWLEDGMENTS

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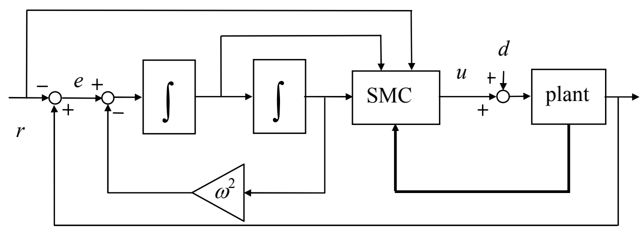


Fig. 1. Controller structure for the proposed SMC based on IMP.

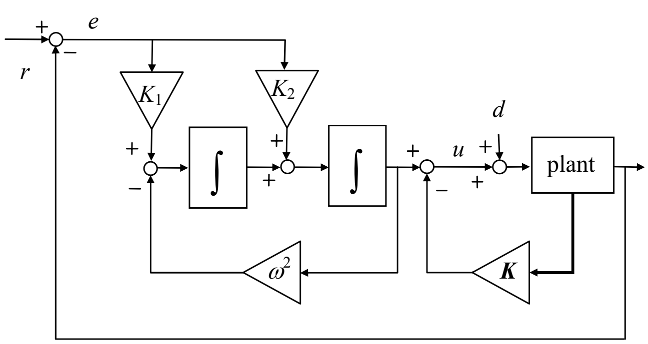


Fig. 2. Controller structure for the IMP controller.

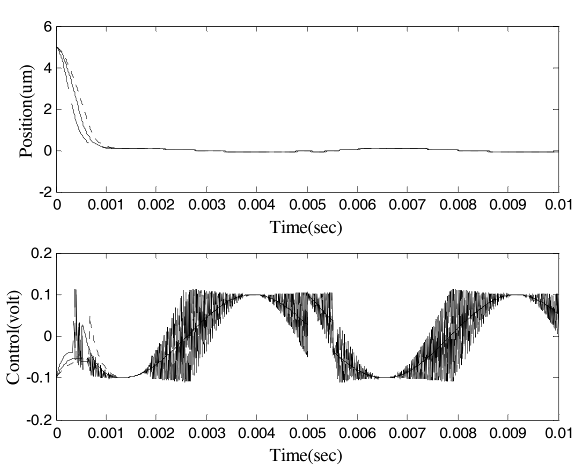


Fig. 3. Dynamic response with the conventional SMC.
Solid line: Case 0. Dashed line: Case 1. Dotted line: Case 2.

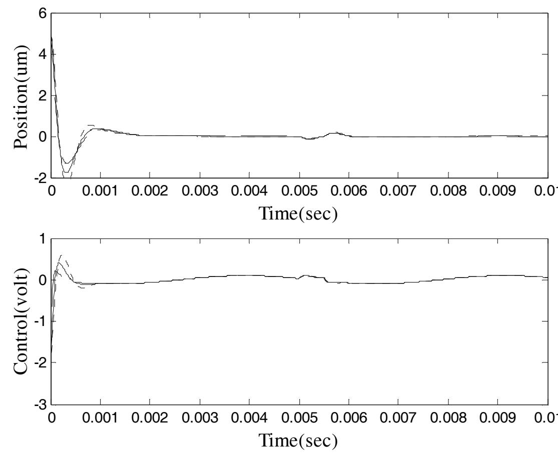


Fig. 4. Dynamic response with the IMP controller.
Solid line: Case 0. Dashed line: Case 1. Dotted line: Case 2.

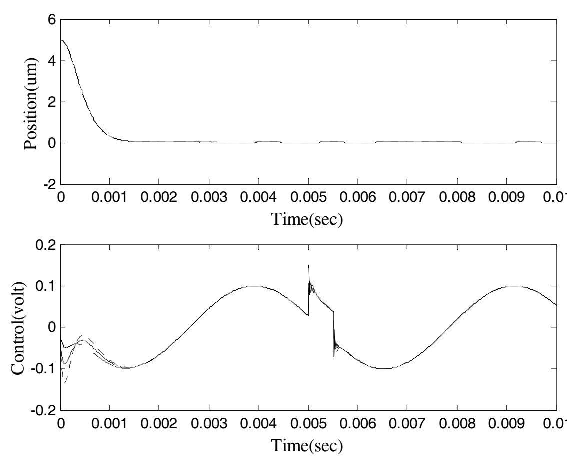


Fig. 5. Dynamic response with the proposed SMC based on IMP.
Solid line: Case 0. Dashed line: Case 1. Dotted line: Case 2.

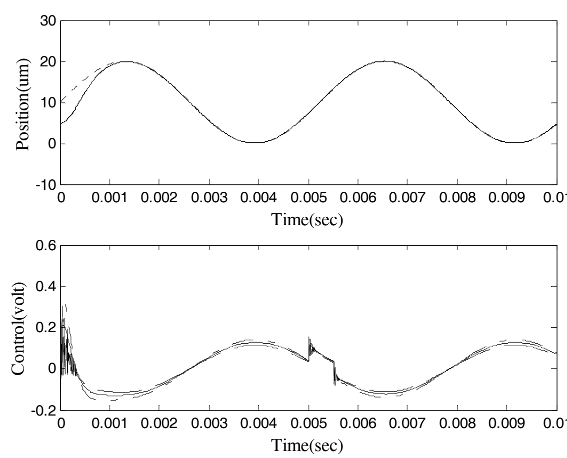


Fig. 6. Tracking response with the proposed SMC based on IMP.
Solid line: Case 0. Dashed line: Case 1. Dash-dot line: Case 2.
Dotted line: reference signal.