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# Cluster Optimization for Takagi & Sugeno Fuzzy Models and Its Application to a Combined Cycle Power Plant Boiler

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*Abstract*- **In this paper, a new method for cluster number optimization of Takagi & Sugeno models is proposed. Also, a general identification methodology is described, including a sensitivity analysis for input variable selection. The new method is exemplified using a benchmark problem, i.e., Chen series. After that, the fuzzy models of a combined cycle power plant boiler, using the proposed methodology, are derived.**

## I. Introduction

COMBINED cycle power plants are of great interest for many countries due to their high efficiencies and their low investment costs [1].

Also, due to the highly non linear behavior of thermal power plants boilers, non linear modeling is necessary to represent the process operation. Particularly, the non linear multivariable fuzzy models developed in this work will be used for a future control design of supervisory controller in order to economic optimize the plant performance [2].

Identification of fuzzy models is a complex problem given by many steps. A relevant step is determining the optimal number of clusters [3].

In that area there are some previous works. Reference [4] propose the cluster validity measures based on the performance of the obtained partition using criteria like the within cluster distance, the partition density, the entropy, etc. This approach implies a high computational effort as clustering must be repeated several times.

Krishnapuram & Freg [5] describe a Compatible Cluster Merging (CCM) for finding the number of linear or planar clusters. The algorithm starts with a maximum number of clusters, and then the number of clusters is reduced until some threshold is reached and no more clusters can be merged. Kaymak & Babuska, [6] propose a modified CCM algorithm based on less conservative criteria.

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In this work, we propose a new method for determining the optimal number of the clusters. Also, a general method for non linear systems based on fuzzy models is described, including a benchmark example.

After that, a combined cycle power plant boiler is described. Then, fuzzy models of a thermal power plant boiler are developed from simulation data, using the proposed method. Finally, the work conclusions are presented.

## II. Non linear identification methodology based on fuzzy models

### A. Non linear multivariable fuzzy models

In this work, the use of the Non Linear Autoregressive with eXogenous variable (NARX) models is considered, where its structure is given by the following equation:

$$y(k) = f(y(k-1), y(k-2), ..., y(k-na), u(k-nk), ..., u(k-nb-nk+1)) \quad (1)$$

where y(k) is a output vector of model, f is the non linear function to be estimated and u(k) correspond to a vector of manipulated variables.

The Takagi&Sugeno fuzzy model has the following f non-linear function:

$$\begin{aligned} &\text{if } y(k-1) \text{ is } A_1^r \text{ and} \cdots \text{and} \\ &u(k-nb-nk+1) \text{ is } A_{na+nb-1}^r \\ &\text{then } y_r(k) = g_0^r + g_1^r y(k-1) + g_2^r y(k-2) + ... \\ &+ g_{na}^r y(k-na) + g_{na+1}^r u(k-nk) + ... \\ &+ g_{na+nb-1}^r u(k-nb-nk+1) \end{aligned} \quad (2)$$

where $A_i^r$ is the fuzzy set of variable i of rule r, $g_i^r$ is consequence parameter of rule r and $y_r$ is the output of rule r. The output of the fuzzy model is:

$$y(t) = \frac{\sum_{r=1}^{Nr} w_r y_r}{\sum_{r=1}^{Nr} w_r} \quad (3)$$

where Nr is the rules number and $w_r$ is the activation degree of rule r given by:

$$w_r = \mu_1^r \cdots \mu_i^r \cdots \mu_{na+nb}^r \tag{4}$$

with $\mu_i^r$ the membership degree of fuzzy set $A_i^r$:

$$\mu_i^r = \exp\left(-0.5 \times \left(a_i^r \times \left(x_i - b_i^r\right)\right)^2\right) \tag{5}$$

where $a_i^r$ and $b_i^r$ are membership parameters and $x_i$ is one of the following input variables: $(y(k-1),\ \ldots,\ y(k-na),$ $u(k-nk),\ \ldots,\ \text{or } u(k-nb-nk+1))$.

### B. Identification procedure

The main steps of the non linear identification methodology are presented in Figure 1. For model identification it is necessary to select real data from the process. Those data must include enough information to represent the different normal operation conditions of the process. Next, the relevant input variables of the non linear model are selected. After that, a structural optimization is made. Then, non linear model parameters using only relevant input variables and optimal structure are calculated. Finally, the non linear model is validated. In the next sections, the main steps of this method are described in detail.

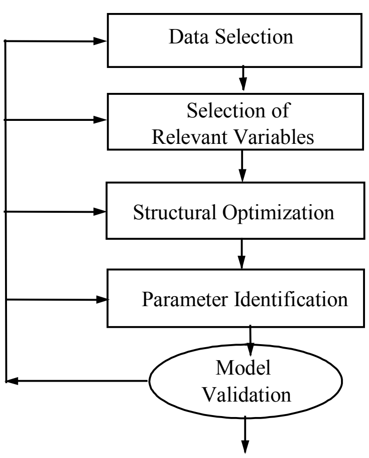


Fig. 1. Flow diagram

#### 1) Data selection

Necessary data sets for non linear fuzzy modeling are:

<u>Training set</u>. From these data, the fuzzy model structure and model parameters are obtained.

<u>Test set</u>. An additional test set is defined. This set is not directly used in the training algorithm; however, it allows to evaluate the model generalization capacity given by the fuzzy model behavior under a new data set.

<u>Validation set</u>. Necessary new data to evaluate the appropriate behavior of adjusted model.

#### 2) Selection of relevant input variables

For any process modeling, one of the most important points is the appropriate selection of the relevant input variables ($x_i$ : $y(k-1),\ \ldots,\ y(k-na),\ u(k-nk),\ \ldots,$ $u(k-nb-nk+1)$) that must be included in the model. To solve this problem the sensitivity analysis method is considered [7].

The methodology consists in adjusting an initial model with the maximum possible input variables, looking for limiting the problem complexity. Then, the influences or sensibilities for each input variable are determined. Finally, the optimum model that uses only the input variables with biggest associated sensibilities is obtained.

The input variable sensitivity $\xi_i$ of the NARX model (equation (1)) is defined by:

$$\xi_i = \frac{\partial f}{\partial x_i} \tag{6}$$

where f is the non linear function and $x_i$ is an input variable.

The sensibilities ($\xi_i$) depend on input variables $x_i$, and they are evaluated using training set. Then, it is necessary to calculate the mean of sensibilities and after that, the input variables with lower mean values are eliminated.

In this case, the input variable sensitivity of fuzzy model (equations (2) to (5)) is given by the following equations [2]):

$$\frac{\partial y(t)}{\partial x_i} = \frac{\sum_{r=1}^{Nr}\left(\frac{\partial w_r}{\partial x_i} y_r + \frac{\partial y_r}{\partial x_i} w_r\right)\sum_{r=1}^{Nr} w_r - \sum_{r=1}^{Nr}\frac{\partial w_r}{\partial x_i}\sum_{r=1}^{Nr} w_r y_r}{\left(\sum_{r=1}^{Nr} w_r\right)^2}$$

$$\frac{\partial w_r}{\partial x_i} = \frac{\partial \mu_i^r}{\partial x_i} \times \mu_1^r \times \cdots \times \mu_{i-1}^r \times \mu_{i+1}^r \times \cdots \times \mu_{Nx}^r$$

$$\frac{\partial \mu_i^r}{\partial x_i} = \mu_i^r \times c_i^r$$

$$c_i^r = -\left(a_i^r \times \left(x_i - b_i^r\right)\right) \times a_i^r$$

$$\frac{\partial y_r}{\partial x_i} = g_i^r \tag{7}$$

#### 3) Parameters identification

In general, the parameters of the non-linear models are obtained minimizing the training set mean squared error.

Sugeno & Yasukawa [8] proposed a method that minimizes the number of rules making a partition of the output variables universe that is projected to the input universe finding the optimal fuzzy sets and rules. This partition is based on a fuzzy clustering method. Using this partition, the premise parameters are obtained. Next, the consequence parameters are obtained using the Takagi & Sugeno method based on least squares, described in [9].

#### 4) Structural optimization

In general, the structural optimization of a non linear multivariable model is a searching procedure consisting in proposing different architectures, increasing complexity. Then, for each proposed structure, the parametric optimization is made minimizing the training set error and evaluating the test set error. Finally, both optimizations conclude when the test error is either increased or stabled.

*Determining the optimal number of clusters.*

In order to determine the optimal number of clusters, a structural optimization of fuzzy models is proposed. Then, the new method is based on a searching procedure consisting in proposing different number of clusters, increasing complexity. Then, for each cluster number proposed, the parametric optimization is made minimizing the training set error and evaluating the test set error. Finally, both optimizations conclude when the test error is either increased or stabled.

The root mean square error is considered as index error for training and test data sets, given by:

$$RMS = \sqrt{\frac{\sum_{i=1}^{N}\left(y(i) - \hat{y}(i)\right)^2}{N}} \quad (8)$$

where y(i) is the system output, $\hat{y}(i)$ is the estimated output by fuzzy model and N is the data number.

#### 5) Model validation

The adjusted fuzzy model is evaluated using a validation set. Then, if the adjusted model evaluation is appropriate, the model identification procedure finishes; otherwise it is convenient to review the previous steps.

### C. Example: Chen series

This example of dynamic system identification was presented by Chen in [10]. The example is given by the following equation:

$$\begin{aligned} y(k) &= \left(0.8 - 0.5\exp\left(-y^2(k-1)\right)\right)y(k-1) \\ &- \left(0.3 + 0.9\exp\left(-y^2(k-1)\right)\right)y(k-2) + u(k-1) \\ &+ 0.2u(k-2) + 0.1u(k-1)u(k-2) + \varepsilon(k) \end{aligned} \quad (9)$$

where y(k) is the output variable, u(k) is the input variable given by uniform distribution ($\mu = 0,\ \sigma = 1$) and $\varepsilon(k)$ is white noise ($\mu = 0,\ \sigma = 0.2$).

250 training data, 250 test data and 250 validation data are considered. Also, the premise and consequence parameters are determined using the method described in section 2.2.3.

For selection of significant input variables, the sensitivity method, described in section 2.2.2, is used.

*Determining the optimal number of clusters*

For the selection of the optimal number of clusters, the proposed method is considered. In Figure 2, the training and test error for different number of clusters is shown. The training error decrease, by increasing the cluster number. Otherwise, the test error has a minimum before, in order to generalize the system dynamic of Chen series. Then, as show Figure 2, the optimal number of clusters for the fuzzy model is four.

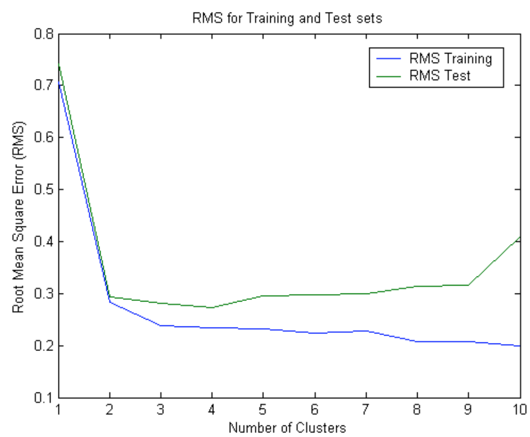


Fig. 2. Training and test errors for Chen series

## III. FUZZY MODELS OF A COMBINED CYCLE POWER PLANT BOILER

### A. Process description

The combined cycle power plants consist in a gas turbine, a boiler and a steam turbine to generate electricity [1].

In Figure 3, the boiler configuration is presented. The feedwater is supplied to the drum, where the thermal energy of combustion products is transferred to be condensed. Then, the feedwater enters the risers, where the furnace heat is used to increase the water temperature and eventually it causes its evaporation. Thus, the circulation of water, steam and water and steam mixture takes place in the drum and risers. Steam generated in the risers is separated in the drum, from where it flows through the superheater to the high pressure turbine. Then, this steam is recycled to the boiler in the reheater where its energy content is increased.

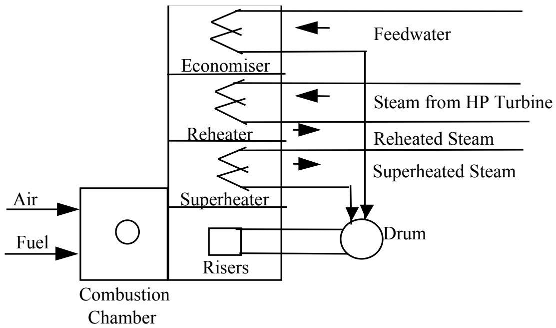

Fig. 3. Boiler configuration

The phenomenological simulator was developed for boiler of a combined cycle power plant (50 MW). The boiler simulator is based on the phenomenological equations, and their parameters are determinated and adapted from [1]. The simulator consists on 15 non linear differential equations and 40 non linear algebraic equations aprox.

## *B. Fuzzy modeling*

As an example of fuzzy modeling proposed, a non linear fuzzy multivariable model of superheated steam pressure ($p_s$) is developed, using fuel flow ($w_f$) and feedwater flow ($w_e$) as manipulated variables.

In Figure 4, the $p_s$ model identification data are presented. The excitation signals ($w_f$) and ($w_e$) are discrete white noises.

For model adjusting and evaluation, the error index (RMS), defined by equation (7), is considered.

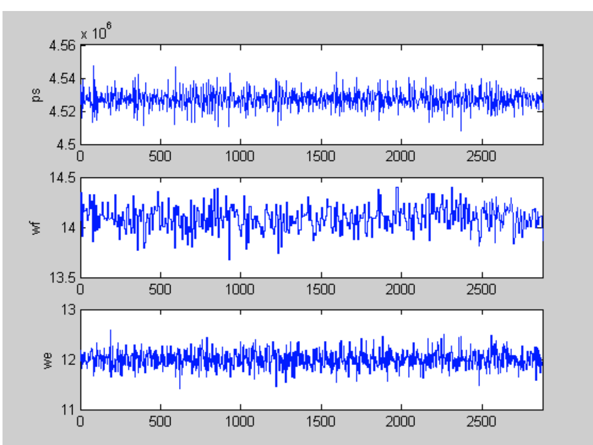

Fig. 4. Identification data

### *1) Data selection*

One thousand data for training, test and validation sets, using 30 second sampling period, are considered. Figure 5 presents the training, test and validation data sets for $p_s$.

### *2) Parameter identification*

The parameters of the fuzzy model proposed for $p_s$ are determinate using the fuzzy clustering and least mean square method. (see section II.B.3).

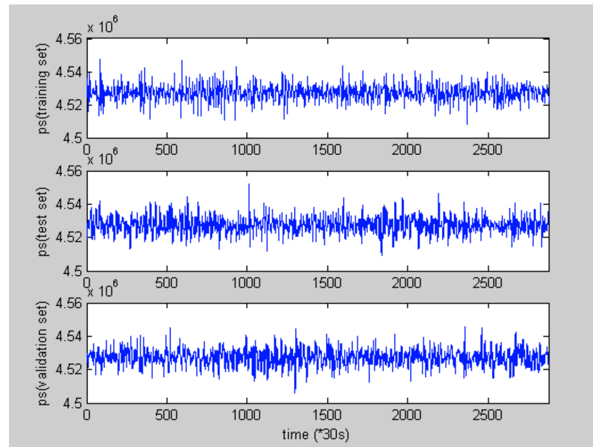

Fig.5. Training, test and validation data sets.

### *3) Selection of relevant input variables*

Due to the computational requirement given by adjusting an initial model with the highest order, the following initial fuzzy model structure is proposed:

$$\begin{aligned}
&\text{if } p_s(k-1) \text{ is } A_1^r \text{ and} \cdots \text{and } p_s(k-5) \text{ is } A_5^r \\
&\text{and } w_f(k-1) \text{ is } A_6^r \text{ and} \cdots \text{and } w_f(k-5) \text{ is } A_{10}^r \\
&\text{and } w_e(k-1) \text{ is } A_{11}^r \text{ and} \cdots \text{and } w_e(k-5) \text{ is } A_{15}^r \\
&\text{then} \\
&p_s(k) = g_0^r + g_1^r p_s(k-1) + \cdots + g_5^r p_s(k-5) \\
&\qquad + g_6^r w_f(k-1) + \cdots + g_{10}^r w_f(k-5) \\
&\qquad + g_{11}^r w_e(k-1) + \cdots + g_{15}^r w_e(k-5)
\end{aligned} \tag{10}$$

Figure 6 presents the mean sensibilities graphic of the proposed initial model, with 15 input variables. In the graphic, the variables with the least statistical values of the sensibilities are candidate to be eliminated. Then, it is possible to conclude that these variables must not be included in the process model. By this way, the following optimal structure is obtained with only model relevant input variables:

$$\begin{aligned}
&\text{if } p_s(k-1) \text{ is } A_1^r \text{ and } p_s(k-2) \text{ is } A_2^r \text{ and} \\
&p_s(k-3) \text{ is } A_3^r \text{ and } p_s(k-4) \text{ is } A_4^r \text{ and} \\
&p_s(k-5) \text{ is } A_5^r \text{ and } w_f(k-1) \text{ is } A_6^r \text{ and} \\
&w_f(k-2) \text{ is } A_7^r \text{ and } w_f(k-4) \text{ is } A_8^r \text{ and} \\
&w_f(k-5) \text{ is } A_9^r \text{ and } w_e(k-1) \text{ is } A_{10}^r \text{ and then} \\
&p_s(k) = g_0^r + g_1^r p_s(k-1) + g_2^r p_s(k-2) \\
&+ g_3^r p_s(k-3) + g_4^r p_s(k-4) + g_5^r p_s(k-5) \\
&+ g_6^r w_f(k-1) + g_7^r w_f(k-2) + g_8^r w_f(k-4) \\
&+ g_9^r w_f(k-5) + g_{10}^r w_e(k-1)
\end{aligned} \tag{11}$$

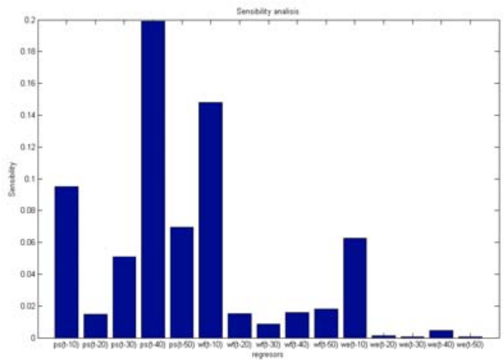

Fig. 6. Sensitivity analysis

*4) Structural Optimization: Determining the optimal number of clusters.*

In order to determine the optimal number of clusters, fuzzy models with different number of clusters are adjusted. As shown Figure 7, the training error decrease by increasing the cluster number and the test error has a minimum. Therefore, the optimal of cluster number is two for the fuzzy model obtained in equation (11).

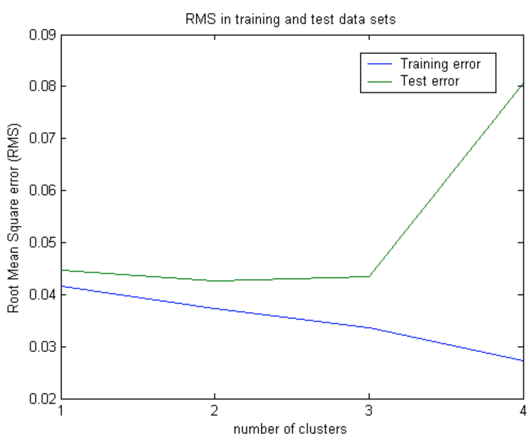


Fig.7. Training and test error for fuzzy model

*5) Model Validation*

Figure 8 presents the prediction of superheated steam pressure $p_s$ using the fuzzy model defined by equation (11) and the optimal number of cluster obtained in section III.B.4.

Table 1 shows the RMS errors for the training, test and validation data sets of a linear model, the initial fuzzy model (equation (10)) and optimal fuzzy model (equation (11) and 2 clusters). The least RMS error is obtained, using the optimal fuzzy model, for test and validation sets.

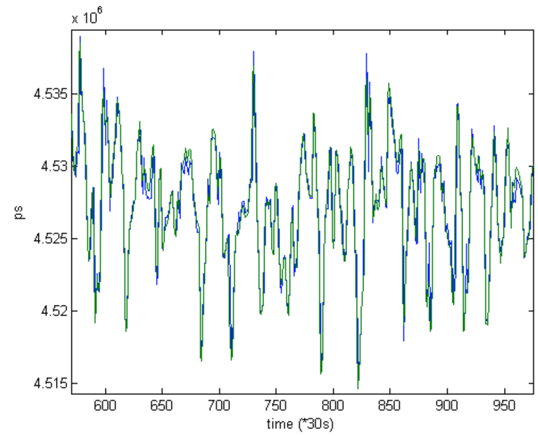


Fig. 8. Prediction of $p_s$ using optimal fuzzy model.

TABLE 1. RMS ERRORS

| | Training | Test | Validation |
|---|---|---|---|
| Linear Model | 1807.50 | 1844.00 | 1740.30 |
| Initial Fuzzy Model | 1503.42 | 1708.18 | 1589.35 |
| Optimal fuzzy model | 1571.14 | 1699.73 | 1560.07 |

## IV. CONCLUSIONS

This paper presents a new method for determining the optimal number of clusters of fuzzy models.

Also, a complete identification methodology for non linear fuzzy models is described, including a sensitivity analysis. For the determining the optimal number of clusters, a Chen series example is presented.

The proposed identification methodology was applied to the modeling of a combined cycle power plant boiler.

The optimal fuzzy model for superheated steam pressure was favorably compared versus an initial fuzzy model and a linear model.

The non linear multivariable fuzzy models for a thermal power plant developed in this work will be used for a future control design of supervisory controller in order to economic optimize the plant performance.

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