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Control Allocation for the X-33 using Existing and Novel Quadratic Programming Techniques †

Adam T. Simmons*
Department of Electrical and Computer Engineering
Auburn University
200 Broun Hall, Auburn, AL 36849-5201
simmoat@auburn.edu

A. Scottedward Hodel**
Department of Electrical and Computer Engineering
Auburn University
200 Broun Hall, Auburn, AL 36849-5201
a.s.hodel@eng.auburn.edu

***Abstract*— Control allocation (CA) is the distribution of control effort among a set of (possibly redundant) effectors to achieve a desired objective. In the case of aerospace vehicles, CA is the selection of effector commands in order to induce a set of commanded moments (torques) on an aerospace vehicle. These moment commands are typically generated by an autopilot/attitude control law that operates in closed loop with a guidance (attitude command) law as well as a CA law and the vehicle itself.**

In this paper several quadratic programming (QP) based CA laws in the context of control for an X-33 based next generation aerospace vehicle model are examined. These laws include Least Squares (no inequality constraints in QP), direction preserving CA, sign-preserving CA, frequency weighted CA and combinations thereof. These methods are evaluated in the context of both nominal and failed effector behavior so that reconfiguration behavior can be examined. Our simulation results to date indicate that a CA law based on frequency-weighted sign preserving QP problem provides superior performance to the other methods examined.

I. INTRODUCTION

The role of *control allocation* (CA), in the terms of aerospace engineering, is to compute a vector $\delta_c \in \mathbb{R}^{n_a}$ of effector commands that induce desired body-frame moments $\tau = \begin{bmatrix} L & M & N \end{bmatrix}^T$(roll, pitch and yaw torques) on a vehicle, compensating for and/or responding to inaccuracies in off-line nominal CA calculations, effector failures and/or degradations (reduced effectiveness), or effector limitations (position saturation). The command vector δ_c may govern the behavior of, e.g., aerosurfaces, reaction thrusters, engine gimbals and/or thrust vectoring.

In the past, the CA problem has been approached with simple methods such as *least squares* [1] or ganging [2]. The ganging CA method associates effector combinations to respective moments, such as using opposite left and right aileron deflections to produce a specific roll moment. The least squares method uses a pseudo-inverse of a reference model and determines the effector deflections as a function of the commanded, or desired, moments.

Although these methods are easily implemented and computationally efficient, they do not reconfigure in effector failures, nor do they consider effector command limits. Effector limits result from either or both of (1) effector hardware design and (2) effector degradation or failure. Position and rate limits are an inherent part of hardware design, and are the result of a trade-off between cost, weight, and vehicle capability. These effector limits define the vehicle's attainable moment set (AMS)$\mathcal{T}$ or, the set of all moment vectors that are achievable within the control constraints (e.g., position limits on the control surfaces)[7][11].

Recent advances in computational power and software performance permit the use of on-line solution of *quadratic programming* (QP) optimizations in CA. Quadratic programming maximizes, or minimizes, a quadratic cost function subject to linear equality and/or inequality constraints [2]. Adding the complexity of constraints and the optimization of a cost function, the QP method for CA has the potential to account for redundancy in the moment generating power of the effectors and the constraints that limit each effector. Within QP, CA laws can use the cost function to weight, or penalize certain effectors within the system, while forcing the system to follow other constraints. In this paper, several quadratic programming techniques are examined and compared, e.g. Direction Preserving, Sign Preserving, Frequency Weighted, Frequency Domain Shaping, and combinations of these algorithms.

The results of the previous algorithms are compared using a high fidelity simulation of the X-33 Reusable Launch Vehicle (RLV). The X-33 is a wedge shaped RLV with 12 effectors: four engines, right/left flaps, right/left rudders, right/left inboard elevons, and right/left outboard elevons. Because the original X-33 was not capable of achieving orbit, we doubled the ISP (specific impulse) for our simulation examples. This fictitious vehicle is called the "Super X-33."

II. SOLUTIONS TO THE CA PROBLEM

The control allocation problem is to identify a set of vehicle effector commands δ_c such that the achieved body torques τ_b acting on the vehicle match or closely approximate the torques commanded, τ_c by the autopilot. Fast, reliable methods for CA are becoming more important as controllable vehicles become more complex, and online computation becomes less expensive. There are three general classes of CA in current common use: direct CA,

† This Research was funded by NASA-NGT8-52904
*Graduate Student, Auburn University
**Faculty, Dept. Electrical and Computer Engineering

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generalized inverses, and daisy chaining [2], [5], [8].

A. Direct Control Allocation

Direct control allocation, proposed by Durham [7], is an approach based on the concept of the attainable moment set (AMS). The AMS $\mathcal{T}$ is the set of all moment vectors that are achievable within a set of control constraints. This method, while computationally expensive, allows the entire attainable moment set to be used. Direct control allocation maps the control space, defined by all possible deflection commands $\{\Delta_c\}$, into the attainable moment space $\mathcal{T}$. To do this, three basic steps are followed; (1) Determine the AMS, (2)Scale the desired, or commanded torque vector to the boundary of the AMS, (3) Solve for the intersection of the AMS and the scaled desired torque vector, and (4)Scale the result by the inverse of the original scaling coefficient from step (2). Computation of the AMS is the most computationally expensive step in direct control allocation. Methods for computation of the AMS are discussed further in [6], [7] and [8].

B. Generalized Inversion Solution

A generalized inverse solution to CA is described by Durham and Bordignon [3] as the selection of a constant matrix $F(x)$ that satisfies $G(x)F(x) = I$ according to $\tau_b = G(x)\delta$, where τ_b are the three orthogonal vehicle moments and δ is the vector of effector commands[8]. The effector commands are then directly obtained from a generalized inverse in response to a desired moment, τ_c where $\delta_c = F(x)\tau_c$. The generalized inverse is more flexible and preferred with modern aircraft, since most vehicle effectors do not act on the vehicle in just one axis. We can further define this generalized inverse solution as a matrix $F(x) : \mathbb{R}^n \rightarrow \mathbb{R}^m$ such that $F(x)G(x) = I_n$ [3], [4].

The most common technique for choosing the generalized inverse matrix $F(x)$ is to choose an inverse such that, in certain specified directions of the moment space, the inverse will yield an inversion allowing the solution to coincides with the attainable moment set (AMS). This can be done by the computation of the solution to a geometric problem where an intersection of the control space and the subset of a constrained control space maps to the required point(s) in moment space. Durham describes this technique and a solution to this geometric problem in [6].

Another method for the computation of the generalized inverse use in this solution, is using a weighted pseudo-inverse. This is done by introducing a weighting matrix when calculating a pseudo-inverse. First, a pseudo-inverse is described by $F = G(x)^T[G(x)G(x)^T]^{-1}$. After placing the arbitrary weighting matrix N into the pseudo-inverse, the problem is described as

$$F = NG(x)N^T[G(x)N(G(x)N)^T]^{-1}$$

where the values of the matrix N are tailored to emphasize, or to deemphasize, a certain set of controls [13]. The pseudo-inverse is often used because it results in a minimum Euclidean norm solution (vector of effector commands), therefore the use of a pseudo-inverse minimizes the movement of the effector deflection commands.

C. Daisy Chaining Solution

Daisy Chaining is a fairly new method for the solution of the control allocation problem[3]. The method works off of a solution obtained by partitioning the control vector, δ and the linear effector authority matrix, $G(x)$ into different groups. These groups are then employed in succession as each previous group of controls are saturated [8]. If you split the two vectors (δ and $G(x)$) into k groups, then the new vectors appear as

$$G(x) = [G(x)_1 G(x)_2 \cdots G(x)_k] \text{ and } \delta^T = [\delta_1 \delta_2 \cdots \delta_k]$$

where $G(x)_i, i = 1, \ldots, k$ are considered full rank and invertible. Each inverted $G(x)_i$ has a corresponding $F(x)_i$ such that $G(x)_i F(x)_i = I$. The controls are taken similarly as in the generalized inversion solution, but each control, $\delta_{\mathrm{j}}, 1 \leq j \leq k$, are only employed if one or more control in the previous $i < j$ groups are saturated. The subsequent groups only make up for saturation from the preceding groups [6].

III. More recent developments

Because the CA problem has become more pertinent for new technological developments in aircraft and the issue of computational expense is becoming a less prominent challenge, mathematical programming techniques have made their way into CA approaches. Two of the most prominent methods of mathematical programming are linear programming (LP) and quadratic programming (QP). These techniques are similar in that they both minimize a cost function subject to a set of equality and inequality constraints.

IV. Infeasible CA problems

LP and QP based CA techniques are gaining increasing attention due to their ability to respond on-line to effector limits (position, rate) through adjustment of the inequality limits δ^-, δ^+, and vehicle health information (such as degraded or unresponsive effectors) through adjustment of the gain-scheduled linear affine effector model matrix $G(x)$. An LP or QP problem is considered *feasible* if there is a vector $\delta \in \Delta$ that satisfies the linear constraint $G(x)\delta = \tau_c$. Otherwise the problem is considered to be *infeasible*. An infeasible problem arises when τ_c is outside of the AMS $\mathcal{T}$. Two techniques are presented in this paper that address infeasible QP problems: the direction preserving method and the sign preserving method [12].

V. CA Problem Specification

We will assume the existence of an off-line computed nominal linear affine CA function

$$\delta_c = F(x)\tau_c + \delta_0(x) \tag{1}$$

where τ_c is the commanded torque vector, x is a vehicle state vector, δ_0 is a trim (neutral torque) vector and $F(x)$ is a matrix of nominal CA gains. One may interpret the columns of $F(x)$ as a set of gains defining "ganged" effectors for each control axis. Ideally, the control allocation matrix $F(x)$ would be chosen to be the pseudo-inverse $G(x)^\dagger$ of the Jacobian Matrix

$$G(x) = \left[\begin{array}{c} \frac{\partial \tau_{b,i}}{\partial \delta_j} \end{array} \right] \Big|_x \in \mathbb{R}^{3 \times n_a} \tag{2}$$

where n_a is the number of effectors; that is the optimal design of $F(x)$ produces

$$G(x)F(x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

so that the induced body-frame vehicle torques τ_b match the commanded body torques τ_c.

The linear affine nominal CA law (1) by itself is inadequate for the CA problem for four reasons:

1) It fails to respond to torque allocation errors that can be detected on-line.
2) It fails to take into account saturation issues.
3) It fails to respond to on-line detected failures in effectors, and,
4) It fails to provide a framework to work with discrete-valued (on-off) effectors such as reaction thrusters.

We use Hodel and Callahan's dynamic CA (DCA) approach [10] to compensate for the effects of failures that are *unknown* to the CA system. Our simulations use a PI control law for the DCA component. CA reconfiguration for *known* failures and degradations is achieved by modification of the appropriate entries of the effector linear-affine model, e.g., by scaling the appropriate column of the Jacobian matrix $G(x)$). For complete failure of an effector, the corresponding column of $G(x)$ is zeroed out to eliminate the effector from the CA solution. For partial failure, each column is scaled by a percentage of failure (i.e. 100% scales a column of $G(x)$ by 0). This is proved in [10] as a viable solution to reconfiguration in an unconstrained CA solution (e.g. no effector limits).

Within this paper, we will use the reconfiguration routine mentioned above and several novel and existing quadratic programming techniques to prove the advantages of a reconfigurable quadratic programming control allocation solutions in a variety of optimization perspectives, e.g. frequency component reduction, guaranteed feasibility. The Least Squares algorithm, which is the least complicated CA solution, will be compared to existing and new QP algorithms, e.g. Direction Preserving, Sign Preserving, Frequency Weighted, Frequency Domain Shaping, and a combination of Frequency Weighted Sign Preserving.

A. Least Squares Problem

The Least Squares (LS) solution is determined using an optimization of some cost function subject to an equality constraint.

$$\begin{array}{ll} \min\limits_{\delta_c} & \frac{1}{2}\delta_c{}^T Q \delta_c + c^T \delta_c \\ \text{subject to} & G(x)\delta_c = \tau_c \end{array} \tag{4}$$

As defined in the aerospace CA problem, the unknown parameter in the LS solution is the command vector δ_c. The weighting matrices, Q and c correspond to penalties on the quadratic and linear components of the cost function. Without inequality constraints (i.e. effector position limits), we consider the LS solution to be the most simplistic programming CA solution. In fact, the LS solution is reduced to a weighted pseudo-inverse (a.k.a Generalized Inverse Solution) for specific weighting matrices (c is zero). For the purposes of this paper, the LS solution is clipped with the assumption the simulation will only accept effector commands within the position constraints (δ_- and δ_+). More recent solutions to the Least Squares active constraint problem have been proposed by Härkegard which incorporates the inequality constraints in a Least Squares solution by iteration [9]. For this paper, the inequality constraints are implemented in a Quadratic Programming solution, which allows for specific weighting and optimizations.

B. Quadratic Programming

A Quadratic Programming solution minimizes a quadratic cost function subject to both equality and inequality constraints. For the aerospace problem, the determination of the command vector δ_c is determined by using QP.

$$\begin{array}{ll} \min\limits_{\delta_c} & \frac{1}{2}\delta_c{}^T Q \delta_c + c^T \delta_c \\ \text{subject to} & G(x)\delta_c = \tau_c \\ \text{and} & \delta_c^- \leq \delta_c \leq \delta_c^+ \end{array} \tag{5}$$

Where the weighting matrices Q and c are the weights on the quadratic and linear functions of the cost respectively. The equality constraint $G(x)\delta_c = \tau_c$ guarantees the solution δ_c matches the commanded torque moment vector τ_c. Finally, the inequality constraints force the solution δ_c to be within the effector high and low position limits δ_c^- and δ_c^+.

The solutions to QP will always be feasible as long as the commanded torque vector τ_c is within the attainable moment set (AMS). When the commanded torque vector τ_c is outside of the AMS, the QP solution is infeasible, or outside of the effector position limits. Two existing approaches have been used to modify the QP solution in which the solution is guaranteed feasible, e.g. Direction Preserving and Sign Preserving.

1) Direction Preserving: In order to guarantee feasibility of the QP problem in CA applications, we elect to modify the QP problem, Equations (5), to include variables that have some kind of guaranteed feasibility.

$$\begin{array}{ll} \min\limits_{\delta,\sigma} & \frac{1}{2}\delta_c{}^T Q_\delta \delta_c + c_\delta{}^T \delta_c + Q_\sigma (1-\sigma)^2/2 \\ s.t. & G(x)\delta_c - \sigma\tau_c = 0 \\ & \begin{bmatrix} \delta_c^- \\ 0 \end{bmatrix} \leq \begin{bmatrix} \delta_c \\ \sigma \end{bmatrix} \leq \begin{bmatrix} \delta_c^+ \\ 1 \end{bmatrix} \end{array} \tag{6}$$

The direction preserving (DP) method preserves the direction of the moment vector τ_c, while allowing the minimization to scale the moment vector magnitude with a scaling factor σ. By using a much higher weight on sigma Q_σ than on the deflection commands Q_δ the minimization utilizes the deflection commands over the scaling factor σ. Figure 1 represents graphically the scaling factor's σ effect on the magnitude of an infeasible moment vector. By scaling

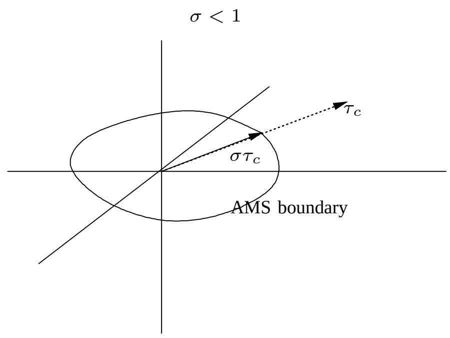

Fig. 1. Direction Preserving Method representation

all three moment vector components (τ_{roll},τ_{pitch},τ_{yaw}) together, the direction of the total moment vector is maintained.

2) Sign Preserving: The sign preserving (SP) method of QP solves the minimization similarly, but allows the scaling σ to be split amongst all three torque vector components individually $\sigma_{roll}, \sigma_{pitch}$, and , σ_{yaw} corresponding to τ_{roll},τ_{pitch}, and τ_{yaw} respectively. Also the minimization weights Q_σ and Q_δ are chosen such that the scaling factors are more heavily weighted (i.e. $Q_\sigma >> Q_\delta$). For the purposes of this paper, the scaling factor weights Q_σ are chosen to be the same for $\sigma_{roll}, \sigma_{pitch}$, and , σ_{yaw}. Figure 2 graphically represents the scaling effect of an infeasible commanded moment vector.

$$\begin{aligned}
\min_{\delta,\sigma} \quad & \tfrac{1}{2}{\delta_c}^T Q_\delta \delta_c + {c_\delta}^T \delta_c + \tfrac{1}{2} Q_\sigma (1-\sigma_{roll})^2 \\
& + \tfrac{1}{2} Q_\sigma (1-\sigma_{pitch})^2 \\
& + \tfrac{1}{2} Q_\sigma (1-\sigma_{yaw})^2 \\
s.t. \quad & G(x)\delta_c - \Sigma\tau_c = 0 \\
& \begin{bmatrix} \delta_c^- \\ 0 \\ 0 \\ 0 \end{bmatrix} \leq \begin{bmatrix} \delta_c \\ \sigma_{roll} \\ \sigma_{pitch} \\ \sigma_{yaw} \end{bmatrix} \leq \begin{bmatrix} \delta_c^+ \\ 1 \\ 1 \\ 1 \end{bmatrix} \\
where \quad & \Sigma = \begin{bmatrix} \sigma_{roll} & 0 & 0 \\ 0 & \sigma_{pitch} & 0 \\ 0 & 0 & \sigma_{yaw} \end{bmatrix}
\end{aligned} \tag{7}$$

By scaling all three moment vector components (τ_{roll},τ_{pitch},τ_{yaw}) individually, the sign of the total moment vector is maintained, but the direction may be modified. This allows the CA solution to scale independently the components of the moment vector with the least control authority (function of states x in "Jacobian" $G(x)$). In

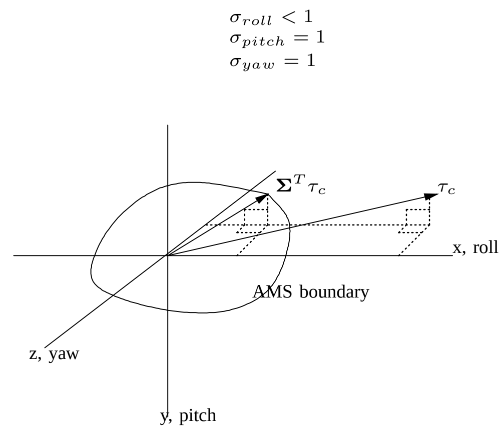

Fig. 2. Sign Preserving Method representation

Figure 2, the $roll$ control authority is small, therefore the sign preserving algorithm maintains 100% of both $pitch$ and yaw while scaling $roll$ to achieve a feasible solution.

Not only is feasibility an important issue in CA, but wear and tear on the vehicle's hardware, or effector, needs to be considered. An effector command with a low frequency component is much more desirable than a high frequency effector command. High frequency effector commands can produce unnecessary wear on specific hardware devices (i.e. electromechanical actuators). One new method of QP which addresses the issue of frequency components in effector commands is Frequency Weighted Quadratic Programming.

3) Frequency Weighted: We have mentioned so far the aspects of CA which consider the absolute effector deflection limits and the rate at which they are achieved, but we can also consider the frequency content, or bandwidth, of the commanded effector deflections. By building on existing QP, LP and LS algorithms, we examine modifications to cost functions in order to reflect the frequency content of the effector commands. The goal of this approach is to reduce physical wear on effector actuators caused by rapid change in the direction of motion due to "jittery" commanded signals.

Modifying the weights of the QP problem, the solution combines frequency content into the optimization.

$$\begin{aligned}
\min_{\delta} \quad & \tfrac{1}{2}{\delta_{lo}}^T Q_{lo} \delta_{lo} + {c_{lo}}^T \delta_{lo} + \tfrac{1}{2}{\delta_{hi}}^T Q_{hi} \delta_{hi} \\
& + {c_{hi}}^T \delta_{hi} \\
\text{s.t.} \quad & G(x)\delta_c(n) = \tau_c(n) \\
& \delta_c^- \leq \delta_c \leq \delta_c^+ \\
\text{where} \quad & \delta_{lo} = \textstyle\sum_{i=0}^m a_i \delta_c(n-i) \\
& \delta_{hi} = \textstyle\sum_{i=0}^m b_i \delta_c(n-i)
\end{aligned} \tag{8}$$

The frequency-weighted cost function approach is developed as follows. Given two FIR filters, a low pass filter $\delta_{lo}(n) = a_0\delta_c(n) + a_1\delta_c(n-1) + \cdots + a_m\delta_c(n-m)$ and a high pass filter $\delta_{hi}(n) = b_0\delta_c(n) + b_1\delta_c(n-1) + \cdots + b_m\delta_c(n-m)$, we modify the quadratic cost function to penalize δ_{hi} more heavily than δ_{lo}. At each time step, the $\delta_c(n-1)\ldots\delta_c(n-m)$ terms are known, and can be restated in the quadratic cost function as a constant, w_{lo} and w_{hi} respectively. This can be written in the form of our standard QP (5) by setting the quadratic weighting matrix $Q = \left[\ a_o^2 Q_{lo} + b_o^2 Q_{hi}\ \right]$, the linear cost vector $c = \left[\ a_o {w_{lo}}^T Q_{lo} + a_o c^T + b_o {w_{hi}}^T Q_{hi} + b_o {c_{hi}}^T\ \right]^T$, and maintaining the same variables. This method only modifies the weighting matrices.

4) Frequency Domain Shaping: Another method for introducing a filtering term in the problem is to introduce it in the equality constraint and not the cost function alone.

$$\begin{aligned} \min_{\delta} \quad & \tfrac{1}{2}{\delta_c}^T Q \delta_c + c^T \delta_c + Q_\sigma (1-\sigma)^2 \qquad (9) \\ s.t. \quad & G(x)[\delta_c(n) - \delta_c(n-1)] = \Sigma[\tau_c - G(x)\delta_c(n-1)] \\ & \delta_c^- \le \delta_c \le \delta_c^+ \end{aligned}$$

To do this, a difference equation is used as the parameter in the equality constraint, $\delta_c(k) - \delta_c(k-1)$ This incorporates frequency-domain shaping behavior in the equality constraints. With the addition of the new term, the equality constraint manages the difference between the actual torque implemented on the previous time step $G(x)\delta_c(k-1)$, with the σ scaled version of the same torque $\Sigma G(x)\delta_c(k-1)$ (where Σ is a vector of the three sigma values from the sign preserving method previously mentioned). The frequency domain shaping equality constraint can be rewritten in the form of our standard QP problem (5) by setting the unknown vector $\delta_c = \left[\ \delta_c(n) \quad \Sigma\ \right]^T$, the quadratic weighting matrix $Q = \begin{bmatrix} Q_\delta & 0 \\ 0 & Q_\Sigma \end{bmatrix}$, the linear cost vector $c = \left[\ {c_\delta}^T \quad -2 * Q_\Sigma\ \right]^T$, $G(x) = \left[\ G(x) \quad -\tau_c + G(x)\delta_c(n-1)\ \right]$, $\tau_c = G(x)\delta_c(n-1)$, $\delta_c^- = \begin{bmatrix} \delta_c^- \\ 0 \end{bmatrix}$ and $\delta_c^+ = \begin{bmatrix} \delta_c^+ \\ 1 \end{bmatrix}$.

VI. Implementation

The use of a "Super X-33" simulation from NASA's Marshal Space Flight Center, allowed for testing and comparison of the different algorithms. The X-33 is a single stage vehicle with twelve effectors (right/left rudder, right/left in/out elevons, right/left flaps, four engines). The "Super X-33" simulation is a modified version of the X-33, which allows the vehicle to complete the ascent phase of flight. The ascent phase of flight goes from launch to 74,000 ft. and reaches Mach 27. A failure of an algorithm resulted in a loss of altitude, drastic reduction in velocity, and/or an attitude angle exceeding 180 degrees (conservative criteria).

VII. Results

Each algorithm discussed in the previous section was implemented in the "Super X-33" simulation through the ascent phase of flight, with and without failures. The ascent phase of flight takes the simulated vehicle from launch to Mach 27 and 74,000 ft. The results of each algorithm were reduced to the mean squared error of the attitude angles, and therefore could be easily compared.

TABLE I

MSE of Algorithms which completed Nominal Ascent Phase of flight, $cmd - experienced$

Algorithm	*Attitude Angle Mean Squared Error* $MSE = E\{[cmd - experienced]^2\}$			
	φ	θ	ψ	Average MSE (φ, θ, ψ)
FWSP	36.83	56.39	19.41	37.54
SP	60.62	20.58	38.71	39.98
FW	60.43	46.60	20.09	42.37
QP	70.32	56.27	12.40	46.33
LS (clipped)	44.57	138.82	21.45	68.28

A. Without Failures

In the nominal ascent phase of flight, without failures, the vehicle successfully flew with all but one of the previously described algorithms. The frequency domain shaping (FDS) algorithm failed for all cases of flight. The effector commands produced by FDS remained saturated due to the addition of error in the equality constraint ($\Sigma G(x)\delta_c(n-1)$). A list of all successful algorithms in the nominal ascent phase of flight can be seen in Table I. From Table I the lowest mean squared error occurs in the frequency weighted sign preserving (FWSP) algorithm.

The purpose of the FW and FWSP methods was to match the equality constraint $G(x)\delta_c = \tau_c$ and to reduce the use of high frequency components in the solution δ_c. For this paper, the FIR filters were chosen with a cutoff frequency of 10Hz for both 5 tap lowpass and highpass FIR filters. The frequency components of the nominal ascent phase effector commands for Engine 2 were examined by taking their Fast Fourier Transform (FFT). The FFT of the engine$_2$ commands can be seen in Figure 3. The QP solution δ_{eng2} has a high frequency component (FFT indicated a 22Hz High Frequency Component). For the FWSP solution δ_{eng2} the frequency at 22Hz is suppressed by the FIR filters (cutoff of 10Hz Low- and High-pass filters).

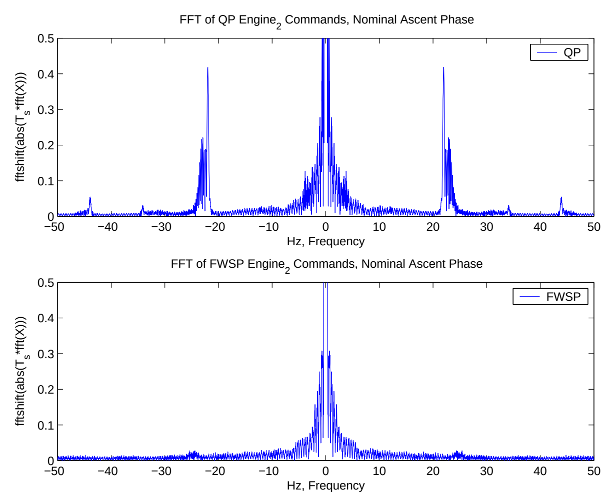

Fig. 3. FFT of Engine$_2$ commands for QP and FWSP, Ascent Phase

TABLE II
MSE OF ALGORITHMS,
RIGHT ELEVON FAILURE FROM 50SECONDS – END OF SIMULATION,
F = FLIGHT FAILURE S = SUCCESSFUL FLIGHT

Algorithm	*Attitude Angle Mean Squared Error* $MSE = E\{[cmd - experienced]^2\}$				
	φ	θ	ψ	Average MSE (φ, θ, ψ)	
SP	42.4	85.5	21.2	49.67	S
FWSP	39.7	89.5	20.1	49.80	S
QP	98.3	56.0	20.5	58.23	S
FW	783.2	125.6	728.5	545.8	F
LS (clipped)	249.9	217.1	2,036	834.3	F

B. With Failures

A 50 percent degradation in both inner and outer Right elevons occurred at 50 seconds for the remainder of the flight. The corresponding Mean Square Error results for the attitude angles are shown in Table II. The Frequency Weighted Sign Preserving and Least Squares methods were the only algorithms which actually failed (F) with a right elevon failure. The other algorithms succeeded (S) performed comparable to their nominal case. Although the Sign Preserving method did out-perform the FWSP method, the results of the FWSP method still reduced the higher frequency component. The reduction of the this high frequency component is an acceptable result for a very small difference in the total Mean Square error (MSE).

VIII. CONCLUSIONS AND FUTURE WORK

Quadratic programming, and more specifically Frequency Weighted Sign Preserving (FWSP) QP allows the user to define weights which give the algorithm a decision making procedure. If the CA solution is feasible, the optimal solution is found according to frequency components only. FWSP method uses the slack scaling variables of the SP method both to ensure that the QP problem is feasible and to adjust the frequency content of the resulting effector commands. The FWSP algorithm gives the user control over the decision process in quadratic programming CA. FWSP quadratic programming produces a solution with lower frequency content, and lower Mean Square Attitude Angle Error than other quadratic programming techniques.

IX. ACKNOWLEDGMENTS

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