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Stable multi–model switching control of a class of nonlinear systems

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Abstract— **The objective of the paper is the design of a stabilizing switching control scheme for a class of nonlinear systems. Such systems are relevant to nonlinear plants represented by means of a finite set of nonlinear discrete–time models. A finite set of receding–horizon control laws is defined for each of the nonlinear discrete–time models representing the plant. A rigorous stability analysis is carried out yielding theoretical constraints to be satisfied by the switching strategies to guarantee stability properties. Simulation results on a case study of practical relevance are also presented showing the effectiveness of the multi-model switching control scheme.**

Keywords: Multi–model Control, Receding–Horizon Control, Hybrid Control

I. INTRODUCTION

The development of a control scheme for hybrid nonlinear systems is the objective of the present paper. Many researchers have been engaged in the modelling and control of hybrid systems; in [1] a general framework for hybrid modelling and control is defined and a review of results and bibliographical references is provided. Survey papers mainly relevant to the stability analysis of hybrid dynamical systems are [2], [3] and [4]. As regards the development of hybrid control schemes, some approaches adopting different control methodologies have been investigated up to now; examples can be found in [5], [6], and [7].

The hybrid plants considered in this work fall into the class of *switched systems*, i.e., systems consisting of a combination of finitely many dynamic systems; indeed these hybrid plants will be referred to as hybrid or switched systems making no difference between the two terms ([8]). The dynamic behaviour of switched systems is characterized by the fact that, in specific time instants denoted as *switching time instants*, the system operating conditions or requirements make it necessary to change the system model and/or the control action to be applied to the system itself.

The stability analysis of switched systems has been thoroughly investigated by many authors, starting from the works [2] and [3]. Some approaches can be found in the literature dealing with switched systems in which the system model is unique, but many regulators are available for the system. This is the case addressed, for example, in [7] and [9], in which switched systems composed of a nonlinear discrete–time plant controlled by a set of *Receding-Horizon* regulators are considered. The proposed hybrid control scheme is made up by the juxtaposition of two control levels. The first control level consists in the definition of a finite set of receding-horizon regulators for the system under concern. The second control level is a discrete–event supervisor that, depending on the system operating conditions and on possible occurred external events, chooses the best control action to be applied to the plant. Moreover, the stability and asymptotic stability of the origin as an equilibrium point of the considered systems have been established by introducing suitable rules to be adopted in switching time instants.

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The present paper is a major generalization of [7] and [9]. The same control scheme is here adopted, but a very significant innovative aspect is introduced: the class of switched systems here considered presents the possibility of switching not only between different controllers, but also between different system models. Moreover, the possibility of adopting approximated receding–horizon regulators in place of optimal ones at the first control level is also taken into account in the present work.

II. THE HYBRID CONTROL SCHEME

Let us consider a nonlinear discrete-time dynamic system described by

$$x_{t+1} = f_i(x_t, u_t)\,, \quad t = 0, 1, \dots \tag{1}$$

where $x_t \in \mathbb{R}^n$ and $u_t \in \mathbb{R}^m$ are the state and the control vectors, respectively, and the sets X and U belong to class $\mathcal{Z} = \bigcup_{k=1}^{\infty} \mathcal{Z}_k, \mathcal{Z}_k \subset \mathbb{R}^q$, where $\mathcal{Z}_k$ is a compact set containing the origin as an internal point. A finite number of dynamic sub–models described by $f_i \colon \mathbb{R}^n \times \mathbb{R}^m \longrightarrow \mathbb{R}^n$, $i = 1, \dots, N$, with $f_i \in \mathcal{C}^1[\mathbb{R}^n \times \mathbb{R}^m, \mathbb{R}^n]$, and $f_i(0, 0) = 0$, is defined. Moreover, denote with $\mathcal{F}$ the class of such sub–models (i.e., $f_i \in \mathcal{F} = \{f_1, \dots f_N\}$). Furthermore, assume the control action u_t to be generated by means of a state–feedback control law belonging to a finite class of control functions, that is

$$u_t = \gamma^{ij}(x_t) \quad \text{where} \quad \gamma^{ij} \in \Gamma_i \triangleq \{\gamma^{i,1}, \dots \gamma^{i,M_i}\}\,.$$

where Γ_i denotes a given class of control functions $\gamma^{ij} \colon \mathbb{R}^n \longrightarrow \mathbb{R}^m$, $i = 1, ...N$, $j = 1, \dots, M_i$, which is defined for each sub–model f_i.

According to the above definitions, the kind of switching events that will be considered are: switching to a different model $f_i \in \mathcal{F}$, switching to a different control function $\gamma^{ij} \in \Gamma_i$, or both. The occurrence of these switching events is in general governed by a suitable supervision system which, in the present paper, will not be addressed (the interested reader is referred to [9]). Even if we assume that the switching events are controlled by a supervisor, the problem of asynchronous events' occurrence must be considered. A detailed explanation of this problem can be found in [7], [9]: here only the definition of the time space $\mathcal{T} \triangleq (T, \lambda)$ will be recalled, which is necessary to the well-posedness of this framework.

$$(T, \lambda) \triangleq \left\{(k, \tau) \in \mathbb{Z}^+ \times \mathbb{R}^+ : k = [\tau]\right\}$$

in which the distance λ is defined as:

$$\forall \quad t_1 = (k_1, \tau_1),\, t_2 = (k_2, \tau_2) \quad \lambda(t_1, t_2) = |\tau_1 - \tau_2|$$

In [9], the following result is stated and proved.

Lemma 2.1: The metric space (T, λ) is a time space in the sense of definition 3.1 in [2]

The definition of the time space $\mathcal{T}$ allows the embedding of the proposed hybrid system into general hybrid systems defined on $\mathbb{R}^+$. This allows us to consider, from now on, a generic switching time instant t as belonging to $\mathbb{R}^+$.

It is now necessary to introduce the concept of *switching sequence*, to collect information about the switching instants, but also about the sub–models and regulators which are involved in the switching process. Following [9], this sequence is indexed by an initial state x_0 and is defined as:

$$\begin{aligned} &\Xi \triangleq \{x_0, (i_0, j_0, t_0), \ldots, (i_n, j_n, t_n)\} \\ &\forall (i_k, j_k, t_k) \in (I \times J_i \times \mathbb{R}^+), k \in \mathbb{N} \end{aligned} \tag{2}$$

where the meaning of the three-tuple (i_k, j_k, t_k) is the following: the integers i_k and j_k respectively denote the "active" sub–model f_{ik} and control law γ^{i_k, j_k} which are active between the "switching-on" time instant t_k and the "switching-off" time instant t_{k+1} (the reader is addressed to [9] for more details).

A further step consists in identifying the switching times at which a specific pair (f_i, γ^{ij}) is switched on and off:

$$\Xi_i^j = \left\{t_0^{ij}, t_1^{ij}, \ldots, t_{2k}^{ij}, t_{2k+1}^{ij}, \ldots, \quad k \in \mathbb{N}\right\}$$

Note that Ξ_i^j is made of time-instants whose indexes are alternatively even and odd positive integers. The even indexes identify switching on time instants for the pair (f_i, γ^{ij}), while the odd ones correspond to the switching off of the same pair. Another sequence that will be useful in the stability analysis with reference to a generic strictly increasing sequence of times Π, is the *even sequence* $E(\Pi)$ ([3], [2]), defined as the sequence of times in Π with even indexes.

The control scheme here proposed makes use of Receding-Horizon (RH) regulators. A detailed statement of the RH control problem referred to system (1) can be found in [10]. For the sake of clarity we recall the basic problem to be solved for each pair (f_i, γ^{ij}), which is:

Problem 1: For any $i \in I$ and any $j \in J_i$, find the RH optimal control law

$$u_t^{RH_{ij}^o} = \gamma^{RH_{ij}^o}(x_t) \in U_i$$

where $u_t^{RH_{ij}^o}$ is the first vector of the control sequence $u_t^{FH_{ij}^o}, \ldots, u_{t+N^{ij}-1}^{FH_{ij}^o}$ that minimizes the finite horizon (FH) cost function:

$$\begin{aligned} &J_{FH}^{ij}(x_t, u_{t,t+N^{ij}-1}, N^{ij}, a^{ij}, P^{ij}) = \\ &= \sum_{r=t}^{t+N^{ij}-1} h^{ij}(x_r, u_r) + a^{ij}\|x_{t+N^{ij}}\|_{P^{ij}} \end{aligned} \tag{3}$$

for the state $x_t \in X^i$.

The above problem can be solved *on-line* or *off-line*, according to the system requirements: this topic is discussed in previous works such as [7], [9], [10].

If the FH cost functions (3) are suitably chosen, the control laws to be applied to the active pair (f_i, γ^{ij}) have some very important stabilizing properties. Theorem 3.1 in [10], which is quoted by [11], details the assumptions on the dynamic system and on the cost parameters N^{ij}, P^{ij}, a^{ij}, which are necessary to obtain the following properties:

1) The RH control law asymptotically stabilizes the origin, which is an equilibrium point of the resulting closed-loop system.
2) There exists a positive scalar $\rho^{ij} \in \mathbb{R}^+$ such that the set $\mathcal{X}_{ij}(N^{ij}, a^{ij}, P^{ij}) \in \mathcal{Z}$, with

$$\begin{aligned} &\mathcal{X}_{ij}(N^{ij}, a^{ij}, P^{ij}) \triangleq \\ &\left\{x \in X^i : J^{FH_{ij}^o}(x, N^{ij}, a^{ij}, P^{ij}) \leq \rho^{ij}\right\} \end{aligned}$$

is an invariant set and a domain of attraction for the origin.

For further deepening about this subject we address the reader to [11], [12]. As regards our purposes, the most important results of this theorem stand in the stabilizing property of RH regulators and the possibility of finding some invariant sets that are domains of attraction for the origin, for each sub–model regulated by a specific control action . It has to be remarked here that any other kind of control functions, guaranteeing stability in some state space regions, could be successfully applied to each subsystem. For the sake of simplicity, from now on the optimal control laws $\gamma^{RH_{ij}^o}$ will be denoted simply as γ^{ij}.

Remark 1 The choice to apply RH regulators to the system is motivated also by the fact that Lyapunov functions can be directly and immediately obtained from the RH algorithm, without requiring any further effort. In fact, thanks to Theorem 3.1 in [10], inside the compact sets $\mathcal{X}_{ij}(N^{ij}, a^{ij}, P^{ij})$ we are provided of a Lyapunov function which is given by:

$$V_{ij}(x_t) \triangleq J^{FH_{ij}^o}(x_t, N^{ij}, a^{ij}, P^{ij})$$

where $a^{ij} \geq \tilde{a}^{ij}$, and $N^{ij}, \tilde{a}^{ij}, P^{ij}$ have to be always determined according to the above quoted Theorem. Then if the state belongs to anyone of these invariant sets, we are automatically provided of a suitable Lyapunov function.

III. Stability analysis

The objective of this section is to show that it is possible to guarantee stability under some assumptions about the transitions of the hybrid trajectory, making use of the Lyapunov stability theory. In [3] stability of nonlinear, non controlled switched systems is proved with the aid of a particular kind of Lyapunov functions, known as Lyapunov-like functions, whose definition is recalled as follows:

Definition 3.1: Given a strictly increasing sequence of times Ξ, a function V is a Lyapunov-like function for f_i and a trajectory $x(\cdot)$ over Ξ iff V is monotonically non-increasing on the even sequence $E(\Xi)$.

By following the same reasoning lines as in [7] and [9], suitable constraints to be fulfilled in switching time instants will be defined to guarantee the stability of the overall control scheme. The switching rules are based on some definitions that still have to be given, so let us introduce the set

$$\mathcal{IJ}(x_t) \triangleq \{(i,j) \in (I, J_i) : x_t \in \mathcal{X}_{ij}\}$$

that characterizes the subset of indexes (i,j) such that the corresponding set $\mathcal{X}_{ij}$ contains the current state x_t. Then, let us define an open set $\mathcal{W}$, which must contain the origin and satisfy $\mathcal{W} \subset \bigcap_{i=1}^{N} X_i$, $\forall i \in I$, and $\mathcal{W} \subset \bigcap \mathcal{X}_{ij}$, $\forall i \in I$, $\forall j \in J_i$, as:

$$\mathcal{W} \triangleq \{x_t \in \mathcal{X}_{ij} \,:\, \|x_t\| < \bar{\epsilon}\}$$

The parameter $\bar{\epsilon}$ is a design parameter, and can be chosen arbitrarily small according to the available set of models and control functions. This set must be defined in order to give a correct definition of the following quantities:

$$\begin{aligned}
&\delta V_{ij}(x_t) \triangleq V_{ij}(x_t) - V_{ij}(f_i[x_t, \gamma^{RH^o_{ij}}(x_t)]),\\
&\quad \forall x_t \in \mathcal{X}_{ij}\backslash\{\mathcal{W}\},\ \forall i \in I,\ \forall j \in J_i,\\
&\delta V_{ij} \triangleq \min_{x_t \in \mathcal{X}_{ij}\backslash\{\mathcal{W}\}} \delta V_{ij}(x_t), \quad \forall i \in I,\ \forall j \in J_i
\end{aligned}$$

The reason why these definitions involve the existence of the set $\mathcal{W}$, is that the origin of the state space we are considering must be excluded, or δV_{ij} would always result equal to 0.

We will now introduce some constraints in which $\overline{\mathcal{W}}$ will denote the closure of set $\mathcal{W}$.

Constraint 3.1: $-$**Strong** Consider a switching time τ at which a pair (f_s, γ^{st}) is switched off and a new pair (f_u, γ^{uv}) is switched on:

$$(f_s, \gamma^{st}) \longrightarrow (f_u, \gamma^{uv})$$

The function V_{uv} satisfies the constraint :

$$\begin{aligned}
&(u,v) \in \tilde{\mathcal{IJ}}(x_\tau) \triangleq \{(u,v) \in \mathcal{IJ}(x_\tau) :\\
&V_{st}(x_\tau) - V_{uv}(x_\tau) \geq \delta V_{st}\}
\end{aligned}$$

Moreover, if for any instant $\bar{t}$ we have that $x_{\bar{t}} \in \overline{\mathcal{W}}$, while $x_{\bar{t}-1} \notin \overline{\mathcal{W}}$, then no switching is allowed for $t \geq \bar{t}$, and the pair $(i,j), i \in I, j \in J_i$, which is active for $t = \bar{t}$ remains active for $t \geq \bar{t}$.

Constraint 3.2: $-$**Weak** Consider a switching time τ , in which the following transition occurs:

$$(f_s, \gamma^{st}) \longrightarrow (f_u, \gamma^{uv})$$

The function V_{uv} satisfies the constraint :

$$\begin{aligned}
&(u,v) \in \mathcal{IJ}^\circ(x_\tau) \triangleq \{(u,v) \in \mathcal{IJ}(x_\tau) :\\
&V_{st}(x_\tau) - V_{uv}(x_\tau) \geq -\delta V_{st}\}
\end{aligned}$$

Moreover, if for any instant $\bar{t}$ we have that $x_{\bar{t}} \in \overline{\mathcal{W}}$, while $x_{\bar{t}-1} \notin \overline{\mathcal{W}}$, then no switching is allowed for $t \geq \bar{t}$, and the pair $(i,j), i \in I, j \in J_i$, which is active for $t = \bar{t}$ remains active for $t \geq \bar{t}$.

These two conditions represent some rules that, if respected, guarantee stability of the system. To prove stability it is first of all necessary to state the following lemma whose proof is not reported here for the sake of brevity (the interested reader can find the proof in [10])

Lemma 3.1: Assume that Theorem 3.1 in [10] is true and Constraint 3.1 or Constraint 3.2 is fulfilled for any trajectory $x(\cdot)$ of the hybrid system (1), determined by the switching sequence Ξ_i^j under the action of the optimal control laws γ^{ij} , for any pair (i,j) with $i \in I, j \in J_i$. Then, for each trajectory $x(\cdot)$ determined by the switching sequence, the functions

$$V_{ij} \triangleq J^{FH^o_{ij}}(x_t, N^{ij}, a^{ij}, P^{ij})$$

are Lyapunov-like functions for f_i and the trajectory $x_{\Xi_i^j}$.

Remark 2 The application of switching constraints is useful from a practical point of view: let us suppose that the a priori computation of the value of each Lyapunov function $V_{ij}(x_t)$, $\forall\,(i,j)$, is possible, for any admissible state belonging to the corresponding invariant $\mathcal{X}_{ij}$. Then, from the knowledge or estimation of δV_{ij}, $\forall\,(i,j)$, we have an immediate criterion to decide if the switching from a certain pair to another is possible when the system is in a certain state x_t and in an arbitrary switching instant. We implicitly assume that there always exists a pair which is allowed to be switched on, guaranteeing stability. Namely in the worst case, if a certain subsystem is active and no switch is allowed, stability is anyhow guaranteed keeping the present subsystem as the active one for future instants, until a switch on another pair is allowed .

Let us now consider the application of the above proposed switching rules. Using Lemma 3.1, it is possible to prove the following stability result for the overall hybrid control scheme under the application of the optimal control laws γ^{ij}. Again the proof can be found in [10].

Theorem 3.1: Assume that Theorem 3.1 in [10] holds true $\forall(i,j) \in (I, J_i)$. Moreover assume that Constraint 3.1 or 3.2 is fulfilled, for any hybrid trajectory $x_t(\cdot)$ determined by the switching sequence Ξ. Then, the equilibrium $x_t = 0$ of the hybrid control system (1) is stable.

The origin is, then, a stable equilibrium point for the considered class of systems under the action of the proposed control scheme. More important, it is possible to show that it is also *asymptotically stable* by applying the stronger constraint 3.1. Some preliminary definitions and results are now addressed to detail the analysis of asymptotic stability.

Definition 3.2: Given the hybrid system (1) and a generic sequence Ξ_i^j, for the corresponding Lyapunov function V_{ij} we define:

$$DV_{ij}(x_{t_{2k}^{ij}}) \triangleq \frac{1}{t_{2k+2}^{ij} - t_{2k}^{ij}}[V_{ij}(x_{t_{2k+2}^{ij}}) - V_{ij}(x_{t_{2k}^{ij}})]$$

Remark 3 Assume Theorem 3.1 holds true. The Lyapunov-like functions V_{ij}, $\forall i \in I$, $\forall j \in J_i$, determined according to assumption (ii) of Theorem 3.1 in [10], are limited inside the corresponding set $\mathcal{X}_{ij}$ by $\mathcal{K}$-functions as follows:

$$\begin{aligned} &\phi_{ij}^-(\|x_{t^{ij}}\|) \le V_{ij}(x_{t^{ij}}) \le \phi_{ij}^+(\|x_{t^{ij}}\|) \quad \forall\, x_{t^{ij}} \in \mathcal{B}_{\beta_{ij}} \\ &\Longrightarrow \lambda_{ij}^-\|x_{t^{ij}}\|^2 \le V_{ij}(\|x_{t^{ij}}\|) \le \lambda_{ij}^+\|x_{t^{ij}}\|^2 \end{aligned} \tag{4}$$

where $\mathcal{B}_\sigma = \{x_t \in \mathbb{R}^n : \|x_t\| < \sigma\}$. Moreover, it is important to show that the function $DV_{ij}(x_{t_{2k}^{ij}})$ just defined is limited in every point of its domain. This constitutes a sort of proof that the derivative of V_{ij} calculated on the even sequence is limited.

Lemma 3.2: Assume that Theorem 3.1 holds true, and that the Strong Constraint 3.1 is fulfilled; then for any sequence Ξ_i^j of our hybrid system (1), in which the Lyapunov-like function V_{ij} is switched on, we can find a $\mathcal{K}$-function ϕ_{ij} such that the following inequality is verified:

$$DV_{ij}(x_{t_{2k}^{ij}}) \le -\phi_{ij}(\|x_{t_{2k}^{ij}}\|), \quad \forall x_{t^{ij}} \in \mathcal{B}_{\beta_{ij}}$$

Finally, the following theorem shows the asymptotic stability of system (1).

Theorem 3.2: Assume that Theorem 3.1 in [10] and Theorem 3.1 hold true with the Strong Constraint 3.1, for system (1) controlled through the optimal RH control laws γ^{ij}. If for any trajectory $x_{t^{ij}}$, $\forall i \in I, \forall j \in J_i$ conditions (4) and Lemma 3.2 are valid, then the origin is an asymptotically stable equilibrium point for system (1).

The proofs of the above lemma and theorem can be found in [10].

IV. Approximated RH regulators

It is straightforward to see that the approximation of RH regulators is an important issue. Let us define, for each sub–model $i \in I$, a class of approximate RH control functions $\hat{\Gamma}_i = \{\hat{\gamma}^{RH_{ij}},\, j \in J_i\}$ that can be used in place of optimal ones. The approximation technique to be adopted will not be taken into consideration here, but the interested reader can find more details in [7], where a neural network is tuned to this aim. In this section a bound on the approximation error will be sought, that allows us to consider the already defined cost functions still as Lyapunov functions for our system, even if approximated regulators are applied instead of the optimal ones. From now on this case will be referred to as the approximated one, while the case in which the optimal regulators are applied will be addressed as the optimal one.

The following problem has to be solved:

Problem 2: Find the maximum approximation error ϵ^{ij}, $\forall i \in I$, $\forall j \in J_i$, that can be tolerated in order to fulfil the condition:

$$\begin{aligned} &V_{ij}(x_t) - V_{ij}(f_i[x_t, \hat{\gamma}^{RH_{ij}}(x_t)]) > 0 \\ &\forall x_t \in \mathcal{X}_{ij} \end{aligned}$$

If we redefine for the approximated case the quantity:

$$\begin{aligned} &\delta\hat{V}_{ij}(x_t) \triangleq V_{ij}(x_t) - V_{ij}(f_i[x_t, \hat{\gamma}^{RH_{ij}}(x_t)]) \\ &\forall x_t \in \mathcal{X}_{ij}, \forall i \in I, \forall j \in J_i \end{aligned}$$

we can prove (as reported in [10]) the following lemma, that solves Problem 2.

Lemma 4.1: If Theorem 3.1 in [10] holds true for any (i, j), then for any indexes pair there exist:

- $\epsilon^{ij} \in \mathbb{R}^+$ that solves Problem 2 for the RH control scheme;
- an approximated control law $\hat{\gamma}^{RH_{ij}}$ solving Problem 1

such that:

$$\begin{aligned} &\delta\hat{V}_{ij}(x_t) = V_{ij}(x_t) - V_{ij}(f_i[x_t, \hat{\gamma}^{RH_{ij}}(x_t)]) > 0, \\ &\quad \forall x_t \in \mathcal{X}_{ij} \end{aligned}$$

If the above Lemma is satisfied, we can take advantage of the cost functions calculated with the approximated RH regulators, that still can be considered Lyapunov functions inside the invariant sets $\mathcal{X}_{ij}$. Under this assumption it is possible to impose analogous switching rules to the switching sequence that affects our system, as it has been done in the optimal case. It is to be stressed here that the switching rules, which again are quite restrictive, are more useful in this approximated contest.

For the sake of brevity, we will propose just the Strong Condition suitably redefined, but always based on definitions given in Section 3.

Constraint 4.1: Be τ a generic switching instant, in which there is the transition:

$$(f_s, \hat{\gamma}^{st}) \longrightarrow (f_u, \hat{\gamma}^{uv})$$

we assume that function V_{uv} fulfills the following constraint:

$$\begin{aligned} (u, v) \in \tilde{\mathcal{IJ}}(x_\tau) \triangleq \{&(u, v) \in \mathcal{IJ}(x_\tau) : \\ &V_{st}(x_\tau) - V_{uv}(x_\tau) \ge \delta V_{st}\} \end{aligned}$$

where δV_{st} is the same quantity that was determined in Condition 3.1 in the optimal case.

Moreover, if for any instant $\bar{t}$ we have that $x_{\bar{t}} \in \overline{\mathcal{W}}$, while $x_{\bar{t}-1} \notin \overline{\mathcal{W}}$, then no switching is allowed for $t \ge \bar{t}$, and the pair $(i, j), i \in I, j \in J_i$, which is active for $t = \bar{t}$ remains active for $t \ge \bar{t}$.

If switching rules are imposed to the system, then it can be easily shown that the Lyapunov functions we are using are also Lyapunov-like functions. This further step is here omitted, being analogous to the optimal case one. Moreover stability and asymptotic stability can also be shown, provided that we are still given the suitable Lyapunov functions, and thus also this part is skipped in this contest. The fundamental result of this section is that if the bound on the approximation error is satisfied, all the framework that allowed to prove stability is still valid, with no difference with respect to the optimal case.

V. Numerical results

First of all, it is worth noting that the simulation results presented in this section are illustrative, in that we try to show the effectiveness of the proposed multi–model switching control scheme without an attempt to verify in a strict and complete way the theoretical results. In particular, some more relaxed conditions on the Lyapunov functions have been imposed.

An ideal Continuous Stirred Tank Reactor (CSTR), is the system to be controlled. Let us refer to the scheme depicted in Figure 1. A single irreversible exothermic reaction from A to B is carried out, assuming constant liquid volume; the regulation aim is to keep the reaction degree and the inner temperature of the tank constant.

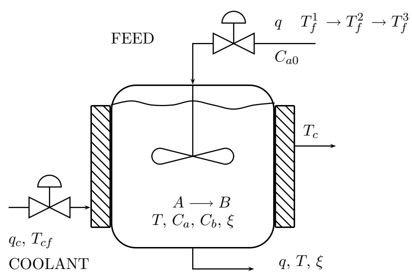

Fig. 1. Stirred tank reactor

The first–principle model equations, which are a component balance of reactant A, and an energy balance, are:

$$\begin{aligned}\frac{d\xi}{dt} &= -\frac{\xi}{\theta} + K(T)(C_{a0} - \xi)\\ \frac{dT}{dt} &= \frac{T_f - T}{\theta} + JK(T)(C_{a0} - \xi) - \frac{Q}{\theta}\end{aligned}$$

where the controlled variables are the reaction degree ξ and the temperature of the tank T; the control variables are the feed and coolant flow q, q_c. The interested reader can find a detailed explanation of the system and all of the involved data in [10].

Under suitable working assumptions, it turns out immediately that the system model undergoes to abrupt changes caused by changes of the reactant feed temperature T_f, controlled by a supervisor that can decide the switching instants. The kind of control action as well can be changed by the supervision system. Three equilibrium points are admissible for this kind of reactor, two of them stable $(\xi_e = 0.091, T_e = 304.95\,K)$, $(\xi_e = 0.897, T_e = 418.95\,K)$ and one unstable $(\xi_e = 0.319, T_e = 337.96\,K)$. The aim of the control action is to keep the reactor in its unstable operating point. This goal can be easily obtained applying a RH control, if all the parameters and data of the system are assumed to be constant, but if the feed of reagent temperature changes discontinuously, the system model changes. Being under the assumption of a perfectly stirred reactor, we tested the behaviour of the CSTR with input feed temperature abrupt changes.

In a few words, this temperature switches among three different values, $290K$, $300K$ and $310K$, and simultaneously there is a change in the control action. The control laws are supposed to change for the change in the parameters of the cost function to be minimized at every current state x_t, according to the receding horizon algorithm. Thus three pairs (f_1, γ_1), (f_2, γ_2) and (f_3, γ_3) are defined, that we will from now on simply address as system 1, 2, 3 respectively. We have to remark that for different feed temperatures, not only the model changes, but also the equilibrium point of the considered system slightly varies. From a practical point of view we preferred to consider the same equilibrium point for the three subsystems, but it has to be noticed that actually this kind of switched system does not fully match the assumptions we made about the common equilibrium point for every pair (f_i, γ^{ij}).

The cost function has the following form:

$$\begin{aligned}&J_{FH}^{ij}(x_t, u_{t,t+N^{ij}-1}, N^{ij}, P_f^{ij}, P^{ij}, R^{ij}) = \\ &\sum_{r=t}^{t+N^{ij}-1} (x_r^T P^{ij} x_r + u_r^T R^{ij} u_r) + x_{t+N^{ij}}^T P_f^{ij} x_{t+N^{ij}} + \\ &+10000 * \sum_{r=t}^{t+N^{ij}-1} (u_{r-1} - u_r)^T * (u_{r-1} - u_r)\end{aligned} \quad (5)$$

As already mentioned, the proposed switching rule is based on the value of the Lyapunov function associated with every specific pair: at every element of a randomly chosen switching sequence Ξ, there is an effort to make an ordered repeated transition from system 1 to 2 and finally system 3. The switching is allowed only if the Lyapunov function of the system to be activated satisfies the definition of Lyapunov like function, namely if it is monotonically nonincreasing on its even sequence. Figures from 2 to 4 are relevant to simulations which have been obtained for $N = 9$ and begun from a different initial point. Stabilization is obtained, the different applied controllers are all stabilizing and the switching rule is always respected with no violation of the expected transition order.

VI. Conclusions

A hybrid control scheme for nonlinear discrete-time switched systems has been described in the paper. The proposed control scheme is composed of a continuous level and a discrete–event supervisor. At the continuous level, which represents the core of the present paper , a finite set of nonlinear discrete-time models represents the system dynamics in different operating conditions. For each of such models a set of receding-horizon regulators is defined at this level. Then, the plant at the continuous control level is viewed as a switched system presenting the possibility of switching between different system models and different controllers. The stability properties of the resulting control scheme are discussed and proved in the paper.

Future research directions are: definition of less restrictive conditions to be adopted in switching time instants, still guaranteeing good stability properties; analysis of systems where not all the state variables are measurable.

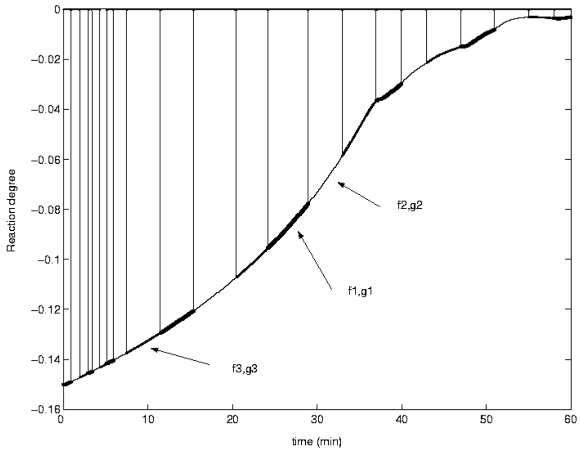

Fig. 2. Reaction degree for $N = 9$

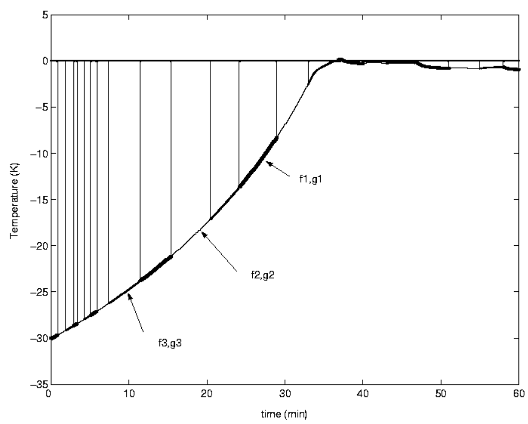

Fig. 3. Temperature for $N = 9$

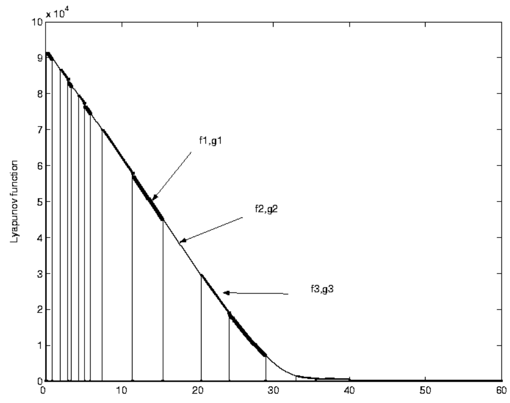

Fig. 4. Global Lyapunov function for $N = 9$

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