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Hybrid Direct Adaptive Stabilization for Nonlinear Uncertain Impulsive Dynamical Systems

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***Abstract*— A direct hybrid adaptive control framework for nonlinear uncertain hybrid dynamical systems is developed. The proposed hybrid adaptive control framework guarantees attraction of the closed-loop system states associated with the hybrid plant states in the face of parametric system uncertainty. A numerical example is provided to demonstrate the efficacy of the proposed approach.**

I. Introduction

The complexity of modern controlled uncertain nonlinear dynamical systems is often exacerbated by the use of hierarchical abstract decision-making units performing logical checks that identify system mode operation and specify a subcontroller within the feedback control architecture to be activated. These multiechelon systems are classified as *hybrid* systems (see [1], [2] and the numerous references therein) and involve an *interacting* countable collection of dynamical systems possessing a hierarchical structure characterized by continuous-time dynamics at the lower-level units and logical decision-making units at the higher-level of the hierarchy. The mathematical description of many of these systems can be characterized by impulsive differential equations [3], [4].

Even though adaptive control algorithms have been extensively developed in the literature for both continuous-time and discrete-time systems, hybrid adaptive control algorithms for hybrid dynamical systems are nonexistent. In this paper we develop a direct hybrid adaptive control framework for nonlinear uncertain impulsive dynamical systems. In particular, using the hybrid invariance principle given in [4], [5] a hybrid adaptive control framework is developed that guarantees attraction of the closed-loop system states associated with the hybrid plant dynamics. Furthermore, the remainder of the state associated with the hybrid adaptive controller gains is shown to be bounded. In the case where the nonlinear hybrid system is represented in a *hybrid normal form*, the nonlinear hybrid adaptive controllers are constructed *without* requiring knowledge of the hybrid system dynamics.

II. Mathematical Preliminaries

In this section we establish definitions, notation, and review some basic concepts on impulsive dynamical systems [3]–[5]. Let $\mathbb{R}$ denote the set of real numbers, $\mathbb{R}^n$ denote the set of $n \times 1$ real column vectors, $(\cdot)^{\mathrm{T}}$ denote transpose, $(\cdot)^\dagger$ denote the Moore-Penrose generalized inverse, $\lambda_{\min}(\cdot)$ denote the minimum eigenvalue of a Hermitian matrix, $\mathcal{N}$ denote the set of nonnegative integers, $\mathbb{N}^n$ (resp., $\mathbb{P}^n$) denote the set of $n \times n$ nonnegative (resp., positive) definite matrices, and let I_n denote the $n \times n$ identity matrix. Furthermore, we write $V'(x)$ for the Fréchet derivative of V at x and $\mathrm{dist}(p, \mathcal{M})$ for the smallest distance from a point p to any point in the set $\mathcal{M}$.

In this paper, we consider controlled *state-dependent* [4] impulsive dynamical systems $\mathcal{G}$ of the form

$$\dot{x}(t) = f_{\mathrm{c}}(x(t)) + G_{\mathrm{c}}(x(t))u_{\mathrm{c}}(t), \quad x(0) = x_0, \quad x(t) \notin \mathcal{Z}_x, \quad (1)$$

$$\Delta x(t) = f_{\mathrm{d}}(x(t)) + G_{\mathrm{d}}(x(t))u_{\mathrm{d}}(t), \quad x(t) \in \mathcal{Z}_x, \quad (2)$$

where $t \geq 0$, $x(t) \in \mathcal{D} \subseteq \mathbb{R}^n$, $\mathcal{D}$ is an open set with $0 \in \mathcal{D}$, $\Delta x(t) \triangleq x(t^+) - x(t)$, $u_{\mathrm{c}}(t) \in \mathcal{U}_{\mathrm{c}} \subseteq \mathbb{R}^{m_{\mathrm{c}}}$, $u_{\mathrm{d}}(t_k) \in \mathcal{U}_{\mathrm{d}} \subseteq \mathbb{R}^{m_{\mathrm{d}}}$, t_k denotes the kth instant of time at which $x(t)$ intersects $\mathcal{Z}_x$ for a particular trajectory $x(t)$, $f_{\mathrm{c}} : \mathcal{D} \to \mathbb{R}^n$ is Lipschitz continuous and satisfies $f_{\mathrm{c}}(0) = 0$, $G_{\mathrm{c}} : \mathcal{D} \to \mathbb{R}^{n \times m_{\mathrm{c}}}$, $f_{\mathrm{d}} : \mathcal{Z}_x \to \mathbb{R}^n$ is continuous, $G_{\mathrm{d}} : \mathcal{Z}_x \to \mathbb{R}^{n \times m_{\mathrm{d}}}$ is such that $\mathrm{rank}\, G_{\mathrm{d}}(x) = m_{\mathrm{d}}$, $x \in \mathcal{Z}_x$, and $\mathcal{Z}_x \subset \mathcal{D}$ is the *resetting set*. Here, we assume that $u_{\mathrm{c}}(\cdot)$ and $u_{\mathrm{d}}(\cdot)$ are restricted to the class of *admissible* inputs consisting of measurable functions such that $(u_{\mathrm{c}}(t), u_{\mathrm{d}}(t_k)) \in \mathcal{U}_{\mathrm{c}} \times \mathcal{U}_{\mathrm{d}}$ for all $t \geq 0$ and $k \in \mathcal{N}_{[0,t)} \triangleq \{k : 0 \leq t_k < t\}$, where the constrained set $\mathcal{U}_{\mathrm{c}} \times \mathcal{U}_{\mathrm{d}}$ is given with $(0, 0) \in \mathcal{U}_{\mathrm{c}} \times \mathcal{U}_{\mathrm{d}}$. We refer to the differential equation (1) as the *continuous-time dynamics*, and we refer to the difference equation (2) as the *resetting law*. In this paper we assume that Assumptions A1 and A2 established in [4] hold for all $u_{\mathrm{d}}(\cdot) \in \mathcal{U}_{\mathrm{d}}$; that is, the resetting set is such that resetting removes $x(t_k)$ from the resetting set and no trajectory can intersect the interior of $\mathcal{Z}_x$. Hence, as shown in [4], the resetting times are well defined and distinct. Since the resetting times are well defined and distinct and since the solution to (1) exists and is unique it follows that the solution of the impulsive dynamical system (1), (2) also exists and is unique over a forward time interval.

Next, we provide a key result from [4], [5] involving an invariant set stability theorem for hybrid dynamical systems. Specifically, consider the impulsive dynamical system (1), (2) with hybrid adaptive feedback controllers $u_{\mathrm{c}}(\cdot)$ and $u_{\mathrm{d}}(\cdot)$ so that the closed-loop hybrid system $\tilde{\mathcal{G}}$ has the form

$$\dot{\tilde{x}}(t) = \tilde{f}_{\mathrm{c}}(\tilde{x}(t)), \quad \tilde{x}(0) = \tilde{x}_0, \quad \tilde{x}(t) \notin \mathcal{Z}_{\tilde{x}}, \quad (3)$$

$$\Delta \tilde{x}(t) = \tilde{f}_{\mathrm{d}}(\tilde{x}(t)), \quad \tilde{x}(t) \in \mathcal{Z}_{\tilde{x}}, \quad (4)$$

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where $t \geq 0$, $\tilde{x}(t) \in \tilde{\mathcal{D}} \subseteq \mathbb{R}^{\tilde{n}}$, $\tilde{x}(t)$ denotes the closed-loop state involving the system state and the adaptive gains, $\tilde{f}_{\rm c} : \tilde{\mathcal{D}} \to \mathbb{R}^{\tilde{n}}$ and $\tilde{f}_{\rm d} : \tilde{\mathcal{D}} \to \mathbb{R}^{\tilde{n}}$ denote the closed-loop continuous-time and resetting dynamics, respectively, with $\tilde{f}_{\rm c}(\tilde{x}_{\rm e}) = 0$, where $\tilde{x}_{\rm e} \in \tilde{\mathcal{D}}\backslash\mathcal{Z}_x$ denotes the closed-loop equilibrium point, and $\tilde{n}$ denotes the dimension of the closed-loop system state. For the statement of the next result the following key assumption is needed.

Assumption 2.1 ([4], [5]): Let $s(t, \tilde{x}_0)$, $t \geq 0$, denote the solution of (3), (4) with initial condition $\tilde{x}_0 \in \tilde{\mathcal{D}}$. Then for every $\tilde{x}_0 \in \tilde{\mathcal{D}}$, there exists a dense subset $\mathcal{T}_{\tilde{x}_0} \subseteq [0,\infty)$ such that $[0,\infty)\backslash\mathcal{T}_{\tilde{x}_0}$ is (finitely or infinitely) countable and for every $\epsilon > 0$ and $t \in \mathcal{T}_{\tilde{x}_0}$, there exists $\delta(\epsilon, \tilde{x}_0, t) > 0$ such that if $\|\tilde{x}_0 - y\| < \delta(\epsilon, \tilde{x}_0, t)$, $y \in \tilde{\mathcal{D}}$, then $\|s(t, \tilde{x}_0) - s(t, y)\| < \epsilon$.

Assumption 2.1 is a generalization of the standard continuous dependence property for dynamical systems with continuous flows to dynamical systems with left-continuous flows. Specifically, by letting $\mathcal{T}_{\tilde{x}_0} = \overline{\mathcal{T}}_{\tilde{x}_0} = [0,\infty)$, where $\overline{\mathcal{T}}_{\tilde{x}_0}$ denotes the closure of the set $\mathcal{T}_{\tilde{x}_0}$, Assumption 2.1 specializes to the classical continuous dependence of solutions of a given dynamical system with respect to the system's initial conditions $\tilde{x}_0 \in \tilde{\mathcal{D}}$ [6]. Since solutions of impulsive dynamical systems are *not* continuous in time and are *not* continuous functions of the system initial conditions, Assumption 2.1 is needed to apply the hybrid invariance principle developed in [4], [5] to hybrid adaptive systems. Henceforth, we assume that the hybrid adaptive feedback controllers $u_{\rm c}(\cdot)$ and $u_{\rm d}(\cdot)$ are such that closed-loop hybrid system (3), (4) satisfies Assumption 2.1. Necessary and sufficient conditions that guarantee that the nonlinear impulsive dynamical system $\tilde{\mathcal{G}}$ satisfies Assumption 2.1 are given in [5]. A sufficient condition that guarantees that the trajectories of the closed-loop nonlinear impulsive dynamical system (3), (4) satisfy Assumption 2.1 are Lipschitz continuity of $\tilde{f}_{\rm c}(\cdot)$ and the existence of a continuously differentiable function $\mathcal{X} : \tilde{\mathcal{D}} \to \mathbb{R}$ such that the resetting set is given by $\mathcal{Z}_{\tilde{x}} = \{\tilde{x} \in \tilde{\mathcal{D}} : \mathcal{X}(\tilde{x}) = 0\}$, where $\mathcal{X}'(\tilde{x}) \neq 0$, $\tilde{x} \in \mathcal{Z}_{\tilde{x}}$, and $\mathcal{X}'(\tilde{x})\tilde{f}_{\rm c}(\tilde{x}) \neq 0$, $\tilde{x} \in \mathcal{Z}_{\tilde{x}}$. The last condition above ensures that the solution of the closed-loop hybrid system is not tangent to the resetting set $\mathcal{Z}_{\tilde{x}}$ for all initial conditions $\tilde{x}_0 \in \tilde{\mathcal{D}}$. For further discussion on Assumption 2.1 see [4], [5].

The following theorem proven in [4], [5] is needed to develop the main results of this paper.

Theorem 2.1 ([4], [5]): Consider the nonlinear impulsive dynamical system $\tilde{\mathcal{G}}$ given by (3), (4), assume $\tilde{\mathcal{D}}_{\rm c} \subset \tilde{\mathcal{D}}$ is a compact positively invariant set with respect to (3), (4), and assume that there exists a continuously differentiable function $V : \tilde{\mathcal{D}}_{\rm c} \to \mathbb{R}$ such that

$$\begin{aligned} V'(\tilde{x})\tilde{f}_{\rm c}(\tilde{x}) &\leq 0, \quad \tilde{x} \in \tilde{\mathcal{D}}_{\rm c}, \quad \tilde{x} \notin \mathcal{Z}_{\tilde{x}}, \quad (5)\\ V(\tilde{x} + \tilde{f}_{\rm d}(\tilde{x})) &\leq V(\tilde{x}), \quad \tilde{x} \in \tilde{\mathcal{D}}_{\rm c}, \quad \tilde{x} \in \mathcal{Z}_{\tilde{x}}. \quad (6)\end{aligned}$$

Let $\mathcal{R} \triangleq \{\tilde{x} \in \tilde{\mathcal{D}}_{\rm c} : \tilde{x} \notin \mathcal{Z}_{\tilde{x}}, V'(\tilde{x})\tilde{f}_{\rm c}(\tilde{x}) = 0\} \cup \{\tilde{x} \in \tilde{\mathcal{D}}_{\rm c} : \tilde{x} \in \mathcal{Z}_{\tilde{x}}, V(\tilde{x} + \tilde{f}_{\rm d}(\tilde{x})) = V(\tilde{x})\}$ and let $\mathcal{M}$ denote the largest invariant set contained in $\mathcal{R}$. If $\tilde{x}_0 \in \tilde{\mathcal{D}}_{\rm c}$, then $\tilde{x}(t) \to \mathcal{M}$ as $t \to \infty$. Finally, if $\tilde{\mathcal{D}} = \mathbb{R}^{\tilde{n}}$ and $V(\tilde{x}) \to \infty$ as $\|\tilde{x}\| \to \infty$, then $\tilde{x}(t) \to \mathcal{M}$ as $t \to \infty$ for all $\tilde{x}_0 \in \mathbb{R}^{\tilde{n}}$.

III. Hybrid Adaptive Controllers for Nonlinear Hybrid Dynamical Systems

In this section we consider the problem of characterizing hybrid adaptive controllers for nonlinear uncertain hybrid systems. Specifically, we consider the controlled state-dependent impulsive dynamical system (1), (2) with $\mathcal{D} = \mathbb{R}^n$, $\mathcal{U}_{\rm c} = \mathbb{R}^{m_{\rm c}}$, and $\mathcal{U}_{\rm d} = \mathbb{R}^{m_{\rm d}}$.

Theorem 3.1: Consider the nonlinear uncertain hybrid dynamical system $\mathcal{G}$ given by (1), (2). Assume there exist a matrix $K_{\rm cg} \in \mathbb{R}^{m_{\rm c}\times s_{\rm c}}$, a continuously differentiable function $V_{\rm s} : \mathbb{R}^n \to \mathbb{R}$, and continuous functions $\hat{G}_{\rm c} : \mathbb{R}^n \to \mathbb{R}^{m_{\rm c}\times m_{\rm c}}$, $F_{\rm c} : \mathbb{R}^n \to \mathbb{R}^{s_{\rm c}}$, and $\ell_{\rm c} : \mathbb{R}^n \to \mathbb{R}^{p_{\rm c}}$ such that $V_{\rm s}(\cdot)$ is positive definite, radially unbounded, $V_{\rm s}(0) = 0$, $\ell_{\rm c}(0) = 0$, $F_{\rm c}(0) = 0$, and, for all $x \in \mathbb{R}^n\backslash\mathcal{Z}_x$,

$$0 = V_{\rm s}'(x)f_{\rm cs}(x) + \ell_{\rm c}^{\rm T}(x)\ell_{\rm c}(x), \quad (7)$$

where

$$f_{\rm cs}(x) \triangleq f_{\rm c}(x) + G_{\rm c}(x)\hat{G}_{\rm c}(x)K_{\rm cg}F_{\rm c}(x). \quad (8)$$

Furthermore, assume there exist a matrix $K_{\rm dg} \in \mathbb{R}^{m_{\rm d}\times s_{\rm d}}$ and continuous functions $\hat{G}_{\rm d} : \mathcal{Z}_x \to \mathbb{R}^{m_{\rm d}\times m_{\rm d}}$ and $F_{\rm d} : \mathcal{Z}_x \to \mathbb{R}^{s_{\rm d}}$ such that $\hat{G}_{\rm d}(x)$, $x \in \mathcal{Z}_x$, is invertible and, for all $x \in \mathcal{Z}_x$,

$$0 > V_{\rm s}(x + f_{\rm ds}(x)) - V_{\rm s}(x), \quad (9)$$

where

$$f_{\rm ds}(x) \triangleq f_{\rm d}(x) + G_{\rm d}(x)\hat{G}_{\rm d}(x)K_{\rm dg}F_{\rm d}(x). \quad (10)$$

Finally, let $c > 0$, $Q_{\rm c} \in \mathbb{P}^{m_{\rm c}}$, $Q_{\rm d} \in \mathbb{P}^{m_{\rm d}}$, $Y \in \mathbb{P}^{s_{\rm c}}$, and $\lambda_{\max}(Q_{\rm d}) < 2$. Then the hybrid adaptive feedback control law

$$\begin{aligned} u_{\rm c}(t) &= \hat{G}_{\rm c}(x(t))K_{\rm c}(t)F_{\rm c}(x(t)), \quad x(t) \notin \mathcal{Z}_x, \quad (11)\\ u_{\rm d}(t) &= \hat{G}_{\rm d}(x(t))K_{\rm d}(t)F_{\rm d}(x(t)), \quad x(t) \in \mathcal{Z}_x, \quad (12)\end{aligned}$$

where $K_{\rm c}(t) \in \mathbb{R}^{m_{\rm c}\times s_{\rm c}}$, $t \geq 0$, and $K_{\rm d}(t) \in \mathbb{R}^{m_{\rm d}\times s_{\rm d}}$, $t \geq 0$, with update laws

$$\begin{aligned} &\dot{K}_{\rm c}(t)\\ &\quad = -\tfrac{1}{2}Q_{\rm c}\hat{G}_{\rm c}^{\rm T}(x(t))G_{\rm c}^{\rm T}(x(t))V_{\rm s}'^{\rm T}(x(t))F_{\rm c}^{\rm T}(x(t))Y,\\ &\qquad\qquad K_{\rm c}(0) = K_{\rm c0}, \quad x(t) \notin \mathcal{Z}_x, \quad (13)\\ &\Delta K_{\rm c}(t) = 0, \quad x(t) \in \mathcal{Z}_x, \quad (14)\\ &\dot{K}_{\rm d}(t) = 0, \quad K_{\rm d}(0) = K_{\rm d0}, \quad x(t) \notin \mathcal{Z}_x, \quad (15)\\ &\Delta K_{\rm d}(t) = -\tfrac{1}{c+F_{\rm d}^{\rm T}(x(t))F_{\rm d}(x(t))}Q_{\rm d}\hat{G}_{\rm d}^{-1}(x(t))G_{\rm d}^{\dagger}(x(t))\\ &\qquad\cdot[\Delta x(t) - f_{\rm ds}(x(t))]F_{\rm d}^{\rm T}(x(t)), \quad x(t) \in \mathcal{Z}_x, \quad (16)\end{aligned}$$

where $\Delta K_{\rm c}(t) \triangleq K_{\rm c}(t^+) - K_{\rm c}(t)$ and $\Delta K_{\rm d}(t) \triangleq K_{\rm d}(t^+) - K_{\rm d}(t)$, guarantees that the solution $(x(t), K_{\rm c}(t), K_{\rm d}(t))$, $t \geq 0$, of the closed-loop hybrid system given by (1), (2), (11)–(16) satisfies $\ell_{\rm c}(x(t)) \to 0$ as $t \to 0$ for all $x_0 \in \mathbb{R}^n$. If, in addition, $\ell_{\rm c}^{\rm T}(x)\ell_{\rm c}(x) > 0$, $x \in \mathbb{R}^n\backslash\mathcal{Z}_x$, $x \neq 0$, then $x(t) \to 0$ as $t \to \infty$ for all $x_0 \in \mathbb{R}^n$.

Proof. First, define $\tilde{K}_{\rm d}(t) \triangleq K_{\rm d}(t) - K_{\rm dg}$ and $\tilde{w}(t) \triangleq G_{\rm d}(x(t))\hat{G}_{\rm d}(x(t))\tilde{K}_{\rm d}(t)F_{\rm d}(x(t))$. Note that with $u_{\rm c}(t)$, $t \geq 0$, and $u_{\rm d}(t_k)$, $k \in \mathcal{N}$, given by (11) and (12), respectively, it follows that the closed-loop hybrid system (1), (2) is given by

$$\begin{aligned} \dot{x}(t) &= f_{\rm c}(x(t)) + G_{\rm c}(x(t))\hat{G}_{\rm c}(x(t))K_{\rm c}(t)F_{\rm c}(x(t)),\\ &\qquad x(0) = x_0, \quad x(t) \notin \mathcal{Z}_x, \quad (17)\\ \Delta x(t) &= f_{\rm d}(x(t)) + G_{\rm d}(x(t))\hat{G}_{\rm d}(x(t))K_{\rm d}(t)F_{\rm d}(x(t)),\\ &\qquad x(t) \in \mathcal{Z}_x, \quad (18)\end{aligned}$$

or, equivalently, using (8) and (10),

$$\begin{aligned}\dot{x}(t) &= f_{\mathrm{cs}}(x(t)) + G_{\mathrm{c}}(x(t))\hat{G}_{\mathrm{c}}(x(t))(K_{\mathrm{c}}(t) - K_{\mathrm{cg}}) \\ &\quad \cdot F_{\mathrm{c}}(x(t)), \quad x(0) = x_0, \quad x(t) \notin \mathcal{Z}_x, \quad (19)\\ \Delta x(t) &= f_{\mathrm{ds}}(x(t)) + G_{\mathrm{d}}(x(t))\hat{G}_{\mathrm{d}}(x(t))(K_{\mathrm{d}}(t) - K_{\mathrm{dg}}) \\ &\quad \cdot F_{\mathrm{d}}(x(t)) \\ &= f_{\mathrm{ds}}(x(t)) + \tilde{w}(t), \quad x(t) \in \mathcal{Z}_x. \quad (20)\end{aligned}$$

Furthermore, note that adding and subtracting K_{dg} to and from (16) and using (20) it follows that

$$\begin{aligned}\tilde{K}_{\mathrm{d}}(t^+) &= \tilde{K}_{\mathrm{d}}(t) \\ &\quad - \tfrac{1}{c + F_{\mathrm{d}}^{\mathrm{T}}(x(t))F_{\mathrm{d}}(x(t))} Q_{\mathrm{d}} \hat{G}_{\mathrm{d}}^{-1}(x(t)) G_{\mathrm{d}}^{\dagger}(x(t)) \\ &\quad \cdot [G_{\mathrm{d}}(x(t))\hat{G}_{\mathrm{d}}(x(t))\tilde{K}_{\mathrm{d}}(t)F_{\mathrm{d}}(x(t))] F_{\mathrm{d}}^{\mathrm{T}}(x(t)) \\ &= \tilde{K}_{\mathrm{d}}(t) - \tfrac{1}{c + F_{\mathrm{d}}^{\mathrm{T}}(x(t))F_{\mathrm{d}}(x(t))} Q_{\mathrm{d}} \tilde{K}_{\mathrm{d}}(t) F_{\mathrm{d}}(x(t)) \\ &\quad \cdot F_{\mathrm{d}}^{\mathrm{T}}(x(t)), \quad x(t) \in \mathcal{Z}_x. \quad (21)\end{aligned}$$

To show convergence of the plant states for the closed-loop hybrid system (13)–(15) and (19)–(21) consider the Lyapunov-like function

$$\begin{aligned}V(x, K_{\mathrm{c}}, K_{\mathrm{d}}) &= V_{\mathrm{s}}(x) \\ &\quad + \mathrm{tr}\, Q_{\mathrm{c}}^{-1}(K_{\mathrm{c}} - K_{\mathrm{cg}}) Y^{-1} (K_{\mathrm{c}} - K_{\mathrm{cg}})^{\mathrm{T}} \\ &\quad + \mathrm{tr} (K_{\mathrm{d}} - K_{\mathrm{dg}})^{\mathrm{T}} Q_{\mathrm{d}}^{-1} (K_{\mathrm{d}} - K_{\mathrm{dg}}). \quad (22)\end{aligned}$$

Note that $V(0, K_{\mathrm{cg}}, K_{\mathrm{dg}}) = 0$ and, since $V_{\mathrm{s}}(\cdot)$, Q_{c}, Q_{d}, and Y are positive definite, $V(x, K_{\mathrm{c}}, K_{\mathrm{d}}) > 0$ for all $(x, K_{\mathrm{c}}, K_{\mathrm{d}}) \neq (0, K_{\mathrm{cg}}, K_{\mathrm{dg}})$. In addition, $V(x, K_{\mathrm{c}}, K_{\mathrm{d}})$ is radially unbounded. Now, using (7), (13), and (15), it follows that the time derivative of $V(x, K_{\mathrm{c}}, K_{\mathrm{d}})$ along the closed-loop system trajectories over the time interval $t \in (t_k, t_{k+1}]$, $k \in \mathcal{N}$, is given by

$$\begin{aligned}&\dot{V}(x(t), K_{\mathrm{c}}(t), K_{\mathrm{d}}(t)) \\ &= V_{\mathrm{s}}'(x(t)) \Big[f_{\mathrm{cs}}(x(t)) + G_{\mathrm{c}}(x(t))\hat{G}_{\mathrm{c}}(x(t))(K_{\mathrm{c}}(t) - K_{\mathrm{cg}}) \\ &\quad \cdot F_{\mathrm{c}}(x(t)) \Big] + 2\mathrm{tr} Q_{\mathrm{c}}^{-1}(K_{\mathrm{c}}(t) - K_{\mathrm{cg}}) Y^{-1} \dot{K}_{\mathrm{c}}^{\mathrm{T}}(t) \\ &= -\ell_{\mathrm{c}}^{\mathrm{T}}(x(t))\ell_{\mathrm{c}}(x(t)) \\ &\quad + \mathrm{tr}\Big[(K_{\mathrm{c}}(t) - K_{\mathrm{cg}})F_{\mathrm{c}}(x(t))V_{\mathrm{s}}'(x(t))G_{\mathrm{c}}(x(t))\hat{G}_{\mathrm{c}}(x(t))\Big] \\ &\quad - \mathrm{tr}\Big[(K_{\mathrm{c}}(t) - K_{\mathrm{cg}})F_{\mathrm{c}}(x(t))V_{\mathrm{s}}'(x(t))G_{\mathrm{c}}(x(t))\hat{G}_{\mathrm{c}}(x(t))\Big] \\ &= -\ell_{\mathrm{c}}^{\mathrm{T}}(x(t))\ell_{\mathrm{c}}(x(t)) \\ &\leq 0, \quad t_k < t \leq t_{k+1}. \quad (23)\end{aligned}$$

Now, suppose there exists $k_{\max} > 0$ such that $k \leq k_{\max}$; that is, the closed-loop system trajectory $x(t)$, $t \geq 0$, intersects the resetting set $\mathcal{Z}_x$ a finite number of times. In this case, the closed-loop hybrid system possesses a continuous flow for all $t > t_{k_{\max}}$ and hence it follows from Theorem 2 of [7] that $\ell_{\mathrm{c}}(x(t)) \to 0$ as $t \to \infty$. If, in addition, $\ell_{\mathrm{c}}^{\mathrm{T}}(x)\ell_{\mathrm{c}}(x) > 0$, $x \in \mathbb{R}^n \backslash \mathcal{Z}_x$, $x \neq 0$, then $x(t) \to 0$ as $t \to \infty$ for all $x_0 \in \mathbb{R}^n$. Alternatively, suppose a trajectory $x(t)$, $t \geq 0$, intersects the resetting set $\mathcal{Z}_x$ infinitely many times. In this case, consider the partial Lyapunov-like function

$$V_{K_{\mathrm{d}}}(K_{\mathrm{d}}) = \mathrm{tr}(K_{\mathrm{d}} - K_{\mathrm{dg}})^{\mathrm{T}} Q_{\mathrm{d}}^{-1} (K_{\mathrm{d}} - K_{\mathrm{dg}}). \quad (24)$$

Note that since Q_{d} is positive definite, $V_{K_{\mathrm{d}}}(K_{\mathrm{d}}) > 0$, $K_{\mathrm{d}} \in \mathbb{R}^{m_{\mathrm{d}} \times s_{\mathrm{d}}}$, $K_{\mathrm{d}} \neq K_{\mathrm{dg}}$. Now, using (21), the difference of $V_{K_{\mathrm{d}}}(K_{\mathrm{d}})$ along the closed-loop system trajectories at the resetting times t_k, $k \in \mathcal{N}$, is given by

$$\begin{aligned}&\Delta V_{K_{\mathrm{d}}}(x(t_k), K_{\mathrm{d}}(t_k)) \\ &\triangleq V_{K_{\mathrm{d}}}(x(t_k^+), K_{\mathrm{d}}(t_k^+)) - V_{K_{\mathrm{d}}}(x(t_k), K_{\mathrm{d}}(t_k)) \\ &= \mathrm{tr}\Big(\tilde{K}_{\mathrm{d}}(t_k) - \tfrac{1}{c + F_{\mathrm{d}}^{\mathrm{T}}(x(t_k))F_{\mathrm{d}}(x(t_k))} Q_{\mathrm{d}} \tilde{K}_{\mathrm{d}}(t_k) F_{\mathrm{d}}(x(t_k)) \\ &\quad \cdot F_{\mathrm{d}}^{\mathrm{T}}(x(t_k))\Big)^{\mathrm{T}} Q_{\mathrm{d}}^{-1} \Big(\tilde{K}_{\mathrm{d}}(t_k) - \tfrac{1}{c + F_{\mathrm{d}}^{\mathrm{T}}(x(t_k))F_{\mathrm{d}}(x(t_k))} Q_{\mathrm{d}} \\ &\quad \cdot \tilde{K}_{\mathrm{d}}(t_k) F_{\mathrm{d}}(x(t_k)) F_{\mathrm{d}}^{\mathrm{T}}(x(t_k))\Big) - \mathrm{tr}\, \tilde{K}_{\mathrm{d}}^{\mathrm{T}}(t_k) Q_{\mathrm{d}}^{-1} \tilde{K}_{\mathrm{d}}(t_k) \\ &= \mathrm{tr}\, \tilde{K}_{\mathrm{d}}^{\mathrm{T}}(t_k) Q_{\mathrm{d}}^{-1} \tilde{K}_{\mathrm{d}}(t_k) - \tfrac{2}{c + F_{\mathrm{d}}^{\mathrm{T}}(x(t_k))F_{\mathrm{d}}(x(t_k))} \mathrm{tr}\Big[\tilde{K}_{\mathrm{d}}^{\mathrm{T}}(t_k) \\ &\quad \cdot \tilde{K}_{\mathrm{d}}(t_k) F_{\mathrm{d}}(x(t_k)) F_{\mathrm{d}}^{\mathrm{T}}(x(t_k))\Big] + \tfrac{1}{(c + F_{\mathrm{d}}^{\mathrm{T}}(x(t_k))F_{\mathrm{d}}(x(t_k)))^2} \\ &\quad \cdot \mathrm{tr}\, F_{\mathrm{d}}(x(t_k)) F_{\mathrm{d}}^{\mathrm{T}}(x(t_k)) \tilde{K}_{\mathrm{d}}^{\mathrm{T}}(t_k) Q_{\mathrm{d}} \tilde{K}_{\mathrm{d}}(t_k) \\ &\quad \cdot F_{\mathrm{d}}(x(t_k)) F_{\mathrm{d}}^{\mathrm{T}}(x(t_k)) - \mathrm{tr}\, \tilde{K}_{\mathrm{d}}^{\mathrm{T}}(t_k) Q_{\mathrm{d}}^{-1} \tilde{K}_{\mathrm{d}}(t_k) \\ &\leq -\tfrac{1}{c + F_{\mathrm{d}}^{\mathrm{T}}(x(t_k))F_{\mathrm{d}}(x(t_k))} F_{\mathrm{d}}^{\mathrm{T}}(x(t_k)) \tilde{K}_{\mathrm{d}}^{\mathrm{T}}(t_k)(2I_{m_{\mathrm{d}}} - Q_{\mathrm{d}}) \\ &\quad \cdot \tilde{K}_{\mathrm{d}}(t_k) F_{\mathrm{d}}(x(t_k)) \\ &\leq 0, \quad k \in \mathcal{N}, \quad (25)\end{aligned}$$

where in (25) we used $\frac{F_{\mathrm{d}}^{\mathrm{T}}(x)F_{\mathrm{d}}(x)}{c + F_{\mathrm{d}}^{\mathrm{T}}(x)F_{\mathrm{d}}(x)} < 1$ and $2I_{m_{\mathrm{d}}} - Q_{\mathrm{d}} > 0$, since by assumption $\lambda_{\max}(Q_{\mathrm{d}}) < 2$. Hence, $V_{K_{\mathrm{d}}}(x(t_k), K(t_k))$, $k \in \mathcal{N}$, is a nonincreasing and bounded function of k. Thus, it follows from the monotone convergence theorem (see Theorem 8.6 of [8]) that $\lim_{k \to \infty} V_{K_{\mathrm{d}}}(x(t_k), K_{\mathrm{d}}(t_k))$ exists which implies that $\Delta V_{K_{\mathrm{d}}}(x(t_k), K_{\mathrm{d}}(t_k)) \to 0$ as $k \to \infty$. Now, it follows from (25) that $\tilde{K}_{\mathrm{d}}(t_k) F_{\mathrm{d}}(x(t_k)) \to 0$ as $k \to \infty$ and hence $\tilde{w}(t_k) \to 0$ as $k \to \infty$. Next, to show that $x(t) \to 0$ as $t \to \infty$, note that, since $\tilde{w}(t_k) \to 0$ as $k \to \infty$, there exists $k^* \geq 0$ such that for all $k \geq k^*$,

$$0 \geq V_{\mathrm{s}}(x(t_k) + f_{\mathrm{ds}}(x(t_k)) + \tilde{w}(t_k)) - V_{\mathrm{s}}(x(t_k)) \quad (26)$$

holds and hence there exist $\hat{\mathcal{Z}}_x \subset \mathcal{Z}_x$ and $\mathcal{K}_{\mathrm{d}} \subset \mathbb{R}^{m_{\mathrm{d}} \times s_{\mathrm{d}}}$ such that

$$\begin{aligned}0 &\geq V_{\mathrm{s}}(x + f_{\mathrm{ds}}(x) + G_{\mathrm{d}}(x)\hat{G}_{\mathrm{d}}(x)\tilde{K}_{\mathrm{d}} F_{\mathrm{d}}(x)) - V_{\mathrm{s}}(x), \\ &\qquad (x, K_{\mathrm{d}}) \in \hat{\mathcal{Z}}_x \times \mathcal{K}_{\mathrm{d}} \subset \mathcal{Z}_x \times \mathbb{R}^{m_{\mathrm{d}} \times s_{\mathrm{d}}}, \quad (27)\end{aligned}$$

and $\mathrm{dist}(x(t_k), \hat{\mathcal{Z}}_x) \to 0$ as $k \to \infty$ and $\mathrm{dist}(\tilde{K}_{\mathrm{d}}(t_k), \mathcal{K}_{\mathrm{d}}) \to 0$ as $k \to \infty$. Hence, it follows that the difference of $V(x, K_{\mathrm{c}}, K_{\mathrm{d}})$ along the closed-loop system trajectories at the resetting times t_k, $k \geq k^*$, is given by

$$\begin{aligned}&\Delta V(x(t_k), K_{\mathrm{c}}(t_k), K_{\mathrm{d}}(t_k)) \\ &\quad \triangleq V(x(t_k^+), K_{\mathrm{c}}(t_k^+), K_{\mathrm{d}}(t_k^+)) \\ &\qquad - V(x(t_k), K_{\mathrm{c}}(t_k), K_{\mathrm{d}}(t_k)) \\ &\quad = V_{\mathrm{s}}(x(t_k) + f_{\mathrm{ds}}(x(t_k)) + \tilde{w}(t_k)) - V_{\mathrm{s}}(x(t_k)) \\ &\qquad + \Delta V_{K_{\mathrm{d}}}(x(t_k), K_{\mathrm{d}}(t_k)) \\ &\quad \leq 0, \quad k \geq k^*. \quad (28)\end{aligned}$$

Next, for $t \geq t_{k^*}$, define the translated closed-loop hybrid system

$$\begin{aligned}\dot{\hat{x}}(\tau) &= f_{\mathrm{c}}(\hat{x}(\tau)) + G_{\mathrm{c}}(\hat{x}(\tau))\hat{G}_{\mathrm{c}}(\hat{x}(\tau))\hat{K}_{\mathrm{c}}(\tau)F_{\mathrm{c}}(\hat{x}(\tau)), \\ &\qquad \hat{x}(0) = x(t_{k^*}^+), \ \hat{x}(\tau) \notin \mathcal{Z}_x, \quad (29)\\ \Delta \hat{x}(\tau) &= f_{\mathrm{d}}(\hat{x}(\tau)) + G_{\mathrm{d}}(\hat{x}(\tau))\hat{G}_{\mathrm{d}}(\hat{x}(\tau))\hat{K}_{\mathrm{d}}(\tau)F_{\mathrm{d}}(\hat{x}(\tau)), \\ &\qquad \hat{x}(\tau) \in \mathcal{Z}_x, \quad (30)\end{aligned}$$

$$\dot{\hat{K}}_{\rm c}(\tau) = -\tfrac{1}{2}Q_{\rm c}\hat{G}_{\rm c}^{\rm T}(\hat{x}(\tau))G_{\rm c}^{\rm T}(\hat{x}(\tau))V_{\rm s}'^{\rm T}(\hat{x}(\tau)) \cdot F_{\rm c}^{\rm T}(\hat{x}(\tau))Y,\ \hat{K}_{\rm c}(0) = K_{\rm c}(t_{k^*}^+),\ \hat{x}(\tau) \notin \mathcal{Z}_x, \quad (31)$$

$$\Delta\hat{K}_{\rm c}(\tau) = 0, \quad \hat{x}(\tau) \in \mathcal{Z}_x, \quad (32)$$

$$\dot{\hat{K}}_{\rm d}(\tau) = 0, \quad \hat{K}_{\rm d}(0) = K_{\rm d}(t_{k^*}^+), \quad \hat{x}(\tau) \notin \mathcal{Z}_x, \quad (33)$$

$$\Delta\hat{K}_{\rm d}(\tau) = -\frac{1}{c+F_{\rm d}^{\rm T}(\hat{x}(\tau))F_{\rm d}(\hat{x}(\tau))}Q_{\rm d}\hat{G}_{\rm d}^{-1}(\hat{x}(\tau))G_{\rm d}^{\dagger}(\hat{x}(\tau)) \cdot[\Delta\hat{x}(\tau) - f_{\rm ds}(\hat{x}(\tau))]F_{\rm d}^{\rm T}(\hat{x}(\tau)),\ \hat{x}(\tau) \in \mathcal{Z}_x, \quad (34)$$

where $\tau \triangleq t - t_{k^*} \geq 0$, $\hat{x}(\tau) \triangleq x(t - t_{k^*})$, $\hat{K}_{\rm c}(\tau) \triangleq K_{\rm c}(t - t_{k^*})$, and $\hat{K}_{\rm d}(\tau) \triangleq K_{\rm d}(t - t_{k^*})$. Furthermore, define $\mathcal{R}_{\rm c} \triangleq \{(\hat{x}, \hat{K}_{\rm c}, \hat{K}_{\rm d}) \in \mathbb{R}^n \times \mathbb{R}^{m_{\rm c}\times s_{\rm c}} \times \mathbb{R}^{m_{\rm d}\times s_{\rm d}} : \hat{x} \notin \mathcal{Z}_x, \dot{V}(\hat{x}, \hat{K}_{\rm c}, \hat{K}_{\rm d}) = 0\} = \{(\hat{x}, \hat{K}_{\rm c}, \hat{K}_{\rm d}) \in \mathbb{R}^n \times \mathbb{R}^{m_{\rm c}\times s_{\rm c}} \times \mathbb{R}^{m_{\rm d}\times s_{\rm d}} : \hat{x} \notin \mathcal{Z}_x, \ell_{\rm c}^{\rm T}(\hat{x})\ell_{\rm c}(\hat{x}) = 0\}$ and $\mathcal{R}_{\rm d} \triangleq \{(\hat{x}, \hat{K}_{\rm c}, \hat{K}_{\rm d}) \in \mathbb{R}^n \times \mathbb{R}^{m_{\rm c}\times s_{\rm c}} \times \mathbb{R}^{m_{\rm d}\times s_{\rm d}} : \hat{x} \in \mathcal{Z}_x, \Delta V(\hat{x}, \hat{K}_{\rm c}, \hat{K}_{\rm d}) = 0\}$. Now, let $\mathcal{M}$ denote the largest invariant set contained in $\mathcal{R} \triangleq \mathcal{R}_{\rm c} \cup \mathcal{R}_{\rm d}$ and note that since $\tilde{w}(t_k) \to 0$ as $k \to \infty$ it follows that for $(\hat{x}, \hat{K}_{\rm c}, \hat{K}_{\rm d}) \in \mathcal{M} \cap (\hat{\mathcal{Z}}_x \times \mathbb{R}^{m_{\rm c}\times s_{\rm c}} \times \mathcal{K}_{\rm d})$, $G_{\rm d}(\hat{x})\hat{G}_{\rm d}(\hat{x})\tilde{K}_{\rm d}F_{\rm d}(\hat{x}) = 0$, $\tilde{K}_{\rm d}F_{\rm d}(\hat{x}) = 0$, and $V_{\rm s}(\hat{x} + f_{\rm ds}(\hat{x})) - V_{\rm s}(\hat{x}) = 0$. However, since (27) holds for all $x \in \mathcal{Z}_x$, $\mathcal{M} = \mathcal{R}_{\rm c} \cup \emptyset$ and hence it follows from Theorem 2.1 that the solution $(\hat{x}(\tau), \hat{K}_{\rm c}(\tau), \hat{K}_{\rm d}(\tau))$, $\tau \geq 0$, to (29)–(34) satisfies $\ell_{\rm c}(\hat{x}(\tau)) \to 0$ as $\tau \to \infty$ and hence $\ell_{\rm c}(x(t)) \to 0$ as $t \to \infty$. Furthermore, if $\ell_{\rm c}^{\rm T}(x)\ell_{\rm c}(x) > 0$, $x \in \mathbb{R}^n\backslash\mathcal{Z}_x$, $x \neq 0$, then $x(t) \to 0$ as $t \to \infty$ for all $x_0 \in \mathbb{R}^n$. □

Remark 3.1: Note that in the case where $\ell_{\rm c}^{\rm T}(x)\ell_{\rm c}(x) > 0$, $x \in \mathbb{R}^n\backslash\mathcal{Z}_x$, $x \neq 0$, the conditions in Theorem 3.1 imply that $x(t) \to 0$ as $t \to \infty$ and hence it follows from (13) that $(x(t), K_{\rm c}(t), K_{\rm d}(t)) \to \mathcal{M} \triangleq \{(x, K_{\rm c}, K_{\rm d}) \in \mathbb{R}^n \times \mathbb{R}^{m_{\rm c}\times s_{\rm c}} \times \mathbb{R}^{m_{\rm d}\times s_{\rm d}} : x = 0, \dot{K}_{\rm c} = 0\}$ as $t \to \infty$. Furthermore, if $x(t)$, $t \geq 0$, intersects $\mathcal{Z}_x$ infinitely many times, then $(x(t), K_{\rm c}(t), K_{\rm d}(t)) \to \mathcal{M} \triangleq \{(x, K_{\rm c}, K_{\rm d}) \in \mathbb{R}^n \times \mathbb{R}^{m_{\rm c}\times s_{\rm c}} \times \mathbb{R}^{m_{\rm d}\times s_{\rm d}} : x = 0, \dot{K}_{\rm c} = 0, K_{\rm d}(t^+) = K_{\rm d}(t)\}$ as $t \to \infty$.

Remark 3.2: In the case where $u_{\rm d}(t) \equiv 0$, Condition (9) can be replaced by

$$0 \geq V_{\rm s}(x + f_{\rm d}(x)) - V_{\rm s}(x). \quad (35)$$

Furthermore, taking $F_{\rm d}(x) = 0$, $x \in \mathcal{Z}_x$, and $K_{\rm d}(t) \equiv 0$, (26) holds for all $k \in \mathcal{N}$. In this case, since $V(x(t), K_{\rm c}(t), K_{\rm d}(t))$ is nonincreasing for all $t \geq 0$, $V(x, K_{\rm c}, K_{\rm d})$ is a Lyapunov function and hence the closed-loop hybrid system (13)–(15) and (19)–(21) is Lyapunov stable and $x(t) \to 0$ as $t \to \infty$. For further details see [9].

It is important to note that the hybrid adaptive control law (11)–(16) does *not* require explicit knowledge of the gain matrices $K_{\rm cg}$ and $K_{\rm dg}$; even though Theorem 3.1 requires the existence of $K_{\rm cg}$, $K_{\rm dg}$, $F_{\rm c}(x)$, $F_{\rm d}(x)$, $\hat{G}_{\rm c}(x)$, $\hat{G}_{\rm d}(x)$, and $V_{\rm s}(x)$ such that (7) and (9) hold. Furthermore, no specific structure on the nonlinear dynamics $f_{\rm c}(x)$ and $f_{\rm d}(x)$ is required to apply Theorem 3.1. However, if (1) and (2) are such that

$$f_{\rm c}(x) = \tilde{A}_{\rm c}x + \tilde{f}_{\rm cu}(x), \quad G_{\rm c}(x) = \begin{bmatrix} 0_{(n-m_{\rm c})\times m_{\rm c}} \\ G_{\rm cs}(x) \end{bmatrix}, \quad (36)$$

$$f_{\rm d}(x) = (\tilde{A}_{\rm d} - I_n)x + \tilde{f}_{\rm du}(x), G_{\rm d}(x) = \begin{bmatrix} 0_{(n-m_{\rm d})\times m_{\rm d}} \\ G_{\rm ds}(x) \end{bmatrix}, \quad (37)$$

where

$$\tilde{A}_{\rm c} = \begin{bmatrix} A_{\rm c0} \\ 0_{m_{\rm c}\times n} \end{bmatrix}, \quad \tilde{f}_{\rm cu}(x) = \begin{bmatrix} 0_{(n-m_{\rm c})\times 1} \\ f_{\rm cu}(x) \end{bmatrix}, \quad (38)$$

$$\tilde{A}_{\rm d} = \begin{bmatrix} A_{\rm d0} \\ 0_{m_{\rm d}\times n} \end{bmatrix}, \quad \tilde{f}_{\rm du}(x) = \begin{bmatrix} 0_{(n-m_{\rm d})\times 1} \\ f_{\rm du}(x) \end{bmatrix}, \quad (39)$$

$A_{\rm c0} \in \mathbb{R}^{(n-m_{\rm c})\times n}$ and $A_{\rm d0} \in \mathbb{R}^{(n-m_{\rm d})\times n}$ are known matrices of zeros and ones capturing a multivariable controllable canonical form representation [10], $f_{\rm cu} : \mathbb{R}^n \to \mathbb{R}^{m_{\rm c}}$ and $f_{\rm du} : \mathbb{R}^n \to \mathbb{R}^{m_{\rm d}}$ are unknown functions, $G_{\rm cs} : \mathbb{R}^n \to \mathbb{R}^{m_{\rm c}\times m_{\rm c}}$, and $G_{\rm ds} : \mathbb{R}^n \to \mathbb{R}^{m_{\rm d}\times m_{\rm d}}$, then we can always construct functions $F_{\rm c} : \mathbb{R}^n \to \mathbb{R}^{s_{\rm c}}$ and $F_{\rm d} : \mathbb{R}^n \to \mathbb{R}^{s_{\rm d}}$, with $F_{\rm c}(0) = 0$, such that the zero solution $x(t) \equiv 0$ to (8) and (10) is globally asymptotically stable *without* requiring knowledge of the hybrid system dynamics. To see this assume that $f_{\rm cu}(x)$ and $f_{\rm du}(x)$ are unknown and are parameterized as $f_{\rm cu}(x) = \Theta_{\rm c}f_{\rm cn}(x)$ and $f_{\rm du}(x) = \Theta_{\rm d}f_{\rm dn}(x)$, where $f_{\rm cn} : \mathbb{R}^n \to \mathbb{R}^{q_{\rm c}}$ and $f_{\rm dn} : \mathbb{R}^n \to \mathbb{R}^{q_{\rm d}}$ with $f_{\rm cn}(0) = 0$, and $\Theta_{\rm c} \in \mathbb{R}^{m_{\rm c}\times q_{\rm c}}$ and $\Theta_{\rm d} \in \mathbb{R}^{m_{\rm d}\times q_{\rm d}}$ are matrices of uncertain constant parameters.

Next, to apply Theorem 3.1 to the uncertain nonlinear hybrid system (1) and (2) with $f_{\rm c}(x)$, $f_{\rm d}(x)$, $G_{\rm c}(x)$, and $G_{\rm d}(x)$ given by (36) and (37), let $K_{\rm cg} \in \mathbb{R}^{m_{\rm c}\times s_{\rm c}}$ and $K_{\rm dg} \in \mathbb{R}^{m_{\rm d}\times s_{\rm d}}$, where $s_{\rm c} = q_{\rm c} + r_{\rm c}$ and $s_{\rm d} = q_{\rm d} + r_{\rm d}$, be given by

$$K_{\rm cg} = [\,\Theta_{\rm cn} - \Theta_{\rm c},\ \Phi_{\rm cn}\,], \quad K_{\rm dg} = [\,\Theta_{\rm dn} - \Theta_{\rm d},\ \Phi_{\rm dn}\,], \quad (40)$$

where $\Theta_{\rm cn} \in \mathbb{R}^{m_{\rm c}\times q_{\rm c}}$, $\Theta_{\rm dn} \in \mathbb{R}^{m_{\rm d}\times q_{\rm d}}$, $\Phi_{\rm cn} \in \mathbb{R}^{m_{\rm c}\times r_{\rm c}}$, and $\Phi_{\rm dn} \in \mathbb{R}^{m_{\rm d}\times r_{\rm d}}$ are known matrices, and let

$$F_{\rm c}(x) = \begin{bmatrix} f_{\rm cn}(x) \\ \hat{f}_{\rm cn}(x) \end{bmatrix}, \quad F_{\rm d}(x) = \begin{bmatrix} f_{\rm dn}(x) \\ \hat{f}_{\rm dn}(x) \end{bmatrix}, \quad (41)$$

where $\hat{f}_{\rm cn} : \mathbb{R}^n \to \mathbb{R}^{r_{\rm c}}$ and $\hat{f}_{\rm dn} : \mathbb{R}^n \to \mathbb{R}^{r_{\rm d}}$, satisfying $\hat{f}_{\rm cn}(0) = 0$, are arbitrary functions. In this case, it follows that, with $\hat{G}_{\rm c}(x) = G_{\rm cs}^{-1}(x)$ and $\hat{G}_{\rm d}(x) = G_{\rm ds}^{-1}(x)$,

$$\begin{aligned} f_{\rm cs}(x) &= f_{\rm c}(x) + G_{\rm c}(x)\hat{G}_{\rm c}(x)K_{\rm cg}F_{\rm c}(x) \\ &= \tilde{A}_{\rm c}x + \begin{bmatrix} 0_{(n-m_{\rm c})\times 1} \\ \Theta_{\rm cn}f_{\rm cn}(x) + \Phi_{\rm cn}\hat{f}_{\rm cn}(x) \end{bmatrix} \end{aligned} \quad (42)$$

and

$$\begin{aligned} f_{\rm ds}(x) &= f_{\rm d}(x) + G_{\rm d}(x)\hat{G}_{\rm d}(x)K_{\rm dg}F_{\rm d}(x) \\ &= (\tilde{A}_{\rm d} - I_n)x + \begin{bmatrix} 0_{(n-m_{\rm d})\times 1} \\ \Theta_{\rm dn}f_{\rm dn}(x) + \Phi_{\rm dn}\hat{f}_{\rm dn}(x) \end{bmatrix}. \end{aligned} \quad (43)$$

Now, since $\Theta_{\rm cn} \in \mathbb{R}^{m_{\rm c}\times q_{\rm c}}$, $\Theta_{\rm dn} \in \mathbb{R}^{m_{\rm d}\times q_{\rm d}}$, $\Phi_{\rm cn} \in \mathbb{R}^{m_{\rm c}\times r_{\rm c}}$, and $\Phi_{\rm dn} \in \mathbb{R}^{m_{\rm d}\times r_{\rm d}}$ are arbitrary constant matrices and $\hat{f}_{\rm cn} : \mathbb{R}^n \to \mathbb{R}^{r_{\rm c}}$ and $\hat{f}_{\rm dn} : \mathbb{R}^n \to \mathbb{R}^{r_{\rm d}}$ are arbitrary functions we can always construct $K_{\rm cg}$, $K_{\rm dg}$, $F_{\rm c}(x)$, and $F_{\rm d}(x)$ without knowledge of $f_{\rm c}(x)$ and $f_{\rm d}(x)$ such that (7) and (9) are satisfied. In particular, choosing $\Theta_{\rm cn}f_{\rm cn}(x) + \Phi_{\rm cn}\hat{f}_{\rm cn}(x) = \hat{A}_{\rm c}x$ and $\Theta_{\rm dn}f_{\rm dn}(x) + \Phi_{\rm dn}\hat{f}_{\rm dn}(x) = \hat{A}_{\rm d}x$, where $\hat{A}_{\rm c} \in \mathbb{R}^{m_{\rm c}\times n}$ and $\hat{A}_{\rm d} \in \mathbb{R}^{m_{\rm d}\times n}$, it follows that (42) and (43) have the form $f_{\rm cs}(x) = A_{\rm c}x$ and $f_{\rm ds}(x) = (A_{\rm d} - I_n)x$, respectively, where $A_{\rm c} = \left[A_0^{\rm T}, \hat{A}_{\rm c}^{\rm T}\right]^{\rm T}$ and $A_{\rm d} = \left[A_0^{\rm T}, \hat{A}_{\rm d}^{\rm T}\right]^{\rm T}$ are in multivariable controllable canonical form. Hence, we can choose $\hat{A}_{\rm c}$ and $\hat{A}_{\rm d}$ such that $A_{\rm c}$ is Hurwitz and $A_{\rm d}$ is Schur. Now, it follows from standard converse Lyapunov theory that there exists a positive-definite matrix P satisfying the Lyapunov equation

$$0 = A_{\rm c}^{\rm T} P + P A_{\rm c} + R_{\rm c}, \qquad (44)$$

where $R_{\rm c}$ is positive definite. If, in addition, P satisfies

$$0 = A_{\rm d}^{\rm T} P A_{\rm d} - P + R_{\rm d}, \qquad (45)$$

where $R_{\rm d}$ is positive definite, then (9) holds with $V_{\rm s}(x) = x^{\rm T} P x$. Hence, the hybrid adaptive feedback controller (11) and (12) with update laws (13), or, equivalently,

$$\dot{K}_{\rm c}(t) = -Q_{\rm c} \hat{G}_{\rm c}^{\rm T}(x(t)) G_{\rm c}^{\rm T}(x(t)) P x(t) F_{\rm c}^{\rm T}(x(t)) Y, \quad (46)$$

and (14)–(16) guarantees global attraction of the *nonlinear* hybrid uncertain dynamical system (1) and (2) where $f_{\rm c}(x)$, $f_{\rm d}(x)$, $G_{\rm c}(x)$, and $G_{\rm d}(x)$ are given by (36) and (37). Note that since $R_{\rm c}$ and $R_{\rm d}$ are arbitrary, (44) and (45) can be cast as a linear matrix inequality feasibility problem involving $P > 0$, $A_{\rm c}^{\rm T} P + P A_{\rm c} < 0$, and $A_{\rm d}^{\rm T} P A_{\rm d} - P < 0$. Finally, as mentioned above, it is important to note that it is not necessary to utilize a feedback linearizing function $F_{\rm c}(x)$ and $F_{\rm d}(x)$ to produce a linear $f_{\rm cs}(x)$ and $f_{\rm ds}(x)$. However, as shown above, when the hybrid system is in a *hybrid normal form* given by (36), (37), the feedback linearizing functions $F_{\rm c}(x)$ and $F_{\rm d}(x)$ provide considerable simplification in constructing $V_{\rm s}(x)$ necessary in computing the hybrid update law (13).

Note that by choosing $\Theta_{\rm dn} = \Phi_{\rm dn} = 0$ considerable simplification occurs in the update law (16). Specifically, in this case it follows that

$$\begin{aligned} G_{\rm d}^{\dagger}(x) f_{\rm ds}(x) &= \begin{bmatrix} 0_{m\times(n-m)}, & G_{\rm ds}^{-1}(x) \end{bmatrix} \begin{bmatrix} A_0 \\ 0_{m\times n} \end{bmatrix} x \\ &= 0, \end{aligned} \qquad (47)$$

and hence the update law (16) can be simplified as

$$\begin{aligned} \Delta K_{\rm d}(t) &= \tfrac{1}{c + F_{\rm d}^{\rm T}(x(t)) F_{\rm d}(x(t))} Q_{\rm d} \hat{G}_{\rm d}^{-1}(x(t)) G_{\rm d}^{\dagger}(x(t)) \\ &\quad \cdot \Delta x(t) F_{\rm d}^{\rm T}(x(t)), \quad x(t) \in \mathcal{Z}_x. \end{aligned} \qquad (48)$$

Next, we consider the case where $f_{\rm c}(x)$, $f_{\rm d}(x)$, $G_{\rm c}(x)$, and $G_{\rm d}(x)$ are uncertain. Specifically, we assume that $G_{\rm c}(x)$ and $G_{\rm d}(x)$ are such that $G_{\rm cs}(x)$ and $G_{\rm ds}(x)$ are unknown and are parameterized as $G_{\rm cs}(x) = B_{\rm cu} G_{\rm cn}(x)$ and $G_{\rm ds}(x) = B_{\rm du} G_{\rm dn}(x)$, where $G_{\rm cn} : \mathbb{R}^n \to \mathbb{R}^{m_{\rm c}\times m_{\rm c}}$ and $G_{\rm dn} : \mathbb{R}^n \to \mathbb{R}^{m_{\rm d}\times m_{\rm d}}$ are known and satisfy $\det G_{\rm cn}(x) \neq 0$, $x \in \mathbb{R}^n \backslash \mathcal{Z}_x$, $\det G_{\rm dn}(x) \neq 0$, $x \in \mathcal{Z}_x$, and $B_{\rm cu} \in \mathbb{R}^{m_{\rm c}\times m_{\rm c}}$ and $B_{\rm du} \in \mathbb{R}^{m_{\rm d}\times m_{\rm d}}$, with $\det B_{\rm cu} \neq 0$ and $\det B_{\rm du} \neq 0$, are unknown symmetric sign-definite matrices but a bound α for the maximum singular value of $B_{\rm du}$ is known and the sign definiteness of $B_{\rm cu}$ and $B_{\rm du}$ are known. For the statement of the next result define $B_{\rm c0} \triangleq \left[0_{m_{\rm c}\times(n-m_{\rm c})}, I_{m_{\rm c}}\right]^{\rm T}$ for $B_{\rm cu} > 0$, $B_{\rm c0} \triangleq \left[0_{m_{\rm c}\times(n-m_{\rm c})}, -I_{m_{\rm c}}\right]^{\rm T}$ for $B_{\rm cu} < 0$, $B_{\rm d0} \triangleq \left[0_{m_{\rm d}\times(n-m_{\rm d})}, I_{m_{\rm d}}\right]^{\rm T}$ for $B_{\rm du} > 0$, and $B_{\rm d0} \triangleq \left[0_{m_{\rm d}\times(n-m_{\rm d})}, -I_{m_{\rm d}}\right]^{\rm T}$ for $B_{\rm du} < 0$.

Corollary 3.1: Consider the nonlinear uncertain hybrid dynamical system $\mathcal{G}$ given by (1) and (2) with $f_{\rm c}(x)$, $f_{\rm d}(x)$, $G_{\rm c}(x)$, and $G_{\rm d}(x)$ given by (36), (37), and $G_{\rm cs}(x) = B_{\rm cu} G_{\rm cn}(x)$ and $G_{\rm ds}(x) = B_{\rm du} G_{\rm dn}(x)$, where $B_{\rm cu} \in \mathbb{R}^{m_{\rm c}\times m_{\rm c}}$ and $B_{\rm du} \in \mathbb{R}^{m_{\rm d}\times m_{\rm d}}$ are unknown symmetric matrices and the sign definiteness of $B_{\rm cu}$ and $B_{\rm du}$ are known and $\sigma_{\max}(B_{\rm du}) < \alpha$, $\alpha > 0$. Assume there exist a matrix $K_{\rm cg} \in \mathbb{R}^{m_{\rm c}\times s_{\rm c}}$, a continuously differentiable function $V_{\rm s} : \mathbb{R}^n \to \mathbb{R}$, and continuous functions $\hat{G}_{\rm c} : \mathbb{R}^n \to \mathbb{R}^{m_{\rm c}\times m_{\rm c}}$, $F_{\rm c} : \mathbb{R}^n \to \mathbb{R}^{s_{\rm c}}$, and $\ell_{\rm c} : \mathbb{R}^n \to \mathbb{R}^{p_{\rm c}}$ such that $V_{\rm s}(\cdot)$ is positive definite, radially unbounded, $V_{\rm s}(0) = 0$, $\ell_{\rm c}(0) = 0$, $F_{\rm c}(0) = 0$, and, for all $x \in \mathbb{R}^n \backslash \mathcal{Z}_x$, (7) holds with $f_{\rm cs}(x)$ given by (8). Furthermore, assume that there exist a matrix $K_{\rm dg} \in \mathbb{R}^{m_{\rm d}\times s_{\rm d}}$ and continuous functions $\hat{G}_{\rm d} : \mathcal{Z}_x \to \mathbb{R}^{m_{\rm d}\times m_{\rm d}}$ and $F_{\rm d} : \mathcal{Z}_x \to \mathbb{R}^{s_{\rm d}}$ such that $\hat{G}_{\rm d}(x)$, $x \in \mathcal{Z}_x$, is invertible and, for all $x \in \mathcal{Z}_x$, (9) holds with $f_{\rm ds}(x)$ given by (10). Finally, let $c > 0$ and $Y \in \mathbb{P}^{s_{\rm c}}$. Then the hybrid adaptive feedback control law

$$u_{\rm c}(t) = G_{\rm cn}^{-1}(x(t)) K_{\rm c}(t) F_{\rm c}(x(t)), \quad x(t) \notin \mathcal{Z}_x, \qquad (49)$$

$$u_{\rm d}(t) = \hat{\alpha}^{-1} G_{\rm dn}^{-1}(x(t)) K_{\rm d}(k) F_{\rm d}(x(t)), \quad x(t) \in \mathcal{Z}_x, \quad (50)$$

where $K_{\rm c}(t) \in \mathbb{R}^{m\times s_{\rm c}}$, $t \geq 0$, $K_{\rm d}(t) \in \mathbb{R}^{m\times s_{\rm d}}$, $t \geq 0$, and $\hat{\alpha} \geq \alpha/2$, with update laws

$$\begin{aligned} \dot{K}_{\rm c}(t) &= -B_{\rm c0}^{\rm T} V_{\rm s}'^{\rm T}(x(t)) F_{\rm c}^{\rm T}(x(t)) Y, \\ &\qquad K_{\rm c}(0) = K_{\rm c0}, \quad x(t) \notin \mathcal{Z}_x, & (51) \\ \Delta K_{\rm c}(t) &= 0, \quad x(t) \in \mathcal{Z}_x, & (52) \\ \dot{K}_{\rm d}(t) &= 0, \quad K_{\rm d}(0) = K_{\rm d0}, \quad x(t) \notin \mathcal{Z}_x, & (53) \\ \Delta K_{\rm d}(t) &= -\tfrac{1}{c + F_{\rm d}^{\rm T}(x(t)) F_{\rm d}(x(t))} B_{\rm d0}^{\rm T} [\Delta x(t) - f_{\rm ds}(x(t))] \\ &\qquad \cdot F_{\rm d}^{\rm T}(x(t)), \quad x(t) \in \mathcal{Z}_x, & (54) \end{aligned}$$

guarantees that the solution $(x(t), K_{\rm c}(t), K_{\rm d}(t))$ of the closed-loop hybrid system given by (1), (2), (49)–(54) satisfies $\ell_{\rm c}(x(t)) \to 0$ as $t \to \infty$. If, in addition, $\ell_{\rm c}^{\rm T}(x)\ell_{\rm c}(x) > 0$, $x \in \mathbb{R}^n \backslash \mathcal{Z}_x$, $x \neq 0$, then $x(t) \to 0$ as $t \to \infty$ for all $x_0 \in \mathbb{R}^n$.

Proof. The result is a direct consequence of Theorem 3.1. First, let $\hat{G}_{\rm c}(x) = G_{\rm cn}^{-1}(x)$ and $\hat{G}_{\rm d}(x) = \hat{\alpha}^{-1} G_{\rm dn}^{-1}(x)$ so that $G_{\rm c}(x)\hat{G}_{\rm cn}(x) = [0_{m\times(n-m)}, B_{\rm cu}]^{\rm T}$ and $G_{\rm d}(x)\hat{G}_{\rm dn}(x) = [0_{m\times(n-m)}, \hat{\alpha}^{-1} B_{\rm du}]^{\rm T}$, and let $K_{\rm cg} = B_{\rm cu}^{-1}[\Theta_{\rm cn} - \Theta_{\rm c}, \Phi_{\rm cn}]$ and $K_{\rm dg} = \hat{\alpha} B_{\rm du}^{-1}[\Theta_{\rm dn} - \Theta_{\rm d}, \Phi_{\rm dn}]$. Next, since $Q_{\rm c}$ and $Q_{\rm d}$ are arbitrary positive-definite matrices with $\lambda_{\max}(Q_{\rm d}) < 2$, $Q_{\rm c}$ in (13) and $Q_{\rm d}$ in (16) can be replaced by $q_{\rm c}|B_{\rm cu}|^{-1}$ and $\hat{\alpha}^{-1}|B_{\rm du}|^{-1}$, respectively, where $q_{\rm c}$ is a positive constant, $|B_{\rm cu}| = (B_{\rm cu}^2)^{\frac{1}{2}}$, and $|B_{\rm du}| = (B_{\rm du}^2)^{\frac{1}{2}}$, where $(\cdot)^{\frac{1}{2}}$ denotes the (unique) positive-definite square root. Now, since $B_{\rm cu}$ and $B_{\rm du}$ are symmetric and sign definite it follows from the Schur decomposition that $B_{\rm cu} = U_{\rm c} D_{B_{\rm cu}} U_{\rm c}^{\rm T}$ and $B_{\rm du} = U_{\rm d} D_{B_{\rm du}} U_{\rm d}^{\rm T}$, where $U_{\rm c}$ and $U_{\rm d}$ are orthogonal and $D_{B_{\rm cu}}$ and $D_{B_{\rm du}}$ are real diagonal. Hence, $|B_{\rm cu}|^{-1} \hat{G}_{\rm c}^{\rm T}(x) G_{\rm c}^{\rm T}(x) = [0_{m_{\rm c}\times(n-m_{\rm c})}, \mathcal{I}_{m_{\rm c}}] = B_{\rm c0}^{\rm T}$ and $\hat{\alpha}^{-1}|B_{\rm du}|^{-1} \hat{G}_{\rm d}^{\rm T}(x) G_{\rm d}^{\rm T}(x) = [0_{m_{\rm d}\times(n-m_{\rm d})}, \mathcal{I}_{m_{\rm d}}] = B_{\rm d0}^{\rm T}$, where $\mathcal{I}_{m_{\rm c}} = I_{m_{\rm c}}$ for $B_{\rm cu} > 0$, $\mathcal{I}_{m_{\rm c}} = -I_{m_{\rm c}}$ for $B_{\rm cu} < 0$, $\mathcal{I}_{m_{\rm d}} = I_{m_{\rm d}}$ for $B_{\rm du} > 0$, and $\mathcal{I}_{m_{\rm d}} = I_{m_{\rm d}}$ for $B_{\rm du} < 0$. Now, (13) and (16) imply (51) and (54), respectively. □

IV. Illustrative Numerical Example

In this section we present a numerical example to demonstrate the utility of the proposed hybrid adaptive control framework for hybrid adaptive stabilization. Specifically, consider the nonlinear uncertain controlled hybrid system given by (1), (2) with $n = 2$, $x = [x_1, x_2]^{\mathrm{T}}$,

$$\begin{aligned} f_{\mathrm{c}}(x) &= \begin{bmatrix} x_2 \\ -\beta x_1 - \mu(x_1^2 - \alpha)x_2 \end{bmatrix}, \quad G_{\mathrm{c}}(x) = \begin{bmatrix} 0 \\ b_{\mathrm{c}} \end{bmatrix}, \\ f_{\mathrm{d}}(x) &= \begin{bmatrix} -x_1 + x_2 \\ -x_2 - a_1 x_1^2 - a_2 \frac{x_2^3}{1+x_2^2} - a_3 x_2^3 \end{bmatrix}, \\ G_{\mathrm{d}}(x) &= [0,\, b_{\mathrm{d}}]^{\mathrm{T}}, \end{aligned}$$

where $\mu, \alpha, \beta, a_1, a_2, a_3, b_{\mathrm{c}}, b_{\mathrm{d}} \in \mathbb{R}$ are unknown. Furthermore, we assume that the resetting set $\mathcal{Z}_x$ is given by

$$\mathcal{Z}_x = \{x \in \mathbb{R}^2 : \mathcal{X}(x) = 0,\, x_2 > 0\}, \tag{55}$$

where $\mathcal{X} : \mathbb{R}^2 \to \mathbb{R}$ is a continuously differentiable function given by $\mathcal{X}(x) = x_1$. It can be easily verified that the resetting set $\mathcal{Z}_x$ satisfies Assumptions A1 and A2 given in [4]. Furthermore, $\mathcal{X}'(x) \neq 0, x \in \mathcal{Z}_x$, and for the closed-loop hybrid system corresponding to the continuous-time dynamics given by (1) and (11), $\mathcal{X}'(x)\dot{x} = x_2 \neq 0, x \in \mathcal{Z}_x$, and hence the closed-loop hybrid system satisfies Assumption 2.1. Here, we assume that $f_{\mathrm{c}}(x)$ and $f_{\mathrm{d}}(x)$ are unknown and can be parameterized as $f_{\mathrm{c}}(x) = [x_2,\, \theta_{\mathrm{c}1}x_1 + \theta_{\mathrm{c}2}x_2 + \theta_{\mathrm{c}3}x_1^2x_2]^{\mathrm{T}}$ and $f_{\mathrm{d}}(x) = \left[-x_1 + x_2,\, -x_2 + \theta_{\mathrm{d}1}x_1^2 + \theta_{\mathrm{d}2}\frac{x_2^3}{1+x_2^2} + \theta_{\mathrm{d}3}x_2^3\right]^{\mathrm{T}}$, where $\theta_{\mathrm{c}1}$, $\theta_{\mathrm{c}2}$, $\theta_{\mathrm{c}3}$, $\theta_{\mathrm{d}1}$, $\theta_{\mathrm{d}2}$, and $\theta_{\mathrm{d}3}$ are unknown constants. Furthermore, we assume that $\operatorname{sign} b_{\mathrm{c}}$ and $\operatorname{sign} b_{\mathrm{d}}$ are known and $|b_{\mathrm{d}}| < 2$. Next, let $\hat{G}_{\mathrm{c}}(x) = 1$, $\hat{G}_{\mathrm{d}}(x) = 1$, $F_{\mathrm{c}}(x) = \left[x_1, x_2, x_1^2x_2\right]^{\mathrm{T}}$, $F_{\mathrm{d}}(x) = \left[x_1^2, \frac{x_2^3}{1+x_2^2}, x_2^3, x_1, x_2\right]^{\mathrm{T}}$, $K_{\mathrm{cg}} = \frac{1}{b_{\mathrm{c}}}\left[\theta_{\mathrm{cn}_1} - \theta_{\mathrm{c}1}, \theta_{\mathrm{cn}_2} - \theta_{\mathrm{c}2}, -\theta_{\mathrm{c}3}\right]$, and $K_{\mathrm{dg}} = \frac{1}{b_{\mathrm{d}}}[-\theta_{\mathrm{d}1}, -\theta_{\mathrm{d}2}, -\theta_{\mathrm{d}3}, \phi_{\mathrm{dn}_1}, \phi_{\mathrm{dn}_2}]$, where θ_{n_1}, θ_{n_2}, ϕ_{dn_1}, ϕ_{dn_2} are arbitrary scalars, so that

$$\begin{aligned} f_{\mathrm{cs}}(x) &= f_{\mathrm{c}}(x) + \begin{bmatrix} 0 \\ b_{\mathrm{c}} \end{bmatrix} \frac{1}{b_{\mathrm{c}}} \\ &\quad \cdot \begin{bmatrix} \theta_{\mathrm{cn}_1} - \theta_{\mathrm{c}1}, \theta_{\mathrm{cn}_2} - \theta_{\mathrm{c}2}, -\theta_{\mathrm{c}3} \end{bmatrix} F_{\mathrm{c}}(x) \\ &= \begin{bmatrix} 0 & 1 \\ \theta_{\mathrm{cn}_1} & \theta_{\mathrm{cn}_2} \end{bmatrix} x \end{aligned} \tag{56}$$

and

$$\begin{aligned} x + f_{\mathrm{ds}}(x) &= x + f_{\mathrm{d}}(x) + \begin{bmatrix} 0 \\ b_{\mathrm{d}} \end{bmatrix} \frac{1}{b_{\mathrm{d}}} \\ &\quad \cdot \begin{bmatrix} -\theta_{\mathrm{d}1}, -\theta_{\mathrm{d}2}, -\theta_{\mathrm{d}3}, \phi_{\mathrm{dn}_1}, \phi_{\mathrm{dn}_2} \end{bmatrix} F_{\mathrm{d}}(x) \\ &= \begin{bmatrix} 0 & 1 \\ \phi_{\mathrm{dn}_1} & \phi_{\mathrm{dn}_2} \end{bmatrix} x. \end{aligned} \tag{57}$$

Now, with the proper choice of θ_{cn_1}, θ_{cn_2}, ϕ_{dn_1}, and ϕ_{dn_2}, it follows from Corollary 3.1 that the hybrid adaptive feedback controller (49) and (50) guarantees that $x(t) \to 0$ as $t \to \infty$. Specifically, here we choose $\theta_{\mathrm{cn}_1} = -1$, $\theta_{\mathrm{cn}_2} = -2$, $\phi_{\mathrm{dn}_1} = -0.1$, $\phi_{\mathrm{dn}_2} = -0.1$, so that (7) and (9) are satisfied with

$$V_{\mathrm{s}}(x) = x^{\mathrm{T}}Px, \quad P = \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}, \quad \ell_{\mathrm{c}}(x) = \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix} x.$$

With $\mu = 2$, $\alpha = 1$, $\beta = 1$, $a_1 = -5$, $a_2 = -2$, $a_3 = 3$, $\gamma = 1$, $b_{\mathrm{c}} = 3$, $b_{\mathrm{d}} = 1.4$, $\hat{\alpha} = 1$, $Y = 0.1I_3$, and initial conditions $x(0) = [1, 1]^{\mathrm{T}}$, $K_{\mathrm{c}}(0) = [0, 0, 0]$, and $K_{\mathrm{d}}(0) = 0_{1\times 5}$, Figure 1 shows the phase portraits of the uncontrolled and controlled hybrid system. Figures 2 and 3 show the state trajectories versus time and the control signals versus time, respectively.

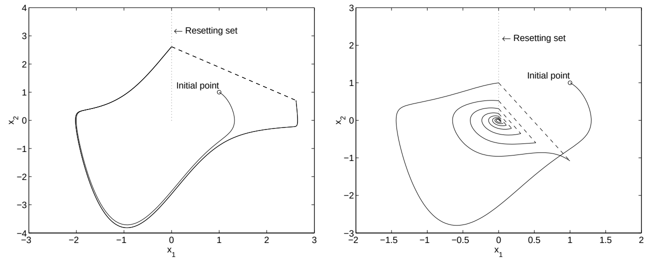

Fig. 1. Phase portraits of uncontrolled and controlled hybrid system

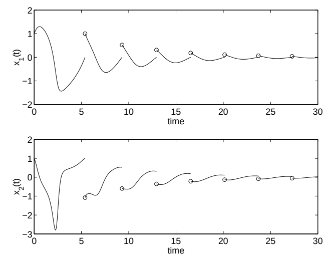

Fig. 2. State trajectories versus time

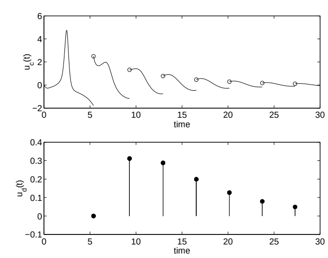

Fig. 3. Control signals versus time

V. Conclusion

A direct hybrid adaptive nonlinear control framework for hybrid nonlinear uncertain systems was developed. Using the the hybrid invariance principle given in [4], [5] the proposed framework was shown to guarantee attraction of the closed-loop system states associated with the hybrid plant dynamics. Furthermore, in the case where the nonlinear hybrid system is represented in a hybrid normal form, the nonlinear hybrid adaptive controllers were constructed without knowledge of the system dynamics. Finally, a numerical example was presented to show the utility of the proposed hybrid adaptive stabilization scheme.

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