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Feedback Control Design for an Elastance-Based Mock Circulatory System

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Abstract – A state-feedback controller was designed to mimic the Starling response of the heart in a mock circulatory system (MCS). Reference volume trajectory of the mock ventricle was determined by the maximum ventricular elastance (E_{max}), end-systolic ventricular pressure, and left atrial pressure. The controller drives a voice coil actuator (VCA) to follow the reference volume, and thus generate the desired chamber pressure, by using position and speed feedbacks. The nonlinear load applied to the VCA was compensated for by using the chamber pressure measurement to improve the controller response. The controller was tested in computer simulation by changing the load conditions and E_{max} of the MCS. The MCS along with the controller was able to reproduce human heart function from healthy to sick conditions. This control algorithm will be implemented in digital signal processor to control the MCS for left ventricular assist device testing.

1. Introduction

As heart disease remains a considerable health problem around the world, the development of both equipment and methodologies for its treatment are of great interest and priority. While the heart transplant is the most widely accepted method for dealing with severe cases of the disease, demand for these transplants exceeds the supply available. Thus, ventricular assist devices (VADs) are being developed as alternatives. Indeed, as the reliability and performance of VADs improves, they are becoming increasingly viable for long-term implants in addition to their traditional role as a bridge to native heart transplantation. There are other encouraging signs for the future of VADs – recently several successful rehabilitations of patients using VADs have been reported[1], implying that the use of such devices is not limited to long-term implantation or as bridge-to-transplant, but also as bridge-to-recovery.

[1] Promising as their future may be, however, there remain several distinct problems in the development, in particular the evaluation, of VADs. Testing of the device and its control strategy is usually performed via animal experimentation or the use of mechanical mock circulation loops. There are several disadvantages associated with both methods – animal testing is costly and time-consuming, while most mock circulation loops available to date have limitations in simulating the native heart in response to the load changes due to VAD intervention. This hemodynamic response is very important in assessing VAD performance under various cardiovascular functions, in particular for evaluating its controller.

The limitations described above originate from the fixed-stroke nature of traditional mock loops. Although ventricular elastance, $E_v(t)$, defined by

$$E_v(t) = \frac{P_v(t)}{[V_v(t) - V_0]}, \tag{1}$$

where $P_v(t)$ and $V_v(t)$ are the ventricular pressure and volume and V_0 is the un-stressed volume, is consistent regardless of the load changes to the ventricle [2] and the maximum of $E_v(t)$ (E_{max}) is a good representation of the contractile state of the ventricle, controlling the MCS to follow a pre-defined $E_v(t)$ trajectory is limited to a small range of load variation [3,4]. Baloa et al [5] designed a pressure controller that drove the mock ventricular chamber pressure to track a reference pressure signal, calculated from (1) using the instantaneous volume measurement, $V_v(t)$, while obtaining $E_v(t)$ from a lookup table. However, it was found that preload (the venous pressure) of the MCS was dependent on its afterload (systemic resistance) [6], which was not physiologically meaningful. It was also determined that the robustness of the controller to pressure disturbances, such as introducing a VAD into the MCS, also needed improvement [6].

This paper presents a state feedback position controller with load compensation to track the chamber volume reference. The reference signal was determined by the heart rate, E_{MAX}, end-systolic chamber pressure, and left atrial pressure (representing the preload of the MCS). The MCS model, proposed by Baloa et al [5], was adopted with the addition of right atrial compliance and a venous return pumping mechanism for controller design and testing. Performance of the controller was evaluated by changing E_{max}, preload, and afterload. The resulting E_{max}, produced by the MCS with the controller, was consistent regardless of the pre- and afterload changes. This implies that the

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controller, along with the MCS, could be suitable for use as a VAD test platform.

2. Mock Circulatory System Model

An electrical analogue of the MCS is shown in Figure 1. This model was adopted from Baloa et al [5] with the addition (inside the dashed line in Fig. 1) of the right atrial compliance (C_{ra}) and the pressure dependent flow (Q_{ra}). The governing equations of the model are listed in (2) to (10). Q_{ra} represents the venous return flow, which is determined by the right atrial pressure in (9) [7]. C_{ra} and Q_{ra} represent a simplified model of the pulmonary circulation in the MCS. These two elements improve the independence of the preload (P_{pv}) from the afterload (R_l), and thus the modified MCS model is a more realistic approximation of the cardiovascular system. Q_{ra} was implemented in the MCS by regulating the speed of a rotary pump to achieve the desired flow rate as determined by (9). Table 1 provides the physical meanings and the values of the model parameters.

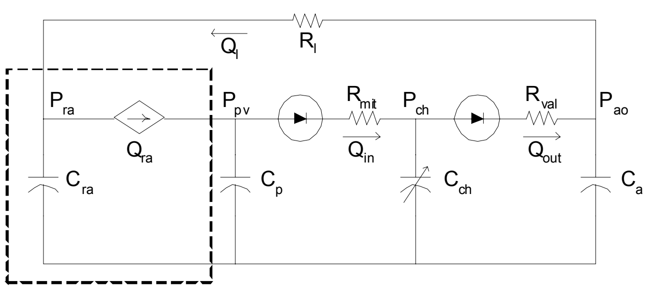

Figure 1. Electrical analogue of the revised MCS model

$$C_a \cdot \dot{P}_{ao} = Q_{out} - Q_l \tag{2}$$

$$C_p \cdot \dot{P}_{pv} = Q_{ra} - Q_{in} \tag{3}$$

$$C_{ch} \cdot \dot{P}_{ch} = Q_{in} - Q_{ch} - Q_{out} \tag{4}$$

$$C_{ra} \cdot \dot{P}_{ra} = Q_l - Q_{ra} \tag{5}$$

$$Q_{out} = \begin{cases} (P_{ch} - P_{ao})/R_{val} & , P_{ch} \geq P_{ao} \\ 0 & , P_{ch} < P_{ao} \end{cases} \tag{6}$$

$$Q_{in} = \begin{cases} (P_{pv} - P_{ch})/R_{mit} & , P_{pv} \geq P_{ch} \\ 0 & , P_{pv} < P_{ch} \end{cases} \tag{7}$$

$$Q_l = (P_{ao} - P_{ra}) / R_l \tag{8}$$

$$Q_{ra} = 240 \cdot [1 - \exp(-\frac{P_{ra} + 0.5}{3})] \tag{9}$$

$$Q_{ch} = \dot{V}_{ch} \tag{10}$$

Table 1. Model parameters

Parameter	Value (unit)	Description
A	60 (cm^2)	Cross-sectional area of pump-head
f_B	40 (N·s/m)	Frictional coefficient
C_2	100 (cm/m)	Conversion factor
C_1	0.013332 (Pa·cm^2/mmHg·m^2)	Conversion factor
C_a	1.37 (mL / mmHg)	Aortic compliance
C_{ch}	0.1832 (mL/mmHg)	Chamber/Left ventricular compliance
C_p	6.74 (mL/mmHg)	Pulmonary venous/Atrial compliance
K_1	23010.31	Controller gain
K_2	65.47851	Controller gain
k_e	3554600	Integrator gain
K_f	27 (N/A)	Current-force factor
K_{sp}	29348 (N/m)	Spring constant
l_1	7946.9	Observer gain
l_2	25019721	Observer gain
M	0.7533 (kg)	Pump-head mass
R_l	1 (mmHg·s/mL)	Systemic resistance
R_{mit}	0.005 (mmHg·s/mL)	Mitral valve resistance
R_{val}	0.005 (mmHg·s/mL)	Aortic valve resistance
V_0	10 (mL)	Residual volume in chamber
V_{bias}	315 (mL)	Maximum volume of chamber

3. Controller Design

The control algorithm was designed to produce the reference chamber volume, $V_{ref}(t)$, and drive the voice coil actuator (VCA) to track it. $V_{ref}(t)$ was determined by

$$V_{ref}(t) = (V_{ed} - SV) + SV \cdot v(t_n) , \tag{11}$$

where SV is the stroke volume of the chamber, V_{ed} is the end-diastolic chamber volume, and $v(t_n)$ is a normalized volume waveform, shown in Figure 2. The normalized time, t_n (defined between 0 and 1), is described by

$$t_n = \begin{cases} t_{S0} \cdot (t/t_S), & 0 \le t < t_S \\ t_{S0} + (1 - t_{S0}) \cdot [(t - t_S)/(t_C - t_S)], & t_S \le t < t_C \end{cases} \quad (12)$$

where t_{S0} is the time point at which $v(t_n)$ reaches 0, t_S is the systolic time interval determined by [8]

$$t_S = 0.14 + 0.2 \cdot t_C \quad (13)$$

and t_C is the cardiac period calculated by

$$t_C = 60 / HR \quad (14)$$

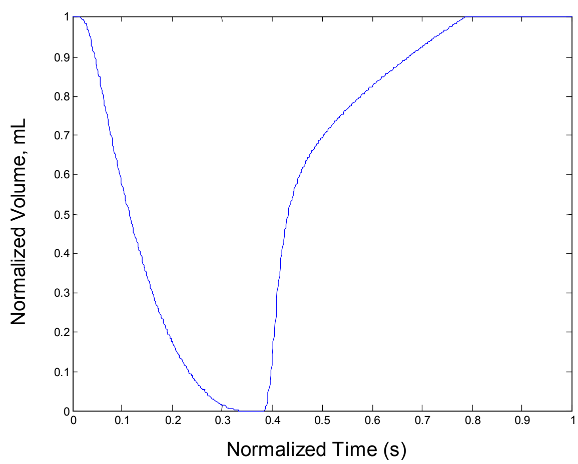

Figure 2. Normalized pre-defined volume waveform

V_{ed} was determined by means of a lookup table from mean left atrial pressure ($\overline{P}_{PV}$), produced by [9]

$$\overline{P}_{PV} = 0.01 \cdot \exp[0.053 \cdot V_{ed}] - 2 \cdot \exp[-0.06 \cdot V_{ed}] \quad (15)$$

Since $E_V(t)$ reaches its maximum value E_{MAX} near the end of systole [2], the end-systolic chamber volume V_{es} can be derived from (1) as

$$V_{es} = P_{es} / E_{max} + V_0, \quad (16)$$

where P_{es} is the end-systolic chamber pressure. By definition, V_{es} is the difference between the end-diastolic volume, V_{ed}, and the stroke volume,

$$V_{es} = V_{ed} - SV. \quad (17)$$

Combining (16) and (17) and solving for SV leads to

$$SV = V_{ed} - \frac{P_{es}}{E_{max}} - V_0 . \quad (18)$$

Since SV in (18) depended on the preload (V_{ed}), afterload (P_{es}) and ventricular contractility (E_{max}), the reference chamber volume obtained from (11) implicitly included the ventricular function in (1).

The mock ventricle was driven by a voice coil actuator. A state-feedback controller, as shown in Figure 3, was designed to control the VCA to track the reference position signal converted from $V_{ref}(t)$,

$$R(t) = [V_{bias} - V_{ref}(t)] / (A^*C_2), \quad (19)$$

where V_{bias} is the maximum chamber volume and C_2 is a unit conversion factor. The dynamics of the VCA can be represented by [5]

$$M \dot{v} = K_f \cdot i - K_{sp} \cdot x - f_B \cdot v - F_l \quad (20)$$

where K_f is the current-force constant and i is the current input to the VCA. K_{sp} is the spring constant, x is the position of the pump-head, f_B is the frictional coefficient and v is the velocity of the pump head. F_l is the load disturbance produced by the chamber pressure P_{ch}, which can be expressed as

$$F_l = A \cdot C_1 \cdot P_{ch}, \quad (21)$$

where A is the area of the pump head and C_1 is a unit conversion factor. F_l is non-linear, and can be compensated for by adding to the current command sent to the VCA. F_l can therefore be ignored, and the state vector $\underline{X} = [x_1 \ x_2]^T$ can be defined, where $x_1 = x$ and $x_2 = v = \dot{x}$, (20) can be written in a state-space form,

$$\begin{aligned} \dot{\underline{X}} &= A \cdot \underline{X} + B \cdot u \\ y &= x = C \cdot \underline{X} \end{aligned} \quad (22)$$

where $A = \begin{bmatrix} 0 & 1 \\ -K_{sp}/M & -f_B/M \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ K_f/M \end{bmatrix}$, $C = [1 \quad 0]$, and u=i. The pole placement method [10] was used to determine the control gains, $K=[k_1 \ k_2]$ and k_e. Integral control was introduced to improve the steady-state error. Since the VCA velocity, v, was not measurable, an observer [10] in Figure 3 was designed to estimate v using the input and output signals.

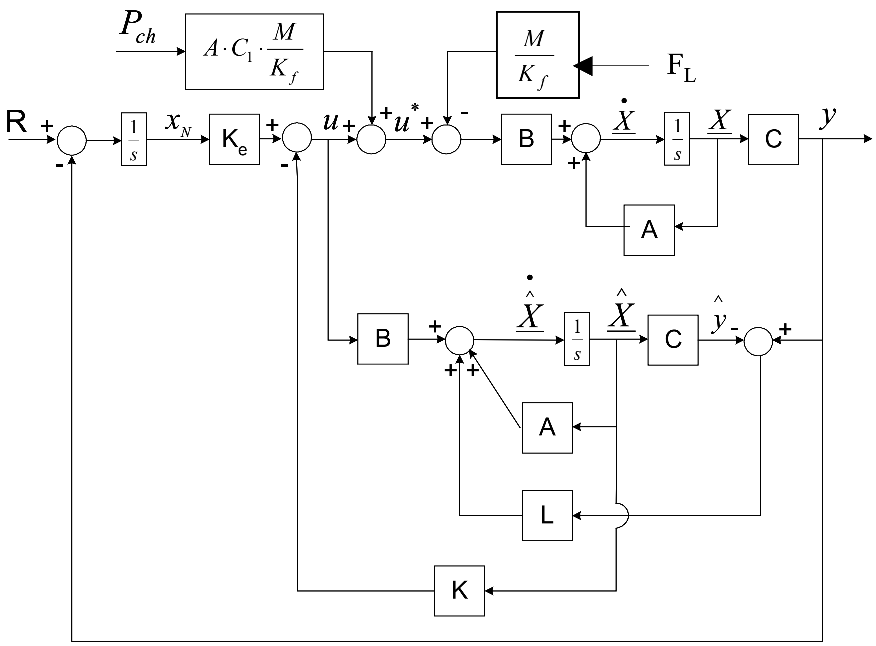

Figure 3. State space block diagram with observer and correction for steady-state error

Assuming that the state estimate, $\hat{\underline{X}}$, is equal to the actual state, $\underline{X}$, from the block diagram in Figure 3,

$$\begin{bmatrix}\dot{\underline{X}}\\ \dot{x}_N\end{bmatrix} = \begin{bmatrix} A-B\cdot K & B\cdot K_e\\ -C & 0\end{bmatrix}\begin{bmatrix}\underline{X}\\ x_N\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}R,$$
$$= A^*\begin{bmatrix}\underline{X}\\ x_N\end{bmatrix}+B^*\cdot R \qquad (23)$$

where $$A^* = \begin{bmatrix} 0 & 1 & 0\\ \dfrac{-K_{sp}-K_f k_1}{M} & \dfrac{-f_B-K_f k_2}{M} & \dfrac{K_f K_e}{M}\\ -1 & 0 & 0\end{bmatrix}.$$

By choosing the desired closed-loop poles at $P_{1,2}=-\zeta\cdot\omega_n \pm j\cdot\omega_n\cdot\sqrt{1-\zeta^2}$ and $P_3=-10\cdot\zeta\cdot\omega_n$ [10], where ζ and ω_n are the desired damping ratio and natural frequency of the closed-loop system, the resulting dominator of the closed-loop transfer function is

$$\det[sI-A^*] = s^3+\frac{f_B+K_f k_2}{M}s^2+\frac{K_{sp}+K_f k_1}{M}s+\frac{K_f K_e}{M}$$
$$\equiv s^3+(12\zeta\cdot\omega_n)s^2+(20\zeta^2+1)\omega_n^2\cdot s+(10\zeta\cdot\omega_n^3). \qquad (24)$$

The control gains, k_1, k_2, and k_e can be determined by comparing with coefficients in (24) with the settling time and percent overshoot (thus, ζ and ω_n) for the closed-loop system. A 0.02 second settling time and 10% overshoot were chosen in the design so that the phase difference between the reference and actual volume was insignificant over the frequency range of 0.83 to 2.5 Hz, corresponding to heart rates of 50 to 150 beats per minute. The resulting control gains are listed in Table 1.

The chamber pressure, P_{ch}, presents a disturbance, in terms of force F_l, against the VCA pump head. F_l is nonlinear due to the opening and closing of the inlet and outlet valves in the pump chamber. Since the chamber pressure is measurable, additional current can be added to the VCA to compensate this nonlinear disturbance. Therefore, the actual current sent into the VCA was

$$U^* = [K_e\cdot\int(R-y)\cdot dt] - K\cdot\hat{\underline{X}}+\frac{M}{K_f}A\cdot C_1\cdot P_{ch}. \qquad (25)$$

Designing the observer entailed the choice of an appropriate gain vector, $L=[l_1\ \ l_2]^T$, such that the state estimate $\hat{\underline{X}}$ approached the actual state $\underline{X}$ in a finite time. This was carried out by choosing the percent overshoot and the settling time for the observer and comparing them with the coefficients in the following equation [10],

$$\det[sI-(A-LC)] = s^2+(l_1+\frac{f_B}{M})\cdot s+(l_1\cdot\frac{f_B}{M}+l_2+\frac{K_{sp}}{M}) \equiv s^2+2\zeta\cdot\omega_n\cdot s+\omega_n{}^2 \quad (26)$$

The settling time for the observer was 0.001 and the overshoot was 5%. The resulting observer gains are listed in Table 1.

4. Controller Testing

Performance of the controller was evaluated in computer simulation. The model of the MCS and the VCA control algorithm was implemented in Simulink (Mathworks Inc., Natick, MA) with the model parameters listed in Table 1. Heart contractility was set to E_{max}=2.2 to simulate a normal healthy heart. Numerical integration in the model was carried out using Runge-Kutta 4th order method with an integration step size of 1 ms. The pump volume tracked $V_{ref}(t)$ well as shown in Figure 4. The simulated pressure waveforms, chamber pressure (P_{ch}), aortic pressure (P_{ao}), right atrial pressure (P_{ra}) and left atrial pressure (P_{la}), are shown in Figure 5. Systolic, mean, and diastolic P_{ao} were 110, 92, and 71 mmHg. Mean P_{ra} and P_{pv} were 2.8 and 6.5 mmHg respectively. These results closely matched the nominal values established by literature [11].

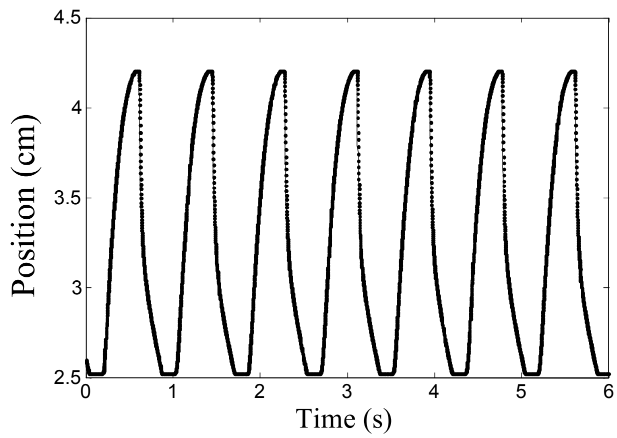

Figure 4. Reference (solid) and actual (dotted) VCA position vs. time

5. Validation

The aim of the validation testing was to access the ability of the MCS controller to produce physiologically meaningful waveforms, such as pressure, volume and flow, in various states of cardiac function ranging from sick to healthy. It has been shown experimentally that the cardiac contractility, represented by E_{max}, should be consistent regardless of the load variations presented to the heart [2]. E_{max} is usually obtained from curve fitting using end-systolic pressure and volume data measured by changing the load conditions of the ventricle [5]. This can be explained by (16). Subtracting V_0 and then multiplying E_{max} on both sides of (16) leads to

$$P_{es} = E_{max}\cdot V_{es} - E_{max}\cdot V_0. \quad (27)$$

End-systolic pressure (P_{es}) and volume (V_{es}) can be changed by varying the load conditions. The slope, obtained from regression using P_{es} and V_{es} data, is the corresponding estimate of E_{max}. The consistency of E_{max} to changing load conditions can be determined statistically by R^2.

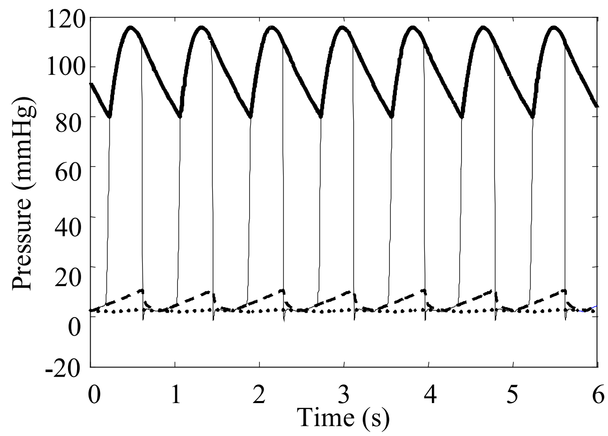

Figure 5. Pressure waveforms obtained from the simulation of the MCS with the controller (P_{ch}: thin solid; P_{ao}: thick solid; P_{la}: dashed line; P_{ra}: dotted line)

For this test, E_{max} in the simulation was set from 0.5 (sick heart, below 25% of its nominal value) to 3.8 (strong heart, beyond 70% of the nominal) to simulate the contractility of the left ventricle. At each given E_{max}, either preload or afterload was varied to produce P_{es} and V_{es} data. These data were then used to produce a linear fit following the equation in (27). The resulting slope should be close to the pre-set E_{max} in simulation with $R^2 \approx 1$.

Preload was varied by gradually changing the output of the venous return pump, Q_{ra}, from 50 to 150% of its nominal value in (9), while holding all other parameters constant. The resulting Pressure-Volume (PV) loops (each loop representing one complete cardiac period) at the nominal $E_{max,}$ along with the regression line, is shown in Figure 6. The results of all the validation tests with preload changes are summarized in Table 2.

Validation testing with afterload change was carried out by gradually varying the value of systemic resistance (R_l) from 50 to 150% of its nominal value, while holding all other parameters constant. E_{max} was varied as described for the previous test. The resulting PV loops and the regression line at the nominal E_{max} setting, are shown in Figure 7. The results of all the validation tests with afterload changes are summarized in Table 2.

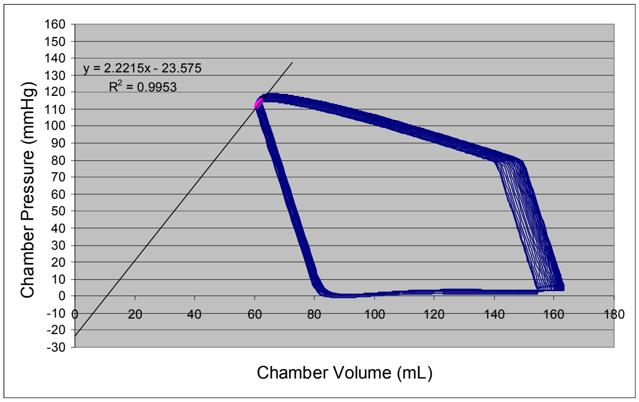

Figure 6. PV Loops with changing preload at E_{max} = 2.2

Table 2. Summary of system response to preload and afterload changes

E_{MAX} Setting	Slope	Error (%)	y-intercept	R^2
For preload variation (+/- 50%)				
0.5	0.508	1.580	-6.019	0.8531
0.8	0.822	2.688	-10.323	0.9749
1.6	1.608	0.525	-16.773	0.9955
2.2	2.222	0.977	-23.575	0.9953
2.8	2.816	0.571	-29.145	0.9977
3.8	3.806	0.161	-38.558	0.9989
For afterload variation (+/- 50%)				
0.5	0.494	-1.160	-4.136	0.9982
0.8	0.789	-1.325	-6.640	0.9989
1.6	1.601	0.087	-16.461	0.9979
2.2	2.189	-0.523	-22.090	0.9954
2.8	2.825	0.900	-30.067	0.9975
3.8	3.856	1.461	-40.917	0.9991

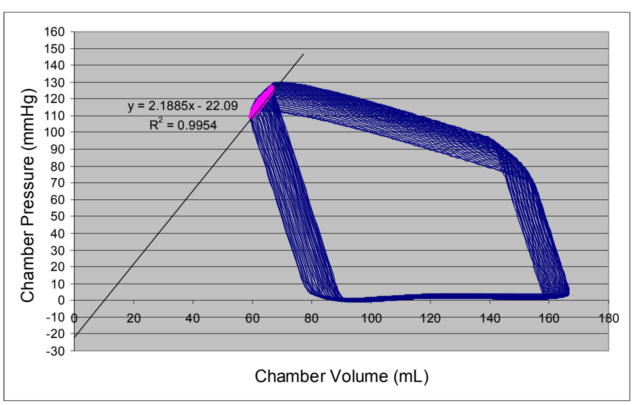

Figure 7. PV Loops with changing afterload at E_{max} = 2.2

6. Discussions and Conclusions

A state feedback controller was designed to control a mock circulatory system and produce physiologically meaningful pressure, volume and flow waveforms based on the contractile state of the left ventricle. A volume reference was generated based on the load conditions (V_{ed} and P_{es}) and contractility (E_{max}) of the mock ventricle. This reference was successfully used to drive a VCA. Performance of the design was evaluated by computer simulation. A series of tests showed that the system can produce a consistent E_{max} ($0.85<R^2<1$) in the range between 25% and 170% of the normal healthy heart condition with less than 3% error regardless of preload and afterload changes. The system remains to be tested in conjunction with a ventricular assist device, following which a physical realization will become feasible.

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