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Neutralization Process Control Using An Adaptive Fuzzy Controller

Feng Wan[1,3], Huilan Shang[1], Li-Xin Wang[2] and You-Xian Sun[3]
[1]School of Engineering, Laurentian University, Sudbury, ON P3E 2H2, Canada
[2]Dept. of Elec. & Electr. Eng., Hong Kong Univ. of Science and Technology, Hong Kong
[3]The Institute of Industrial Process Control, Zhejiang University, Hangzhou, China

***Abstract*— Controlling of neutralization processes is a classical problem beyond many common control techniques available today due to their strong nonlinearity and extreme sensitivity to disturbances. This paper proposes an adaptive fuzzy control scheme for wastewater neutralization process control. A standard Mamdani type fuzzy system is constructed to describe the nonlinearity of the process and the parameters in the fuzzy system are tuned online using the least square algorithm with deadzone. The controller is designed based on the fuzzy model and the control action is computed by exploiting the special features of the fuzzy system. Simulation results are given in control of the acidity in the continuous flow of an industrial wastewater treatment system.**

I. INTRODUCTION

Neutralization is a common and important operation in chemical plants, such as biological, wastewater treatment, electrochemistry and precipitation systems. The purpose of neutralization is to adjust the pH value to neutrality for a certain requirement, for example, to minimize the environmental impact in wastewater treatment systems. In the literature, different approaches to control of neutralization processes have been applied, such as modified PI, linear adaptive, model-based, nonlinear adaptive, and predictive adaptive controllers (see [3], [4], [7] and the references therein). Despite various techniques devoted to improve those classical methods, the performance of neutralization control is not yet adequate due to the inherent nonlinearity of the titration curve and high sensibility to small perturbations near the equivalence point.

Recently, artificial neural networks, radial basis function networks and fuzzy systems have been widely used in the identification and control of nonlinear processes, owing to their abilities to represent nonlinear mappings to arbitrary accuracy and learn on-line from the innovation of the processes [4], [6], [8], [9]. Like many other methods, artificial neural networks and radial basis function networks, unfortunately have some severe weaknesses: they can hardly make use of qualitative heuristic knowledge, they have not clear physical meanings and thus are not explainable to users, their construction and initialization are usually done in a trial-and-error manner and they are complicated to implement. Comparing with their competitors, fuzzy systems are capable to do both nonlinear approximation and human knowledge incorporation. They are rule-based systems constructed from a collection of fuzzy IF-THEN rules; on the other hand, they are nonlinear mappings with nice properties like universal approximation [10]. Therefore, fuzzy systems are good candidates for modeling and controlling of complicated nonlinear processes.

In this paper, an adaptive fuzzy controller is proposed to control a neutralization process. First, a Mamdani type fuzzy system with triangular membership functions is built to describe the nonlinearity of the process and the least square algorithm with deadzone is used to tune the parameters in the fuzzy model. Based on this model a controller is designed using the decomposing properties of the fuzzy system. Performance analysis of the scheme is given as well as numerical results in control of the acidity in the continuous flow of an industrial wastewater treatment system.

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II. ADAPTIVE FUZZY CONTROLLER

A. Control problem

Consider the single-input-single-output nonlinear process given by

$$y(t+1) = g(y(t), \dots, y(t-n+1); u(t), \dots, u(t-m+1)), \tag{1}$$

where $u(t)$ and $y(t)$ are the process input and output, respectively; n and m are the orders which are known and $g(\cdot)$ is an unknown continuous nonlinear function that describes the dynamics of the process. Denote the input vector of $g(\cdot)$ as $x(t) = [y(t), \cdots, y(t-n+1);\ u(t), \cdots, u(t-m+1)]^T$, $x = [x_1, \dots, x_s]^T \in X \subset R^s$ with dimension s, and y_r be the desired trajectory, which is known in advance. The control task is to determine the control action at sampling time t, $u(t)$, which makes the output of the process at time $t+1$, $y(t+1)$, trace the desired value $y_r(t+1)$. In short, the following tracking error cost function is to be minimized:

$$J(t+1) = [y_r(t+1) - y(t+1)]^2. \tag{2}$$

B. Fuzzy model

Since $y(t+1)$ is unknown at time t, to predict it the following fuzzy system model is used:

$$\hat{y}(t+1) = f(y(t), \dots, y(t-n+1); u(t), \dots, u(t-m+1)), \tag{3}$$

where $\hat{y}(t+1)$ denotes the prediction of $y(t+1)$, $f(\cdot)$ is a Mamdani type fuzzy system with the product inference engine, the singleton fuzzifier, the center-average defuzzifier and triangular membership functions and constructed through three steps:

Step 1: Let $X = [\alpha_1, \beta_1] \times \cdots \times [\alpha_s, \beta_s]$. For every j ($j = 1, 2, \ldots, s$), define N_j fuzzy sets in $[\alpha_j, \beta_j]$ with the following triangular membership functions:

$$\begin{aligned} \mu_{A_j^1}(x_j) &= \mu(x_j; \gamma_j^1, \gamma_j^1, \gamma_j^2), \\ \mu_{A_j^{N_j}}(x_j) &= \mu(x_j; \gamma_j^{N_j-1}, \gamma_j^{N_j}, \gamma_j^{N_j}), \\ \mu_{A_j^r}(x_j) &= \mu(x_j; \gamma_j^{r-1}, \gamma_j^r, \gamma_j^{r+1}), \end{aligned} \quad (4)$$

where $r = 2, 3, \ldots, N_j - 1$, $\alpha_j = \gamma_j^1 < \gamma_j^2 < \cdots < \gamma_j^{N_j} = \beta_j$ and

$$\mu(x; a, b, c) = \begin{cases} \frac{x-a}{b-a}, & if \quad a \neq b \ and \ x \in [a, b], \\ \frac{c-x}{c-b}, & if \quad b \neq c \ and \ x \in [b, c], \\ 1, & if \quad a = b \ and \ x \leq a, \\ 1, & if \quad b = c \ and \ x \geq c, \\ 0, & otherwise. \end{cases} \quad (5)$$

Step 2: Construct $N = \prod_{j=1}^{s} N_j$ fuzzy IF-THEN rules in the following form:

$$\text{Rule}^{i_1 \cdots i_s} : \text{IF } x_1 \text{ is } A_1^{i_1} \text{ and ... and } x_s \text{ is } A_s^{i_s} \text{ THEN } y \text{ is } B^{i_1 \cdots i_s}, \quad (6)$$

where $i_1 = 1, 2, \ldots, N_1$; $\ldots$; $i_s = 1, 2, \ldots, N_s$ and $B^{i_1 \cdots i_s}$ is a fuzzy set with center $\theta^{i_1, \ldots, i_s}$, which is free to change.

Step 3: Construct a fuzzy system $f(x)$ from the rules of (6) using the product inference engine, the singleton fuzzifier and the center average defuzzifier. Then the fuzzy system can be expressed as follows:

$$\begin{aligned} f(x) &= \frac{\sum_{i_1=1}^{N_1} \cdots \sum_{i_s=1}^{N_s} \theta^{i_1, \ldots, i_s} [\prod_{j=1}^{s} \mu_{A_j^{i_j}}(x_j)]}{\sum_{i_1=1}^{N_1} \cdots \sum_{i_s=1}^{N_s} [\prod_{j=1}^{s} \mu_{A_j^{i_j}}(x_j)]} \\ &= \sum_{i_1=1}^{N_1} \cdots \sum_{i_s=1}^{N_s} \theta^{i_1, \cdots, i_s}(t) \left[\prod_{j=1}^{s} \mu_{A_j^{i_j}}(x_j(t)) \right]. \end{aligned} \quad (7)$$

The last equality is due to the fact that the denominator on the right hand side of the first equality becomes one if the triangular membership functions (4)-(5) are used.

The following lemmas reveal the approximation capability of the Mamdani type fuzzy system $f(x)$ in the form of (7) in representing an unknown nonlinear function $g(x)$.

Lemma 1: For any given continuous function $g(x)$ on a compact set X, and an arbitrary $\varepsilon > 0$, a standard Mamdani type of fuzzy system in the form of (7) exists such that

$$||f(x) - g(x)||_\infty < \varepsilon, \quad (8)$$

where the infinity norm $|| * ||_\infty$ of $p(x)$ on X is defined as $||p(x)||_\infty = \sup_{x \in X} |p(x)|$.

Lemma 2: (i) If $g(x)$ is continuously differentiable in the domain of interest X, then

$$||f(x) - g(x)||_\infty \leq \sum_{j=1}^{s} \left\| \frac{\partial g(x)}{\partial x_j} \right\|_\infty h_j, \quad (9)$$

where $h_j = \max_{1 \leq r \leq N_j - 1} |\gamma_j^{r+1} - \gamma_j^r|$;

(ii) If $g(x)$ is twice continuously differentiable in X, then

$$||f(x) - g(x)||_\infty \leq \frac{1}{8} \sum_{j=1}^{s} \left\| \frac{\partial^2 g(x)}{\partial x_j^2} \right\|_\infty h_j^2. \quad (10)$$

C. Adaptation law

Since $\mu_{A_j^{i_j}}(x_j)$ are fixed triangular membership functions, $f(x)$ can be written into the following form:

$$f(x) = \phi^T(x)\theta, \quad (11)$$

where θ is the collection of the $\prod_{j=1}^{s} N_j$ parameters $\theta^{i_1 \ldots i_s}$:

$$\begin{aligned} \theta &= [\theta^{1,1,\ldots,1}, \ldots, \theta^{1,1,\ldots,N_s}, \ldots, \\ &\quad \theta^{N_1, N_2, \ldots, 1}, \ldots, \theta^{N_1, N_2, \ldots, N_s}]^T, \end{aligned} \quad (12)$$

and $\phi(x)$ is the corresponding fuzzy basis function vector:

$$\begin{aligned} \phi(x) &= [\phi^{1,1,\ldots,1}(x), \ldots, \phi^{1,1,\ldots,N_s}(x), \ldots, \\ &\quad \phi^{N_1, N_2, \ldots, 1}(x), \ldots, \phi^{N_1, N_2, \ldots, N_s}(x)]^T \end{aligned} \quad (13)$$

with

$$\phi^{i_1, \cdots, i_s}(x) = \prod_{j=1}^{s} \mu_{A_j^{i_j}}(x_j(t)). \quad (14)$$

Equation (11) indicates that $f(x)$ is linear in the parameters θ. Therefore, standard parameter tuning algorithms can be applied to tune the parameters in $f(x)$, here the following least square algorithm with deadzone is adopted:

$$\begin{aligned} \theta(t) &= \theta(t-1) + \\ &\quad \frac{I(t-1)P(t-2)\phi(x(t-1))}{1 + I(t-1)\phi^T(x(t-1))P(t-2)\phi(x(t-1))} e_p(t) \end{aligned} \quad (15)$$

$$\begin{aligned} P(t-1) &= P(t-2) - \\ &\quad \frac{I(t-1)P(t-2)\phi(x(t-1))\phi^T(x(t-1))P(t-2)}{1 + I(t-1)\phi^T(x(t-1))P(t-2)\phi(x(t-1))}, \end{aligned} \quad (16)$$

where the initial estimate of θ is $\theta(0)$, $P(-1)$ is a positive definite matrix, the prediction error $e_p(t)$ is defined as

$$e_p(t) = y(t) - \phi^T(x(t-1))\theta(t-1), \quad (17)$$

and the indication function

$$I(t-1) = \begin{cases} 1, & if \ |\frac{e_p^2(t)}{1+\phi^T(x(t-1))P(t-2)\phi(x(t-1))}| > \Delta^2, \\ 0, & otherwise, \end{cases} \quad (18)$$

with the deadzone $\Delta > 0$ specified by the user.

Remark 1: The reason for using the above tuning algorithm is that it provides good robustness in the presence of measurement noise, inaccurate modeling or computer round-off error: the parameter tuning is turned off when the prediction error is small compared with the disturbances. For the identification with fuzzy system models, if there are not enough fuzzy sets defined in the construction procedure, the approximation error may not be further reduced regardless of how the fuzzy system model is tuned.

In this case, by Lemma 2, the deadzone can be set as $\Delta = \sum_{j=1}^{n} ||\frac{\partial g}{\partial x_j}||_\infty h_j$, so that the tuning mechanism is switched off if the approximation error is already smaller than the structure error due to the construction.

The cost function in (2) can be written as

$$\begin{aligned} & J(t+1) \\ = \; & [(y_r(t+1) - \widehat{y}(t+1)) + (\widehat{y}(t+1) - y(t+1))]^2 \\ \leq \; & 2(y_r(t+1) - \widehat{y}(t+1))^2 + 2(\widehat{y}(t+1) - y(t+1))^2. \end{aligned}$$

The adaptive control scheme consists of two parts: (i) the adaptation law for tuning the parameters in the fuzzy system model on-line in order to make the predicted output $\widehat{y}(t+1)$ follow the process output $y(t+1)$, which is shown in this subsection and (ii) the control law for determining $u(t)$ in order to make $\widehat{y}(t+1)$ follow the desired trajectory $y_r(t+1)$, which will be further discussed in the followings.

D. Control law

Denote the minimal and maximal values of an allowable control action as $u_{\min}$ and $u_{\max}$, respectively. The objective is to find a control $u_c(t)$ within the range $[u_{\min}, u_{\max}]$ that can make the error between the predicted output $\widehat{y}(t+1)$ and the desired output $y_r(t+1)$ as small as possible, i.e.:

$$u_c(t) = \arg\left\{ \min_{u(t) \in [u_{\min}, u_{\max}]} J_c(t+1) \right\}, \qquad (19)$$

where the control error cost function is defined as

$$J_c(t+1) = [y_r(t+1) - \widehat{y}(t+1)]^2, \qquad (20)$$

and $\widehat{y}(t+1)$ is the fuzzy system output in the form of (7):

$$\begin{aligned} \widehat{y}(t+1) &= f(x(t)) \\ &= \sum_{i_1=1}^{N_1} \cdots \sum_{i_s=1}^{N_s} \theta^{i_1,\cdots,i_s}(t) \left[\prod_{j=1}^{s} \mu_{A_j^{i_j}}(x_j(t)) \right] \end{aligned} \qquad (21)$$

At time t, $y_r(t+1)$ and all elements in $x(t) = [y(t), \cdots, y(t-n), u(t), \cdots, u(t-m)]^T$, except $x_{n+1}(t) = u(t)$, are known, and the parameter estimates $\theta^{i_1,\cdots,i_s}(t)$ are provided by the adaptation law (15)-(18). Combining the known terms into a new parameter $\overline{\theta}^{i_{n+1}}(t)$, which is defined as

$$\overline{\theta}^{i_{n+1}}(t) = \sum_{i_1=1}^{N_1} \cdots \sum_{i_n=1}^{N_n} \sum_{i_{n+2}=1}^{N_{n+2}} \cdots \sum_{i_s=1}^{N_s} \theta^{i_1,\cdots,i_s}(t) \left[\prod_{\substack{j=1, \\ j \neq n+1}}^{s} \mu_{A_j^{i_j}}(x_j(t)) \right] \qquad (22)$$

for $i_{n+1} = 1, \cdots, N_{n+1}$, in consequence the fuzzy system can be rewritten in a simplified form:

$$f(x(t)) = \sum_{i_{n+1}=1}^{N_{n+1}} \overline{\theta}^{i_{n+1}}(t) \mu_{A_{n+1}^{i_{n+1}}}(u(t)). \qquad (23)$$

The membership functions $\mu_{A_{n+1}^{i_{n+1}}}(u(t))$ are defined as in (4)-(5) and plotted in Fig. 1. We call $\gamma_{n+1}^{i_{n+1}}$ the center of the fuzzy set $A_{n+1}^{i_{n+1}}$, $i_{n+1} = 1, \cdots, N_{n+1}$. Clearly, for any $u(t)$ covered by the fuzzy sets defined, an index p exists such that $u(t) \in [\gamma_{n+1}^p, \gamma_{n+1}^{p+1}]$, $p = 1, 2, \cdots$, or $N_{n+1} - 1$.

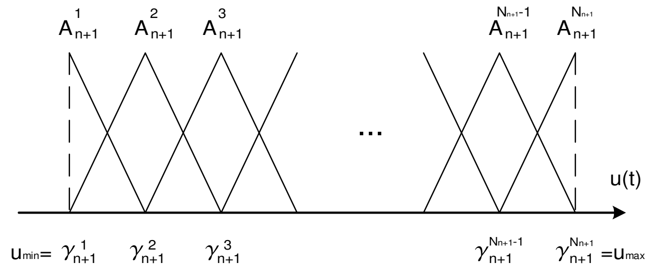

Fig. 1. Membership functions covering the domain of the control $u(t)$.

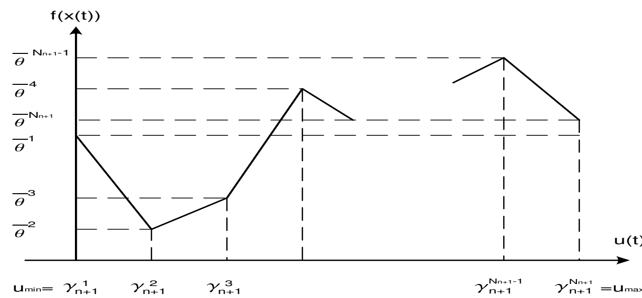

Fig. 2. $f(x(t))$ as a function of $u(t)$ with other components of $x(t)$ fixed.

From Fig. 1, it is seen that if $u(t) \in [\gamma_{n+1}^p, \gamma_{n+1}^{p+1}]$, then fuzzy system (23) can be further simplified to

$$\begin{aligned} f(x(t)) &= \overline{\theta}^p(t) \mu_{A_{n+1}^p}(u(t)) + \overline{\theta}^{p+1}(t) \mu_{A_{n+1}^{p+1}}(u(t)) \\ &= \left(\frac{\overline{\theta}^{p+1} - \overline{\theta}^p}{\gamma_{n+1}^{p+1} - \gamma_{n+1}^p} \right) u(t) + \left(\frac{\overline{\theta}^p \gamma_{n+1}^{p+1} - \overline{\theta}^{p+1} \gamma_{n+1}^p}{\gamma_{n+1}^{p+1} - \gamma_{n+1}^p} \right) \end{aligned} \qquad (24)$$

Hence, the fuzzy system $f(x(t))$ is a continuous piece-wise linear function of $u(t)$ when other components of $x(t)$ are fixed, as shown in Fig. 2. Moreover,

$$\overline{\theta}^p(t) = f(x(t) | u(t) = \gamma_{n+1}^p), \quad p = 1, 2, \cdots, N_{n+1}. \qquad (25)$$

Lemma 3: Let

$$\overline{\theta}^{p_{\min}}(t) = \min(\overline{\theta}^1(t), \cdots, \overline{\theta}^{N_{n+1}}(t)), \qquad (26)$$

$$\overline{\theta}^{p_{\max}}(t) = \max(\overline{\theta}^1(t), \cdots, \overline{\theta}^{N_{n+1}}(t)), \qquad (27)$$

where the index $p_{\min}$(or $p_{\max}$) $= 1, 2, \cdots$, or N_{n+1}. Then the fuzzy system output satisfies

$$f(x(t)) \in [\overline{\theta}^{p_{\min}}(t), \overline{\theta}^{p_{\max}}(t)]. \qquad (28)$$

From (24), it can be seen that if $y_r(t+1) \in [\overline{\theta}^{p_{\min}}(t), \overline{\theta}^{p_{\max}}(t)]$, then the control action $u(t)$ can be determined through

$$u^p(t) = \frac{y_r(t+1) - \left(\frac{\gamma_{n+1}^{p+1} \overline{\theta}^p - \gamma_{n+1}^p \overline{\theta}^{p+1}}{\gamma_{n+1}^{p+1} - \gamma_{n+1}^p} \right)}{\left(\frac{\overline{\theta}^{p+1} - \overline{\theta}^p}{\gamma_{n+1}^{p+1} - \gamma_{n+1}^p} \right)}. \qquad (29)$$

Unfortunately, the control $u(t)$ cannot be calculated from (29) if the index p is not specified. In the the following steps, a searching method is proposed to find a proper control that minimizes the control error cost function $J_c(t+1)$ in the form of (20).

Step 1: At time t, compute parameters $\overline{\theta}^{i_{n+1}}(t)$ (for $i_{n+1} = 1, \cdots, N_{n+1}$) according to (22), and find $\overline{\theta}^{p_{\min}}(t)$ from (26) and $\overline{\theta}^{p_{\max}}(t)$ from (27). If $y_r(t+1) \in [\overline{\theta}^{p_{\min}}(t), \overline{\theta}^{p_{\max}}(t)]$, go to Step 2; otherwise, go to Step 3.

Step 2: If $y_r(t+1) \in [\overline{\theta}^{p_{\min}}(t), \overline{\theta}^{p_{\max}}(t)]$, the control $u(t)$ is determined as follows: Find all intervals represented by index p, which $y_r(t+1)$ falls into, i.e., $y_r(t+1) \in [\overline{\theta}^{p+1}(t), \overline{\theta}^{p}(t)]$ or $y_r(t+1) \in [\overline{\theta}^{p}(t), \overline{\theta}^{p+1}(t)]$, and then use (29) to calculate the control candidate $u^p(t)$ for every such p. If there are more than one candidates, the final control $u(t)$ is chosen as that one that has the smallest $|u^p(t) - u(t-1)|$ (since big change on the control action is usually undesirable).

Step 3: If $y_r(t+1) \notin [\overline{\theta}^{p_{\min}}(t), \overline{\theta}^{p_{\max}}(t)]$, which implies that $J(t+1)$ cannot reach zero by any control $u(t) \in [u_{\min}, u_{\max}]$: (i) when $y_r(t+1) < \overline{\theta}^{p_{\min}}(t)$, the control is chosen as $u(t) = \gamma_{n+1}^{p_{\min}}$; (ii) when $y_r(t+1) > \overline{\theta}^{p_{\max}}(t)$, the control is chosen as $u(t) = \gamma_{n+1}^{p_{\max}}$. If there are multiple of such controls, choose one that has the smallest $|u(t) - u(t-1)|$ as the final control action.

E. Performance analysis

Consider $f(x|\theta)$ representing the fuzzy system (7) with parameter θ. Assume that the unknown process function $g(x)$ is continuous on a compact operating region and, moreover, it has bounded partial derivatives with respect to all components of x. Define the *optimal parameter* θ^* as

$$\theta^* = \arg\min_{\theta} \left\{ \sup_{x\in X} |f(x|\theta) - g(x)| \right\}, \qquad (30)$$

so that the best fuzzy system approximator of $g(x)$ is $f(x|\theta^*)$. Let the *minimum approximation error* $w(x)$ be defined as

$$w(x) = f(x|\theta^*) - g(x). \qquad (31)$$

From Lemma 2, it is clear that $w(x)$ should satisfy the following inequalities:

$$\begin{aligned} |w(x)| &\leq \sup_{x\in X} |f(x|\theta^*) - g(x)| \\ &\leq \sup_{x\in X} |f(x) - g(x)| \leq \sum_{j=1}^{s} \left\| \frac{\partial g}{\partial x_j} \right\|_\infty h_j, \end{aligned} \qquad (32)$$

where the furthest item on the right can be designed in the construction procedure of the fuzzy system. Define the *tracking error* as $e_t(t+1) = y_r(t+1) - y(t+1)$, then

$$\begin{aligned} e_t(t+1) &= y_r(t+1) - y(t+1) \\ &= (y_r(t+1) - \widehat{y}(t+1)) \\ &\quad + \phi^T(x(t))(\theta(t) - \theta^*) + w(x(t)). \end{aligned} \qquad (33)$$

Recall that our ultimate objective is to make the process output $y(t+1)$ track the reference trajectory $y_r(t+1)$. From (33), it is clear that the tracking error $e_t(t+1)$ is influenced by three parts: (i) how close the control action $u(t)$ can bring the model output $\widehat{y}(t+1)$ to the target $y_r(t+1)$, represented by $e_c(t+1)$, which is defined as $e_c(t+1) = y_r(t+1) - \widehat{y}(t+1)$ and is called the *control error* ($|e_c(t+1)| = J_c^{1/2}(t+1)$), (ii) how close the parameter estimate $\theta(t)$ is to the optimal parameter θ^*, described by $\phi^T(x(t))(\theta(t) - \theta^*)$, which is called the *parameter error*, and (iii) how close the best fuzzy model $f(x(t)|\theta^*)$ is to the unknown process nonlinearity $g(x(t))$, characterized by the minimum approximation error $w(x(t))$, which is called the *structure error*. The aim of the control algorithm in subsection D, the adaptation law in subsection C, and the fuzzy system design procedure in subsection B, is to minimize the control error, the parameter error, and the structure error, respectively. The *prediction error*, which is defined as $e_p(t+1) = \widehat{y}(t+1) - y(t+1)$, equals the sum of the parameter error and the structure error. The performance of the adaptive control scheme consisting of the fuzzy system design procedure, the adaptation algorithm, and the control algorithm, is described by the following theorems. Due to the page limitation, all proofs to the lemmata and theorems will be omitted.

Theorem 1: For the identification part, if the fuzzy system model (3) is used for the process (1), the fuzzy system is constructed as in subsection II.B. and its parameters are tuned by using the adaptation law (15)-(18) with deadzone chosen as $\Delta = \sum_{j=1}^{n} ||\frac{\partial g}{\partial x_j}||_\infty h_j$, then:

(i) the parameter discrepancy $\widetilde{\theta}(t) = \theta(t) - \theta^*$ satisfies

$$||\widetilde{\theta}(t)||^2 \leqslant \kappa ||\widetilde{\theta}(0)||^2, \qquad \text{for } t \geqslant 1, \qquad (34)$$

where κ =condition number of $[P(-1)^{-1}] \triangleq \frac{\lambda_{\max} P(-1)^{-1}}{\lambda_{\min} P(-1)^{-1}}$;

(ii) the prediction error $e_p(t+1) = \widehat{y}(t+1) - y(t+1)$ satisfies

$$\limsup_{t\to\infty} \frac{e_p^2(t)}{1 + \phi^T(x(t-1))P(t-2)\phi(x(t-1))} \leqslant \Delta^2. \qquad (35)$$

Remark 2: From the expressions of triangular membership function (4)-(5) and regression vector (13)-(14), it can be seen that in the set X, $\phi(x(t-1))$ is always bounded in terms of Euclidean length. Therefore $1 + \phi^T(x(t-1))P(t-2)\phi(x(t-1))$ and also $\limsup_{t\to\infty} |e_p(t)|$ is bounded. Moreover, since h_j in Lemma 2 can be made arbitrarily small by defining sufficiently many fuzzy sets, Δ can be made less than any given value. However, there is no guarantee that the parameter vector $\theta(t)$ will converge to θ^*.

Theorem 2: For the control part, the control algorithm provides the best control result that can be achieved via a single-step allowable control action. That is to say, using the algorithm, an allowable control $u_c(t)$ can always be found, which globally minimizes the control error function $J_c(t+1)$, i.e.,

$$J_c(t+1|u(t) = u_c(t)) = \min_{u(t)\in[u_{\min}, u_{\max}]} J_c(t+1). \qquad (36)$$

Theorem 3: Consider the overall adaptive control scheme applied to nonlinear process (1). The tracking error $e_t(t+1) = y_r(t+1) - y(t+1)$ satisfies

$$\limsup_{t\to\infty}|e_t(t+1)| \leqslant \limsup_{t\to\infty}|e_p(t+1)| + \limsup_{t\to\infty} J_c^{1/2}(t+1). \quad (37)$$

Remark 3: In Theorem 1: (i) the parameter vector θ is bounded since θ^* is finite and (34) is true; (ii) since the output of the fuzzy system $\widehat{y}$ is bounded (θ and ϕ are both bounded), the prediction error e_p is bounded under the assumption that the process operating region involved is compact and on it the process $g(x)$ is continuous and has bounded partial derivatives with respect to all components of x. In Theorem 2, the one-step-ahead control error function J_c is bounded provided that the desired value y_r is finite since the output of the fuzzy system $\widehat{y}$ is bounded. In Theorem 3, the tracking error e_t is bounded if J_c is bounded since (37) is true.

III. Wastewater Neutralization Control

A. Neutralization process

The process under study is the same as that in [3], [5] and described as follows. A strong acid flows into a tank and is there thoroughly mixed with a strong base whose inward rate of flow is controller in such a way as to produce a neutral outward flow from the tank. Fig. 3 depicts a simple diagram of a standard wastewater neutralization tank.

Because the acid and the base are strong, each is completely dissociated, and also the dissociation of the water can be disregarded. This can be described by the following model:

$$V\dot{z} = d(a-z) - u(b+z), \quad (38)$$

where

V =volume of the tank,
d =rate of flow of the acid,
a =concentration of the acid,
u =rate of flow of the base,
b =concentration of the base, and
$z = [H^+] - [OH^-]$ is the distance from neutrality.

The pH value y can be calculated from the following nonlinear transformation:

$$z = 10^{-y} - 10^{y}K_w, \quad (39)$$

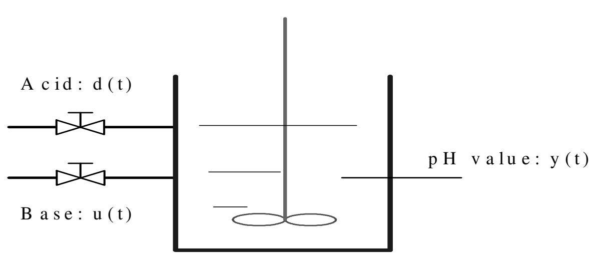

Fig. 3. The wasterwater neutralization system.

where K_w = water equilibrium constant = 10^{-14} (unit: gram-ion/liter).

It is assumed parameters a, b and V are fixed and known. For simulation purposes, their values used here are the same as in [3]: $V = 2$ liter, $a = 10^{-3}$ mol/liter, $b = 10^{-3}$ mol/liter.

The following approximation discrete-time model is developed for the continuous equation (38):

$$z(t+1) \simeq z(t) + \frac{T}{V}\left[d(t)(a - z(t)) - u(t)(b + z(t))\right], \quad (40)$$

where the sampling interval T is 1 minute.

From (39), it can be seen that the pH value $y(t+1)$ is a function of $z(t+1)$:

$$y(t+1) = h(z(t+1)) = \log_{10}\left(\frac{10^{14}z(t+1) + \sqrt{10^{28}z^2(t+1) + 4\times 10^{14}}}{2}\right). \quad (41)$$

By using (40), $y(t+1)$ can be further represented as a function of $y(t)$, $u(t)$ and $d(t)$ in the following N-ARMA form:

$$y(t+1) \simeq g\left(y(t), u(t), d(t)\right), \quad (42)$$

where $g(\cdot)$ is a nonlinear function.

B. Simulation

The objective is to control the pH value y in the tank by manipulating the base flow rate u. Two types of control tasks are considered in the following:

(i) Setpoint tracking. The control objective is to make the pH value follow the following reference trajectory closely:

$$y_r(t) = \begin{cases} 7, & t < 100, \\ 8, & 100 \le t < 150, \\ 9, & 150 \le t < 200, \\ 8, & 200 \le t < 250, \\ 7, & 250 \le t < 300, \\ 6, & 300 \le t < 350, \\ 5, & 350 \le t < 400, \\ 6, & 400 \le t < 450, \\ 7, & 450 \le t < 500 \\ 8, & 500 \le t < 550 \\ 9, & 550 \le t < 600 \\ 8 & 600 \le t. \end{cases} \quad (43)$$

The acid flow rate $d(t) = 0.1125$ liter/minute for the whole time.

(ii) Disturbance rejection. The control objective is to keep the pH value at a preset level ($y(t) = 7$) regardless of the disturbance due to the acid flow variation as follows:

$$d(t) = \begin{cases} 0.1125 \text{ liter/minute}, & t < 100, \\ 0.1125 + 0.0125\sin\frac{\pi t}{25} \text{ liter/minute}, & 100 \le t. \end{cases} \quad (44)$$

The fuzzy prediction model is

$$\hat{y}(t+1) = f\left(y(t), u(t), d(t)\right), \quad (45)$$

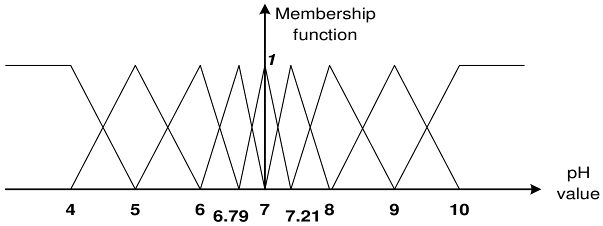

Fig. 4. Membership functions for pH value.

where $f(\cdot)$ is a Mamdani type fuzzy system in the form of (7), with a product inference engine, a singleton fuzzifier, a center-average defuzzifier. The fuzzy sets for the pH value $y(t)$ are described by nine triangular membership functions which are shown in Fig. 4.

For $u(t)$ and $d(t)$, four fuzzy sets with equally-spaced triangular membership functions were chosen to uniformly cover their range, respectively, which are both $[0.1, 0.1125]$. All parameters in the fuzzy system are initialized as zero and tuned on-line based on the input-output observations of the process and $P(-1) = 10^5 I$. Both simulations start at initial states $y(0) = 11$ and $u(0) = 0.1$ liter/minute.

Fig. 5 and 6 show the control results for the setpoint tracking problem and the disturbance rejection problem, respectively. It can be seen that the adaptive fuzzy controller works very well both in tracking a stair-step reference signal and in disturbance rejection, and the control error becomes smaller and smaller as the controller keeps on learning.

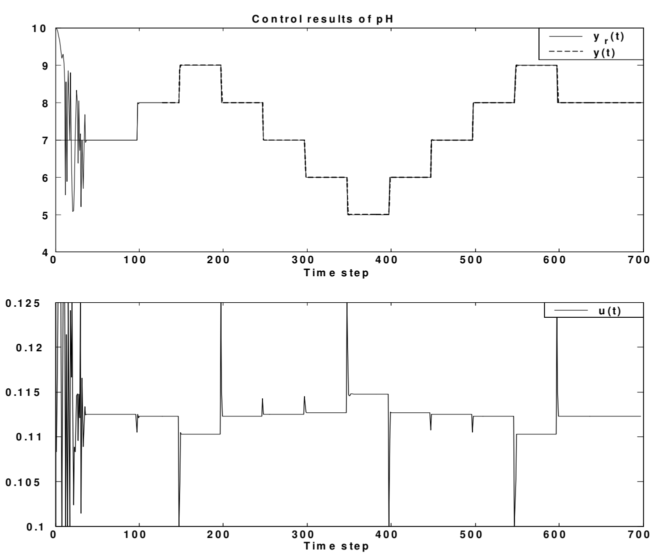

Fig. 5. Control of pH in acidic wastewater using the adaptive fuzzy controller: setpoint tracking problem. Top: Reference trajectory and the controlled outputs of the process. Bottom: Control actions.

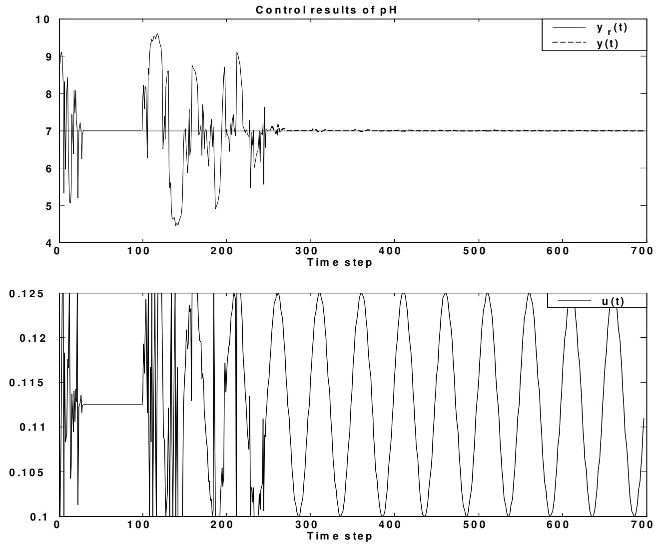

Fig. 6. Control of pH in acidic wastewater using the adaptive fuzzy controller: disturbance rejection problem. Top: Reference trajectory and the controlled outputs of the process. Bottom: Control actions.

IV. CONCLUSION

An adaptive fuzzy control scheme has been developed in this paper for neutralization process control. The process is modeled by a standard Mamdani fuzzy system, and the fuzzy system parameters are adjusted using the least square algorithm with dead to tune. The control action is computed by making use of the special internal properties of the fuzzy system, which globally minimizes the cost function comprising the errors between the predicted outputs of the process and the reference trajectory. Performance analyses and simulation results in control of an industrial wastewater treatment system show the effectiveness of the proposed scheme.

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