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On Parameter Convergence of Adaptive Fully Linearizable Systems

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***Abstract*— Verifiable sufficient conditions for parameter convergence of an adaptive fully linearizable system with unknown parameters, including those affine with the control input, are provided. The adaptive control in [8] has solved the tracking problem for a wider class of systems, however, parameter convergence so far can not be verified *a priori*. By taking advantages of the obtained asymptotic tracking stability and the vanishing time derivatives of the estimated parameters, we found it can indeed be checked beforehand provided some mild assumptions are satisfied. Numerical examples for illustrating the main results are given in the final.**

I. INTRODUCTION

Feedback linearization is well known for its unique ability of rendering the closed-loop system linear by totally cancelling the system's nonlinearity [3]. In reality, complete knowledge of the nonlinearity is hardly available. Hence, it is often incorporated with other schemes to achieve the design goals. Among others, adaptive linearizing control is most suitable for systems with unknown linear-in-parameter nonlinearity and various such schemes have been developed (see a review in [10]). Though asymptotic tracking stability has been obtained by these designs, noticeably, parameter convergence is not guaranteed in general. The major obstacle is that the persistence of excitation (PE) of the regressor depends in a complex way on the closed-loop signals and hence is hard to predict in advance [5],[6]. Thanks to the asymptotic tracking stability and the invariant property of the PE under vanishing perturbations, verifiable conditions for parameter convergence in certain adaptive nonlinear systems have been obtained [2],[4]. The coefficients affine with the control input, however, still need to be known in these derivations, which may not be available in practical applications.

We intend to release such restrictions for fully linearizable systems in this paper. To that end, control schemes ensuring asymptotic tracking stability is indispensable. The remarkable design in [8] fulfill that need and is therefore adopted. The regressor resulting from such a design depends not only on the system states but also on the estimated parameters, rendering the undertaken task difficult. Especially, the latter dependence is tougher to deal with since the ultimate behaviors of the estimated parameters are not known. It is noticed, however, the time derivatives of the estimation errors do vanish, implying the changes of the regressor, due to the variation of the estimated parameter vector, become negligible within a prescribed time period as time passes by long enough. Therefore, prior check of the fulfillment of the PE within that time period is then possible. By repeating such a process consecutively, it is shown that the PE can be verified beforehand provided some mild assumptions on the reference trajectories and the regressor are satisfied. Though the analysis is somewhat involved, the established verifying procedures can be carried out in relatively easy ways.

The remainder of the paper is organized as follows. The closed-loop system under investigation and its properties are introduced in Section II. Despite these nice properties, parameter convergence, as stated, is not addressed so far. Therefore, verifiable conditions are provided in Section III to solve that problem. A numerical example is given in Section IV to demonstrate its usefulness. Concluding remarks are finally made in Section V.

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II. PROBLEM STATEMENT

Consider a fully linearizable system in a normal form of

$$\begin{aligned} \dot{x}_1 &= x_2 \\ &\vdots \\ \dot{x}_n &= \alpha^T f(x) + (\beta^T g(x))u(t) \end{aligned} \tag{1}$$

where $x \in R^n$ is the state, $u(t) \in R$ is the control input, $\alpha \in R^r$ and $\beta \in R^s$ are the *unknown* constant parameter vectors while $f(x) \in R^r$ and $g(x) \in R^s$ are the corresponding *known* basis functions, respectively. The representation (1) may simply be the very original model, such as a one-dimensional servo system, or may result from a global coordinate transformation of a nonlinear system with relative degree n [3]. Whatsoever, we'll develop our main results directly on it without considering its origination.

Given a reference trajectory $x^d(t)$, the adaptive control proposed in [8] can be applied to achieve asymptotic tracking stability. However, as stated, parameter convergence is not guaranteed unless the regressor is persistently excited, which so far can not be checked beforehand. We try to solve this problem in this paper. The adopted control, using the terminologies here, can be written explicitly as

$$u = \frac{1}{\hat{\beta}^T(t)g(x)} \cdot (-\hat{\alpha}^T(t)f(x) + v) \tag{2}$$

where $\hat{\alpha}(t), \hat{\beta}(t)$ are the estimated parameters at the time t and $v(t)$ is the extra control input given by

$$v(t) = \dot{x}_n^d - K^T e \tag{3}$$

with $e = [x_1 - x_1^d, \cdots, x_n - x_n^d]^T$ the tracking error vector and K the corresponding control gain vector. Clearly, to avoid the control in (2) from singularity, the denominator $\hat{\beta}^T(t)g(x)$ must be bounded away from zero for all time.

By substituting (2) and (3) into (1), it yields the following tracking error dynamics,

$$\dot{e}(t) = Ae(t) + B(\tilde{\theta}^T(t)\psi(t)) \tag{4}$$

where $\tilde{\theta}(t) = \hat{\theta}(t) - \theta = [\hat{\alpha}(t)^T, \hat{\beta}(t)^T]^T - [\alpha^T, \beta^T]^T$ is the parameter error vector, $B = [0, \cdots, 0, 1]^T$, and

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 \\ -K_1 & -K_2 & -K_3 & \cdots & -K_n \end{bmatrix},$$

$$\psi(\hat{\theta}(t), x(t), t) = \begin{bmatrix} -f(x(t)) \\ \frac{\hat{\alpha}^T(t)f(x(t)) - v(t)}{\hat{\beta}^T(t)g(x(t))} g(x) \end{bmatrix} \tag{5}$$

The corresponding parameter update law is

$$\dot{\hat{\theta}}(t) = -(e^T PB)\psi(t) \tag{6}$$

where the symmetric positive-definite matrix P comes from the following Lyapunov equation

$$A^T P + PA = -Q, \quad Q > 0. \tag{7}$$

By selecting the Lyapunov function $V(e, \tilde{\theta})$ to be

$$V(e, \tilde{\theta}) = 1/2(e^T Pe + \tilde{\theta}^T \tilde{\theta}) \tag{8}$$

and directly calculating its time derivative, after some manipulations, it yields

$$\dot{V}(e, \tilde{\theta}) \leq -\lambda_{\min}(Q)\|e\|^2 \tag{9}$$

where $\lambda_{\min}(Q)$ is the minimum eigenvalue of the matrix Q. Since the Lyapunov function V is nonincreasing and $e(t) \in L_2$, as can be seen from 9), it can be easily inferred that [8]

- P1) All the signals in the closed-loop system remain bounded $\forall t \geq 0$;
- P2) $e(t), \dot{\hat{\theta}}(t) \to 0$ as $t \to \infty$.

Based on P1)-P2) above, we're going to derive sufficient conditions for ensuring the PE of the regressor $\psi(t)$ in (5), which in turn guarantees the *exponential* stability of the overall closed-loop system. Details are given in the following.

III. Conditions for Persistent Excitation

The major difficulty of checking the fulfillment of PE is that the regressor in (5) depends not only on the tracking errors $e(t)$, but also on the estimated parameter vector $\hat{\theta}(t)$, which is more difficult to predict. Nevertheless, by properly using the properties P1)-P2) above and making some mild assumptions, it is found that such a criteria can actually be checked *a priori* in relatively easy ways established here.

For ease of reference, the definition of PE [7] is first quoted here.

Definition 1 A piecewise continuous signal vector $\phi : R^+ \mapsto R^n$ is PE in R^n with a level of excitation ϵ_0 if there exist constants $t_0, T_0 > 0$ such that

$$\int_{t_1}^{t_1+T_0} | \zeta^T \phi(\tau) | \, d\tau \geq \epsilon_0, \quad \forall t_1 \geq t_0 \tag{10}$$

where ζ is any a unit vector in R^n.

Before introducing the main results, the following assumptions are made.

- A1) The reference trajectory $x^d(t)$ is smooth and T-periodic;
- A2) The basis functions $f(x), g(x)$ are continuous in x;
- A3) The set of functions $f_i(x^d(t))g_j(x^d(t))$, $g_k(x^d(t))\dot{x}_n^d(t), i = 1, \cdots, r, j, k = 1, \cdots, s$ is linearly independent within the time period $[0, T]$;
- A4) A compact set $Q \in R^{n+r+s}$ exists, within which the overall states $[x(t), \hat{\theta}(t)]^T$ are confined and the term $\hat{\beta}^T(t)g(x)$ is bounded away from zero for all time.

Clearly, A4) is indispensable for the control (2) to avoid from singularity. Usually, some prior bounds on the parameter vector θ are sufficient to fulfill A4) [8]. To see this, let's first assume its sustenance and a prior bound S for θ, defined below, is known.

$$S \stackrel{\text{def}}{=} \{(\hat{\alpha}, \hat{\beta}) \mid \|\hat{\alpha} - \alpha_N\| \leq d_1, \|\hat{\beta} - \beta_N\| \leq d_2\} \tag{11}$$

with α_N, β_N the known nominal parameter vector. From (9), it is not hard to obtain that

$$\|\tilde{\beta}(t)\| \leq W,$$

and

$$\|e(t)\| \leq W/\sqrt{\lambda_{min}(P)}, \forall t \geq 0$$

where $W = (\lambda_{max}(P) \parallel e(0) \parallel^2 + d_1^2 + d_2^2)^{1/2}$. It implies that

$$\begin{aligned} \hat{\beta}(t) &\in S_1 \stackrel{\text{def}}{=} \{\hat{\beta} \mid \parallel \hat{\beta} - \beta_N \parallel \leq d_2 + W\}, \\ e(t) &\in S_2 \stackrel{\text{def}}{=} \{e \mid \parallel e \parallel \leq W/\sqrt{\lambda_{min}(P)}\}, \\ &\forall t \geq 0 \end{aligned} \tag{12}$$

Clearly, we can take $Q = S_1 \cup S_2$. If the term $\hat{\beta}^T(t)g(x)$ is really bounded away from zero for all time within Q, then A4) sustains. Thus, given any a reference trajectory and a specific system, fulfillment of A1)-A4) can be checked *a priori*.

After that, the estimated parameters are guaranteed to converge to their true values by the following theorem.

Theorem 1: Sustained A1)-A4), the whole closed-loop system, consisting of the tracking error dynamics in (4) and the update law in (6), is exponentially stable.

Proof: First, since the set $Q \in R^{n+r+s}$ in A4) is compact, by projecting it onto the parameter space, we then have a compact subset $\Omega \in R^{r+s}$, within which A4) is fulfilled. Define the class of vector functions $\bar{\psi}$ as

$$\bar{\psi}(\bar{\theta}, x^d(t), t) \triangleq \left[\begin{array}{c} -f(x^d(t)) \\ \frac{\bar{\alpha}^T f(x^d(t)) - \dot{x}_n^d}{\bar{\beta}^T g(x^d(t))} g(x^d) \end{array} \right] \quad (13)$$

where $\bar{\theta} = [\bar{\alpha}, \bar{\beta}]^T$ is a constant vector in Ω. Next, we'll show that A3) implies the linear independence of the component functions of $\bar{\psi}$ within $[0, T]$. To see this, suppose they are linearly dependent, then by definition, there exists some nonzero constant vector $[a^T \vdots b^T]^T \in R^{r+s}$ such that

$$\begin{aligned} & (\bar{\beta}^T g(x^d)) a^T f(x^d) + (\bar{\alpha}^T f(x^d) - \dot{x}_n^d) b^T g(x^d) \\ = & \sum_{j=1}^{s} \sum_{i=1}^{r} (\bar{\beta}_j a_i + \bar{\alpha}_i b_j) f_i(x^d) g_j(x^d) \\ & + \sum_{j=1}^{s} \dot{x}_n^d g_j(x^d) = 0 \end{aligned}$$

which contradicts A3).

The linear independent property obtained above, together with A2), implies that [1]

$$\int_0^T | \zeta^T \bar{\psi}(\bar{\theta}, x^d(t), t) | dt \geq \epsilon(\bar{\theta}), \quad \forall \bar{\theta} \in \Omega \quad (14)$$

where $\zeta \in R^{r+s}$ is a unit vector and $\epsilon(\bar{\theta})$ is some positive number depending on $\bar{\theta}$. Since the set Ω is compact, the minimum of all those $\epsilon(\bar{\theta}), \forall \bar{\theta} \in \Omega$, denoted by ϵ_m, is well-defined, i.e.,

$$\int_0^T | \zeta^T \bar{\psi}(\bar{\theta}, x^d(t), t) | dt \geq \epsilon_m > 0, \forall \bar{\theta} \in \Omega \quad (15)$$

By the periodicity of the integrand in (15), we have

$$\begin{aligned} & \int_t^{t+T} | \zeta^T \bar{\psi}(\bar{\theta}, x^d(\tau), \tau) | d\tau \geq \epsilon_m > 0, \\ & \forall \bar{\theta} \in \Omega, t \geq 0 \end{aligned} \quad (16)$$

which ensures the PE of the vector function $\bar{\psi}(\bar{\theta}, x^d(t), t)$. It is noted that the point $\bar{\theta}$ is fixed during the time period $[t, t+T]$, but not has to be so for all $t \geq 0$. Such a property is crucial to our subsequent proof.

Based on (16), we are now in a position to establish the PE property of the regressor ψ in (4). The main idea is to first decompose the integral (10) as the part (16) plus the rest terms. Next it'll be shown that the rest terms can be made arbitrarily small within the integration period in (10) as time becomes sufficiently large. Therefore, the integral in (10) will be ultimately dominated by (16) and hence the PE of the regressor can be inferred.

Denote the unit vector $\zeta = [\zeta_a \vdots \zeta_b]^T \in R^{r+s}$ and define $\Delta f(x)$ and $\Delta g(x)$ as

$$\Delta f(x) = f(x) - f(x^d), \quad \Delta g(x) = g(x) - g(x^d) \quad (17)$$

By using these notations and the triangular inequality, the lower bound ϵ_0 in (10) can be estimated as follows

$$\begin{aligned} & \int_t^{t+T} | \zeta^T \psi(\tau) | d\tau \\ = & \int_t^{t+T} | \zeta_a^T f(x(\tau)) + \zeta_b^T g(x(\tau)) \\ & \cdot \frac{\hat{\alpha}^T(\tau) f(x(\tau)) - v(\tau)}{\hat{\beta}^T(\tau) g(x(\tau))} | d\tau \\ \geq & \int_t^{t+T} | \zeta_a^T f(x^d(\tau)) + \zeta_b^T g(x^d(\tau)) \\ & \cdot \frac{\hat{\alpha}^T(\tau)(f(x^d(\tau)) + \Delta f(x(\tau))) - v(\tau)}{\hat{\beta}^T(\tau) g(x(\tau))} | d\tau \\ & - \int_t^{t+T} \{| \zeta_a^T (\Delta f(x(\tau))) | + | \zeta_b^T \Delta g(x(\tau)) \\ & \frac{\hat{\alpha}^T(\tau) f(x(\tau)) - v(\tau)}{\hat{\beta}^T(\tau) g(x(\tau))} |\} d\tau \\ \geq & \int_t^{t+T} | \zeta_a^T f(x^d(\tau)) + \zeta_b^T g(x^d(\tau)) \\ & \cdot \frac{\hat{\alpha}^T(\tau) f(x^d(\tau)) + \dot{x}_n^d(\tau)}{\hat{\beta}^T(\tau) g(x(\tau))} | d\tau \\ & - \int_t^{t+T} D_1(\tau) d\tau \end{aligned} \quad (18)$$

where

$$\begin{aligned} D_1(\tau) = & (1 + \| \frac{\zeta_b^T g(x^d(\tau))}{\hat{\beta}^T(\tau) g(x(\tau))} \hat{\alpha}(\tau) \|) \| \Delta f(\tau) \| \\ & + \| \frac{\zeta_b^T g(x^d(\tau))}{\hat{\beta}^T(\tau) g(x(\tau))} K \| \| e(\tau) \| \\ & + | \frac{\hat{\alpha}^T(\tau) f(x(\tau)) - v(\tau)}{\hat{\beta}^T(\tau) g(x(\tau))} | \| \Delta g(\tau) \| \end{aligned}$$

By substituting the expression

$$\hat{\alpha}(\tau) = \hat{\alpha}(t) + \int_t^{\tau} \dot{\hat{\alpha}}(\tau') d\tau', \quad \tau \in [t, t+T],$$

into (18), it yields

$$\begin{aligned} & \int_t^{t+T} | \zeta^T \psi(\tau) | d\tau \\ & \geq \int_t^{t+T} | \zeta_a^T f(x^d(\tau)) + \zeta_b^T g(x^d(\tau)) \\ & \cdot \frac{\hat{\alpha}^T(t) f(x^d(\tau)) + \dot{x}_n^d(\tau)}{\hat{\beta}^T(\tau) g(x(\tau))} | d\tau \\ & - \int_t^{t+T} \{ D_1(\tau) + | \frac{\zeta_b^T g(x^d(\tau))}{\hat{\beta}^T(\tau) g(x(\tau))} \\ & \cdot [\int_t^{\tau} \dot{\hat{\alpha}}(t') dt']^T f(x^d(\tau)) |\} d\tau \end{aligned}$$

$$
\begin{aligned}
&\geq \int_t^{t+T} \mid \zeta_a^T f(x^d(\tau)) + \zeta_b^T g(x^d(\tau)) \\
&\cdot \frac{\hat{\alpha}^T(t) f(x^d(\tau)) + \dot{x}_n^d(\tau)}{\hat{\beta}^T(t) g(x^d(\tau))} + D_2(\tau) \mid d\tau \\
&- \int_t^{t+T} D_1(\tau) d\tau - T C_M \|\alpha_M(t)\|
\end{aligned} \tag{19}
$$

where

$$
\begin{aligned}
C_M &= \max_{t' \geq 0} \| \frac{\zeta_b^T g(x^d(t'))}{\hat{\beta}^T(t') g(x(t'))} f(x^d(t')) \|, \\
\alpha_M(t) &= \max_{\tau} |\dot{\hat{\alpha}}(\tau)\|, \quad \forall \tau \in [t, t+T] \\
D_2(\tau) &= \zeta_b^T g(x^d(\tau))(\hat{\alpha}^T(t) f(x^d(\tau)) + \dot{x}_n^d(\tau)) \\
&\cdot (\frac{1}{\hat{\beta}^T(\tau) g(x(\tau))} - \frac{1}{\hat{\beta}^T(t) g(x^d(\tau))})
\end{aligned} \tag{20}
$$

Since all signals in the closed-loop system are bounded as guaranteed in P1) and $\hat{\beta}^T(t')g(x(t'))$ is bounded away from zero for all $t' \geq 0$ by assumption, C_M in (20) is therefore well defined. Moreover, from P2) it can be concluded that

$$
\lim_{t' \to \infty} \alpha_M(t') = 0, \quad \lim_{t' \to \infty} D_i(t') = 0, \quad i = 1, 2.
$$

Hence, given any a positive constant ϵ_1, by definition, there exist $t_i > 0, i = 1, 2, 3$, such that

$$
\begin{aligned}
D_1(t') &\leq \epsilon_1, \quad \forall t' \geq t_1 \\
\mid D_2(t') \mid &\leq \epsilon_1, \quad \forall t' \geq t_2 \\
\alpha_M(t') &\leq \epsilon_1, \quad \forall t' \geq t_3
\end{aligned}
$$

Let $t_0 = \max(t_1, t_2, t_3)$. The inequality (19) can be further reduced to

$$
\begin{aligned}
& \int_t^{t+T} \mid \zeta^T \psi(\tau) \mid d\tau \\
\geq & \int_t^{t+T} \mid \zeta_a^T f(x^d(\tau)) + \zeta_b^T g(x^d(\tau)) \\
& \cdot \frac{\hat{\alpha}^T(t) f(x^d(\tau)) + \dot{x}_n^d(\tau)}{\hat{\beta}^T(t) g(x^d(\tau))} \mid d\tau - (2 + T C_M)\epsilon_1 \\
= & \int_t^{t+T} \mid \zeta^T \psi(\hat{\theta}(t), x^d(\tau), \tau) \mid d\tau \\
& -(2 + T C_M)\epsilon_1 \\
= & \int_t^{t+T} \mid \zeta^T \bar{\psi}(\bar{\theta}, x^d(\tau), \tau) \mid d\tau \\
& -(2 + T C_M)\epsilon_1 \quad \forall t \geq t_0
\end{aligned} \tag{21}
$$

where $\bar{\theta} = \hat{\theta}(t)$ is a constant vector within Ω. From (16) and by selecting $\epsilon_1 = (4 + 2TC_M)^{-1}\epsilon_m$, it can finally be concluded that

$$
\begin{aligned}
\int_t^{t+T} \mid \zeta^T \psi(\tau) \mid d\tau &\geq \epsilon_m - \epsilon_m/2 \\
&\geq \epsilon_m/2, \quad \forall t \geq t_0
\end{aligned}
$$

which, by definition, implies the PE of the regressor ψ. ■

IV. Numerical Examples

To illustrate the main results, simulation of a second-order fictitious system is undertaken in this section. The dynamics of the simulated system is

$$
\begin{aligned}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= -\alpha_1 x_1^3 - \alpha_2 x_2 (x_1^2 - 1) \\
&\quad + \beta(1 + \sqrt{x_1^2 + x_2^2}) \cdot u,
\end{aligned} \tag{22}
$$

where $x = [x_1, x_2]^T$ is the system state vector; $\alpha_1, \alpha_2, \beta$ are the three unknown system parameters and u is the control. The reference trajectory is

$$
x_1^d(t) = A \sin(\omega t) \tag{23}
$$

The corresponding basis functions are

$$
\begin{aligned}
f_1(x) &= -x_1^3 \quad f_2(x) = -x_2(x_1^2 - 1) \\
g(x) &= 1 + \sqrt{x_1^2 + x_2^2}
\end{aligned}
$$

which apparently sustain A2). Since $g(x)$ is simply a scalar function in this case, therefore, the linear independence of the following three time functions within $[0, \pi]$, as can be easily verified, is sufficient for fulfilling A3).

$$
\begin{aligned}
\dot{x}_2^d(t) &= 8.0 \sin(2t) \quad f_1(x^d) = -8.0 \sin^3(2t) \\
f_2(x^d) &= -4 \cos(2t)(4 \sin^2(2t) - 1)
\end{aligned}
$$

The control in (2) for this application can be written as

$$
\begin{aligned}
u &= (\hat{\beta}(t) g(x))^{-1}(-\hat{\alpha}_1(t) f_1(x) - \hat{\alpha}_2(t) f_2(x) \\
&\quad + \ddot{x}_1{}^d - K_1 e_1 - K_2 e_2)
\end{aligned}
$$

The numerical values used in this simulation are: $\theta = [1.0, 0.8, 3.0]^T$, $\hat{\theta}(0) = [0.8, 0.7, 1.6]^T$, $e(0) = [0, 0]^T, K_1 = 1, K_2 = 4, A = 2.0, \omega = 2.0$. Assume that $\theta_N = 0.8\theta$ and $d_1 = 0.3\|\alpha_N\|$, $d_2 = 0.3\beta_N$. By substituting these numerical values in (12), we have $0.2971 \leq \hat{\beta}(t) \leq 3.12, \forall t \geq 0$. Hence, A4) is satisfied. Since A1)-A4) are fulfilled, the estimation errors are guaranteed to converge to zero, as depicted in Fig. 1.

V. Concluding Remarks

Verifiable sufficient conditions for parameter convergence of an adaptive fully linearizable system with unknown parameters, including those affine with the control input, have been derived. A numerical example showing how these conditions can be verified is given. The key to the success of the establishment is that all the state variables entering the regressor vector are ensured to track the reference trajectory asymptotically. This is analogous to the necessity of controllability of a linear system to be persistently excited [7]. Besides, full-state measurement is required. Attempting to reducing the sensors, however, will be very difficult since the regressor depends nonlinearly on the overall system states.

In respect to parameter identification, the results obtained here are superior to standard schemes in that it needs no filtering of the system states and its PE property can be easily verified. However, issues of robustness are not taken into account, which will be undertaken in the near future.

VI. ACKNOWLEDGMENTS

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REFERENCES

[1] C. T. Chen, *Introduction to Linear System Theory,* 2nd ed. New York: Holt, Rinehart and Winston, 1986.

[2] J. T. Huang, "Sufficient conditions for parameter convergence in linearizable systems," *IEEE Trans. Automat. Contr.*, vol. 48, pp. 878-890, May 2003.

[3] A. Isidori, *Nonlinear Control Systems, 2nd ed.* Berlin: Springer-Verlag, 1989.

[4] J. S. Lin and I. Kanellakopoulos, " Nonlinearities enhance parameter convergence in strict feedback systems," *IEEE Trans. Automat. Contr.*, vol. 44, pp. 89-94, Jan. 1999.

[5] A. P. Morgan and K. S. Narendra, "On the stability of nonautonomous differential equation $\dot{x} = [A + B(t)]x$ with skew-symmetric matrix $B(t)$," *SIAM J. Contr. Optimiz.*, vol. 15, pp. 163-176, Jan. 1977.

[6] K. S. Narendra and A. M. Annaswamy, "Persistent excitation in adaptive systems," *Int. J. Contr.*, vol. 45, pp. 127-160, 1987.

[7] —-, *Stable Adaptive Systems.* Englewood Cliffs, NJ: Prentice-Hall, 1989.

[8] S. Sastry and A. Isidori, "Adaptive control of linearizable systems," *IEEE Trans. Automat. Contr.*, vol. 34, pp. 1123-1131, Nov. 1989.

[9] S. Sastry, and M. Bodson, *Adaptive control: Stability, Convergence and Robustness.* Englewood Cliffs, NJ: Prentice-Hall, 1989.

[10] R. Marino and P. Tomei, *Nonlinear Control Design - Geometric, Adaptive and Robust.* London: Prentice-Hall, 1995.

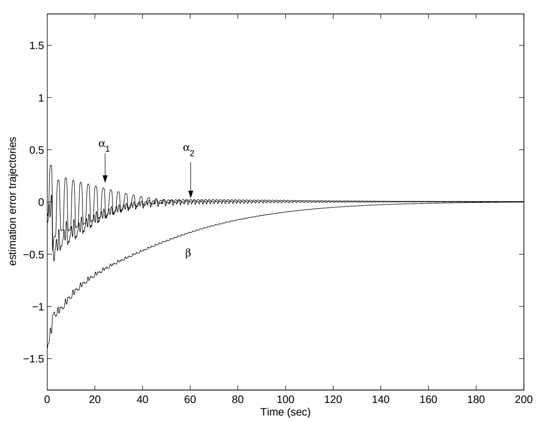

Fig. 1. Estimation errors vs. time