An Indirect Variable Structure Model Reference Adaptive Control Applied to the Speed Control of a Three-Phase Induction Motor

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Abstract**—In this paper an indirect approach to the variable structure model reference adaptive control (VS-MRAC) is proposed. This approach is applied to the speed control of a three-phase induction motor. In the direct approach, the switching laws are designed for the parameters of the controller, and in this case the main difficulty is to find the bounds which must be used in the control laws. On the other hand, in the indirect approach, the switching laws are designed for the plant parameters, which are uncertain, but their bounds can be easily found, since they are related to physical parameters of the system. The idea is to design an indirect variable structure approach and to apply it to the speed control of a three-phase induction motor.**

I. Introduction

In recent days the induction motors have been increasingly taking place of the DC motors in high performance electrical motor drives [11]. In the case of motors with squirrel's cage rotor, its main advantage is the elimination of all sliding electrical contacts, resulting in an exceedingly simple and rugged construction. Induction machines are made in a variety of designs with ratings of a few watts to several megawatts. The induction motors can be used in adverse atmospheres since that they don't have commutator and, consequently, there isn't a possibility of sparking. With the progress of the power electronics and the appearance of low cost and very fast microprocessors, the induction motor drives have reached a competitive position compared to DC machines. For the DC motors, the speed control can be carried out in a simple way, since the torque and the flux can be decoupled. The technique of vectorial control based on the rotor field orientation applied to the induction motors [2], [10], [11], when the motor is fed by ideal current sources, provides the decoupling between the torque and flux in a similar way to the DC machine. This technique is known as Field Orientation Control (FOC). The choice of the rotor flux as reference for the **d** axis facilitates the decoupling between motor torque and flux [3], [10], [11]. In this control strategy, an important element of uncertainty is the value of the rotor time constant that varies with the operation conditions, changing the system behavior. Then there is the necessity of methods of adaptive and/or robust control, which can be applied to systems that present parametric uncertainties.

In adaptive control, a common approach is the model reference adaptive control (MRAC) [12], where is proposed a reference model (transfer function) that describes the desired I/O properties of the closed-loop plant. This plant adaptation to model, generally, is slow, generating an undesirable transient.

On the other hand, the variable structure control approach has its roots in relay control, and consists of use a switching control law as a function of system state variables, and, in its common configuration, in order to restrict the system dynamics to a surface referred as a sliding surface [16]. The variable structure systems have as main characteristics the fast transient and robustness to parameter changes and disturbances (in a range stipulated on project).

Thereby, it was proposed a controller that makes use of the characteristics of both techniques [7], i.e., a controller with very fast transient, robustness to parameter changes and disturbances, utilizing only plant input/output measurements. This controller was named VS-MRAC, that was posteriorly modified by simplifications in the original algorithm proposed by Araújo [1], facilitating the practical application of this controller [8]. The VS-MRAC controller, in its direct approach, has been successfully applied on control of DC machines [15] as well as on control of induction machines [4], [5].

In a recent work was presented the indirect approach to the VS-MRAC [13]. In the present paper, in order to confirm its feasibility, an application on a three-phase induction motor is shown.

II. Model of the Induction Motor

In this section we use a vectorial technique for modeling the induction motor, which is very important to study field orientation control [3], [11]. We define a system of complex orthogonal axis, d and q, where the rotor flux is the reference for de **d** axis. The motor vectorial diagram is presented in Figure 1, where

δ - stator electrical current vector angle related to the rotor flux;
ρ - rotor flux angle related to stator phase 1 axis;
ω_S - stator electrical current vector angular speed;
$\psi_{Rd}(t)$ - rotor flux related to the **d** axis;
ε - angle between axis of stator phase 1 and rotor phase 1;
$i_S(t)$ - stator electrical current vector;
i_{Sd}, i_{Sq} - stator electrical current vector components on direct and quadrature axis, respectively;
$\omega(t) = \dfrac{d\varepsilon(t)}{dt}$ - rotor angular mechanical speed.

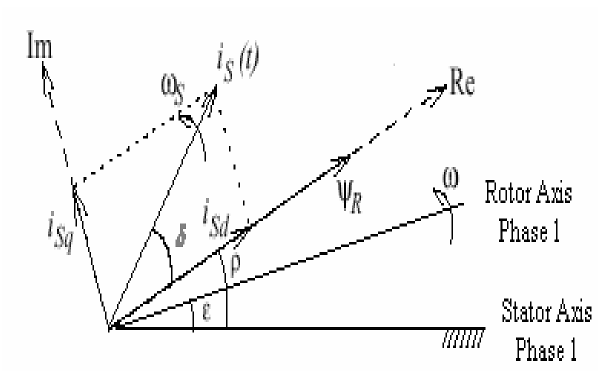

Fig. 1. Motor Vectorial Diagram

From Figure 1 we have

$$i_s(t) = \left(i_{sd}(t) + j i_{sq}(t)\right)e^{j\rho} \qquad (1)$$

$$\psi_R(t) = \psi_{Rd}(t)e^{j(\rho-\varepsilon)} \qquad (2)$$

Using the vectorial analysis with the rotor flux orientation [3], [11], we obtain the following expression for the torque

$$T_e(t) = \frac{2}{3}P\frac{L_m}{L_r}\psi_{Rd}(t)i_{Sq}(t) \qquad (3)$$

where

L_r - rotor inductance by phase;
L_m - magnetization inductance by phase;
P - number of poles pairs.

The equation (3) describes the induction motor torque in a similar way to the DC machine. The component of the rotor flux vector on direct axis is equivalent to the field flux in DC machine and the component of stator electrical current vector on the quadrature axis is equivalent to the armature current in a DC machine. Additionally, if the component of the rotor flux is kept constant, the torque can be controlled only by the component of the stator electrical current vector on the quadrature axis.

III. PARAMETERIZATION AND ASSUMPTIONS

The plant and model reference are defined as follows

Plant

$$y(s) = W(s)u(s)$$

$$W(s) = k_p\frac{n_p(s)}{d_p(s)}$$

$$n_p(s) = s^{n-1} + \sum_{i=1}^{n-1}\beta_i s^{n-1-i} \qquad (4)$$

$$d_p(s) = s^n + \alpha_1 s^{n-1} + \sum_{i=1}^{n-1}\alpha_{i+1}s^{n-1-i} \qquad (5)$$

Reference Model

$$y_m(s) = M(s)r(s)$$

$$M(s) = k_m\frac{n_m(s)}{d_m(s)}$$

$$n_m(s) = s^{n-1} + \sum_{i=1}^{n-1}\beta_{mi}s^{n-1-i} \qquad (6)$$

$$d_m(s) = s^n + \alpha_{m1}s^{n-1} + \sum_{i=1}^{n-1}\alpha_{m,i+1}s^{n-1-i} \qquad (7)$$

In order to simplify the notation, let us define the plant parameter vector as

$$\theta_p^* = [k_p, \beta^T, \alpha_1, \alpha^T]^T \qquad (8)$$

where

- $\beta \in R^{n-1}$ is a vector that contains the elements β_i $(i = n-1,\ldots,1)$ of (4)
- $\alpha_1 \in R$ is the element α_1 of (5)
- $\alpha \in R^{n-1}$ is a vector that contains the elements α_{i+1} $(i = n-1,\ldots,1)$ of (5)

and, likewise, it's defined β_m, α_{m1} and α_m, with respect to reference model.

To obtain the control algorithm, the following assumptions are made:

- the plant is monovariable, controllable, observable and has relative degree $n^* = 1$;
- the degree n of $d_p(s)$ is known;
- k_m and k_p have the same signal (positive, without lost of generality);
- the reference model is strictly real positive (SRP) and has the same plant relative degree;
- $n_p(s)$, $n_m(s)$ and $d_m(s)$ are Hurwitz monic polynomials;
- $d_p(s)$ is a monic polynomial;
- the upper bounds for k_p, β_i $(i = 1,...,n-1)$, $\alpha 1$ and αi $(i = 2,...,n)$ are known.

The adaptation algorithm needs $2n$ variables. Once only the plant input/output are known, it's built a set of state variables from u and y, that are filtered signals of plant input/output [9].

A. Plant Input/Output Filters

The following filters are defined

$$\begin{cases} \dot{v}_1 = \Lambda.v_1 + g.u & (9) \\ \dot{v}_2 = \Lambda.v_2 + g.y & (10) \end{cases}$$

where

$$\Lambda \in R^{(n-1)x(n-1)};\ \det(sI - \Lambda) = n_p(s) \quad (11)$$

$$g = \begin{bmatrix} 0 & . & . & 0 & \gamma \end{bmatrix}^T \in R^{n-1} \quad (12)$$

$\gamma \in R$

$v_1, v_2 \in R^{n-1}$

The vector w with the $2n$ variables is named *regressor vector* and is denoted by

$$w = \begin{bmatrix} v_1^T & y & v_2^T & r \end{bmatrix}^T \quad (13)$$

and, thus, the control law u is given by

$$u = \theta^T(t).w(t) \quad (14)$$

where $\theta(t)$ is the adaptive parameters vector. If the previous assumptions are satisfied, one has that $\exists! \theta^*$ such that $u = \theta^{*T} w = u^*$ implies at $\frac{y(s)}{r(s)} = M(s)$, i.e. , $y \to y_m$ (*matching* condition) where

$$u^* = \theta_{v_1}^{*T} v_1 + \theta_n^* y + \theta_{v_2}^{*T} v_2 + \theta_{2n}^* r \quad (15)$$

With $$\theta^* = \begin{bmatrix} \theta_{v_1}^{*\,T} & \theta_n^* & \theta_{v_2}^{*\,T} & \theta_{2n}^* \end{bmatrix}^T \quad (16)$$

Obviously, θ^* only can be calculated if the plant parameters are known, but when this is not possible, θ^* turns into θ.

B. Control Law to Matching Condition

The expressions to the controller parameters at the *matching* condition [14], when plant parameters are unknown or partially known, are

$$\begin{aligned} \theta_{v_1}(t) &= \frac{\beta_m - \hat{\beta}(t)}{\gamma} \\ \theta_n(t) &= \frac{\hat{\alpha}_1(t) - \alpha_{m1}}{\hat{k}_p(t)} \\ \theta_{v_2}(t) &= \frac{\hat{\alpha}(t) - \alpha_m + (\alpha_{m1} - \hat{\alpha}_1(t))\beta_m}{\hat{k}_p(t).\gamma} \\ \theta_{2n}(t) &= \frac{k_m}{\hat{k}_p(t)} \end{aligned} \quad (17)$$

where $\left(\hat{.}\right)$ is the estimate of $(.)$ at each instant t.

IV. VARIABLE STRUCTURE MODEL REFERENCE DIRECT ADAPTIVE CONTROL (VS-MRAC)

In this section will be reviewed the strategy of adaptive control that needs only plant input/output measurements, with switching laws to the controller parameters [1], [7].

This technique had its origin on the necessity of find a controller that could turn the closed-loop system robust to plant uncertainties and disturbances, with transient performance better than MRAC strategy.

At the scope of this work, the emphasis will be given to plants with $n^* = 1$ and the MRAC adaptation integral laws will be replaced by switching laws.

Hence the adaptive parameters θ_i (16) turn in

$$\begin{aligned} \theta_i &= -\bar{\theta}_i \operatorname{sgn}(e_0 w_i) \\ \bar{\theta}_i &> \left|\theta_i^*\right|; i = 1,2,...,2n \end{aligned} \tag{18}$$

where $e_0 = y - y_m$ is the system output error.

At implementation level, the values of $\bar{\theta}_i$ must be chosen in terms of a determined percentage of nominal values of parameters θ_i^*, to avoid generating a very high control signal. Here is where we may have difficulties to find these $\bar{\theta}_i$, because the evaluation of (17) must be done manually, what it's rather tedious for high order systems. Another option is to use an auxiliary computer program to make this evaluation, but this isn't recommended whereas a good algorithm should be as self-contained as possible.

By Lyapunov's Stability Theory, is shown that the closed-loop system presents asymptotic global stability. Also is proved that the null error, chosen as sliding surface, is reached in a finite time [7].

V. VARIABLE STRUCTURE MODEL REFERENCE INDIRECT ADAPTIVE CONTROL

In a recent work [13], it was developed a controller similar to VS-MRAC, but with the switching laws made on the plant parameters. This was made in order to simplify the controller design, avoiding the problems mentioned above, and whereas the uncertainties at the plant parameters can be known easier than in the traditional VS-MRAC, since they are related with uncertainties in physical parameters, such as resistances, capacitances, inertia moments, friction coefficients, etc. The only remark is that now the algorithm is a bit expensive from the computational point of view, since the evaluation of (17) is made directly in the algorithm

Therefore the innovation of this technique is on the design facility, i.e., once deduced the *matching* condition expressions (17), the work consists on defining the range of pertinence of each plant parameter and on the design of the switching control laws.

Thus, the following switching laws were proposed [13]

$$\begin{aligned} \hat{k}_p &= k_{p,nom} - \bar{k}_p \operatorname{sgn}(e_0 \zeta_p) \\ \hat{\beta}_i &= -\bar{\beta}_i \operatorname{sgn}(e_0 \zeta_{\beta_i} \operatorname{sgn}(k_p)), i = 1,...,n-1 \\ \hat{\alpha}_1 &= -\bar{\alpha}_1 \operatorname{sgn}(e_0 \zeta_1) \\ \hat{\alpha}_i &= -\bar{\alpha}_i \operatorname{sgn}(e_0 \zeta_{\alpha i}), i = 2,...,n \end{aligned} \tag{19}$$

where

$$\begin{aligned} \zeta_p &= \frac{\beta_m{}^T v_1}{\gamma} - u - \frac{\hat{\beta}^T v_1}{\gamma} \\ \zeta_\beta{}^T &= \begin{bmatrix} \zeta_{\beta_1} & ,..., & \zeta_{\beta_{n-1}} \end{bmatrix} \\ \zeta_{\beta_i} &= \frac{-v_{1,i}}{\gamma}, i = 1,...,n-1 \\ \zeta_1 &= y - \frac{\beta_m{}^T v_2}{\gamma} \\ \zeta_\alpha{}^T &= \begin{bmatrix} \zeta_{\alpha_2} & ,..., & \zeta_{\alpha_n} \end{bmatrix} \\ \zeta_{\alpha i} &= \frac{v_{2,i-1}}{\gamma}, i = 2,...,n \end{aligned} \tag{20}$$

The stability proof is shown in [14]. The relays' amplitudes must satisfy

$$\begin{cases} \bar{k}_p > \left|k_p - k_{p,nom}\right| \\ \bar{\alpha}_1 > \left|\alpha_1\right| \\ \bar{\beta}_i > \left|\beta_i\right|, i = 1,...,n-1 \\ \bar{\alpha}_i > \left|\alpha_i\right|, i = 2,...,n \end{cases} \tag{21}$$

The introduction of $k_{p,nom}$ (nominal) in the $\hat{k}_p$ expression (19) is necessary to guarantee that $\hat{k}_p$ doesn't assume negative values and thus guarantee the system stability [14]. Therefore, the switching is made only on parameter uncertainty. This procedure is close to what occur on practice, where one has an uncertainty on the nominal value of the parameter, like, e.g. the uncertainty range of a resistor value.

At implementation level, $\hat{\beta}$ should be calculated before $\hat{k}_p$, and, in the expression of ζ_p (20), instead of use u, it should be used a filtered signal of u (u_{av}).

The control signal obtained in variable structure systems has a high frequency switching and shouldn't be used on practical problems, due to limitations in the driver system. As solutions we could have a change at the relay function by the introduction of a linear region [5] (Figure 2) and/or techniques of smooth control [6]. The first one causes error in steady-state, but should be used together with a PI controller.

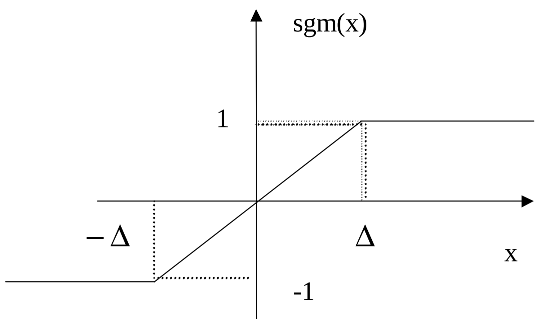

Fig. 2. Linear region

The function $sgm(x)$ is defined as

$$sgm(x) = \begin{cases} \text{sgn}(x); |x| > \Delta \\ \frac{1}{\Delta} x; |x| \leq \Delta \end{cases} \quad (22)$$

VI. EXPERIMENTAL RESULTS

In this section will be presented the results obtained for the application of the technique described above to the speed control of a three-phase induction motor.

A. Driver System

The driver system used to implement the variable structure model reference indirect adaptive control is showed in Figure 3. It is composed by an 0.25 HP induction motor fed by a three-phase VSI/PWM inverter with current control by hysteresis window. In the current control, Hall effect sensors are used to measure the currents of two phases of the motor. One microcomputer receives the motor speed using a tachometer and, by a control software in C language, sends the necessary signal to the inverter.

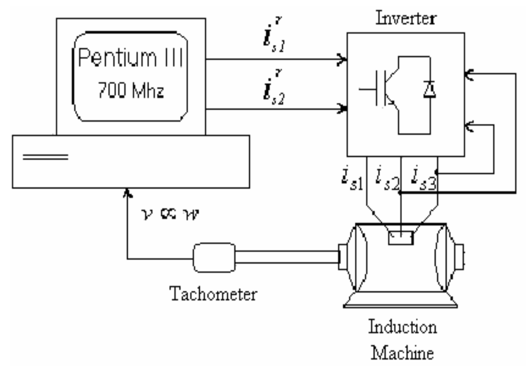

Fig. 3. Driver System

B. Results

The dynamic of the induction motor is represented by the following equation

$$J \frac{d\omega(t)}{dt} = T_e(t) - B\omega(t) - T_l(t) \quad (23)$$

where

J moment of inertia of the rotational mass;
B damping constant;
T_e induction motor torque;
T_l load torque.

The induction motor model introduced here ((3) and (23)), for a certain operating point, considering the rotor flux constant and the motor parameters given in [3], yields to a first order model given by

$$W(s) = \frac{k_p}{s+\alpha} = \frac{3798}{s+11.3} \quad (24)$$

The reference model time constant was chosen near to plant time constant, because high order models with faster dynamics could generate high control signal, without necessity. Thus, the chosen reference model was

$$M(s) = \frac{k_m}{s+\alpha_m} = \frac{12}{s+12} \quad (25)$$

In analyzing (24), the values used in the algorithm were $k_{p,nom} = 3798, \bar{k}_p = 250, \bar{\alpha} = 40$. The sample period was $h = 0.00042s$. A linear region with $\Delta = 1500$ was introduced in order to smooth the control signal. With these values the following graphic was obtained (Figure 4), where the plant and reference model speeds are given in rpm and the control signal $u = i_{Sq}$ .is given in mA.

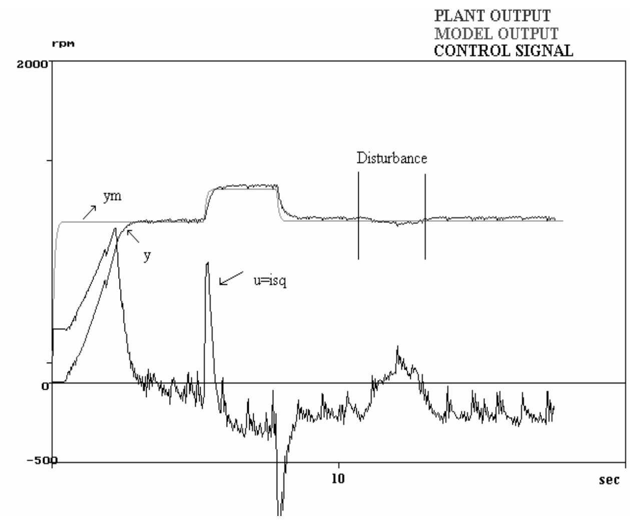

Fig. 4. Experimental result

In the practical test the motor and the reference model outputs start with zero initial speed. The reference is initially assumed as 1000 rpm, after a certain time it's increased to 1200 rpm, and finally it's returned to 1000 rpm. After that, it's introduced an external disturbance. The plant follows the reference in each case (the model gain is one). The disturbance effect is felt only by increasing the control signal during the time interval in which the disturbance is applied. The transient behavior is fast and with no oscillations as it was expected.

VII. CONCLUSION

In this work, the new technique of variable structure model reference adaptive control was applied to the speed control of a three-phase induction motor, a motor with wide range applications in industry. This technique has as objective and main advantage turn the design easier, once that the switching laws are designed to the plant parameters that have uncertainties known easier than in the direct approach, once they are physical parameters. The results have confirmed that the proposed technique presents a performance similar to direct case.

As suggestions to the continuity of this work we could mention the generalization to plants with relative degree bigger than one, a deepest study of linear regions and smooth control, as well as applications in several areas like robotics, process control, etc.

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