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Passivity-based Dynamic Visual Feedback Control for Three Dimensional Target Tracking: Stability and L_2-gain Performance Analysis

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***Abstract*— This paper investigates vision-based robot control based on the passivity for the three dimensional target tracking. Firstly the fundamental representation between the moving target object and the camera is derived from the relation among the three coordinate frames. Next, we consider the observer which is reproduced from the fundamental representation of relative rigid body motion just as Luenberger observer for linear systems. Then, the relationship between the estimation error in the 3D workspace and in the image plane is established. Secondly we derive the passivity of the dynamic visual feedback system by combining the passivity of both the visual feedback system and the manipulator dynamics. The stability via Lyapunov method for the full 3D dynamic visual feedback system is discussed based on the passivity. Finally, the L_2-gain performance analysis for the disturbance attenuation problem is considered via the dissipative systems theory.**

I. Introduction

Robotics and intelligent machines need many information to behave autonomously under dynamical environments. Visual information is undoubtedly suited to recognize unknown surroundings. Vision based control of robotic systems involves the fusion of robot kinematics, dynamics, and computer vision to control the motion of the robot in an efficient manner. The combination of mechanical control with visual information, so-called visual feedback control or visual servoing, should become extremely important, when we consider a mechanical system working under dynamical environments [1], [2].

Classical visual servoing algorithms assume that the manipulator dynamics is negligible and do not interact with the visual feedback loop. However, as stated in [3], this assumption is invalid for high speed tasks, while it holds for kinematic control problems. Though some researches proposed the control law which guarantee the stability of the system based on the Lyapunov method, robot manipulators are unfortunately limited to the planar type [4], [5]. On the other hand, Kelly $et\ al.$ [6] considered a simple image-based controller for dynamic visual feedback system in the three dimensional(3D) workspace under the assumption that the objects' depths are known. Cowan $et\ al.$ [7] addressed the problems of the field of view for the 3D dynamic visual feedback system by using the navigation functions. Although the good solutions to the set-point problems are reported in those papers, few results have been obtained for the tracking problems of the moving target object in the full 3D dynamic visual feedback system which includes not only both the position and the orientation but also the manipulator dynamics.

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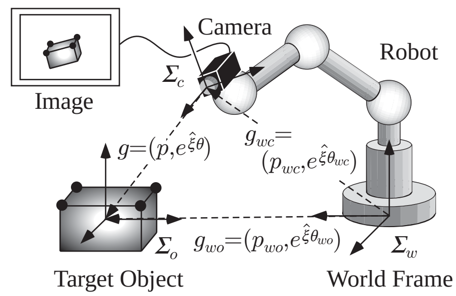

Fig. 1. Eye-in-Hand visual feedback system

This paper deals with the vision-based robot motion control of a moving target object in 3D workspace with the eye-in-hand configuration as depicted in Fig. 1. Firstly the fundamental representation between the moving target object and the camera is derived from the relation among the three coordinate frames. Next, we consider the observer which is reproduced from the fundamental representation of relative rigid body motion just as Luenberger observer for linear systems. Taking into account the manipulator dynamics, the stability via Lyapunov method for the full 3D dynamic visual feedback system will be discussed based on the passivity, which is obtained in our previous works [8], [9], [10]. Moreover, the L_2-gain performance analysis for the disturbance attenuation problem will be considered via the dissipative systems theory.

Throughout this paper, we use the notation $e^{\hat{\xi}\theta_{ab}} \in \mathcal{R}^{3\times 3}$ to represent the change of the principle axes of a frame Σ_b relative to a frame Σ_a. The notation '$\wedge$' (wedge) is the skew-symmetric operator such that $\hat{\xi}\theta = \xi \times \theta$ for the vector cross-product $\times$ and any vector $\theta \in \mathcal{R}^3$. The notation '$\vee$' (vee) denotes the inverse operator to '$\wedge$': i.e., $so(3) \to \mathcal{R}^3$. $\xi_{ab} \in \mathcal{R}^3$ specifies the direction of rotation and $\theta_{ab} \in \mathcal{R}$ is the angle of rotation. Here $\hat{\xi}\theta_{ab}$ denotes $\hat{\xi}_{ab}\theta_{ab}$ for the simplicity of notation. We use the 4×4 matrix

$$g_{ab} = \begin{bmatrix} e^{\hat{\xi}\theta_{ab}} & p_{ab} \\ 0 & 1 \end{bmatrix} \tag{1}$$

as the homogeneous representation of $g_{ab} = (p_{ab}, e^{\hat{\xi}\theta_{ab}}) \in SE(3)$ which is the description of the configuration of a frame Σ_b relative to a frame Σ_a. The adjoint transformation associated with g_{ab} is denoted by $\mathrm{Ad}_{(g_{ab})}$ [11]. Let us define the vector form of the rotation matrix as $e_R(e^{\hat{\xi}\theta_{ab}}) := \mathrm{sk}(e^{\hat{\xi}\theta_{ab}})^\vee$ where $\mathrm{sk}(e^{\hat{\xi}\theta_{ab}})$ denotes $\frac{1}{2}(e^{\hat{\xi}\theta_{ab}} - e^{-\hat{\xi}\theta_{ab}})$.

II. Relative Rigid Body Motion in Visual Feedback System

A. Fundamental Representation for Visual Feedback System

The visual feedback system considered in this paper has the camera mounted on the robot's end-effector as depicted in Fig. 1, where the coordinate frames Σ_w, Σ_c and Σ_o represent the world frame, the camera (end-effector) frame and the object frame, respectively. Let $p_{co} \in \mathcal{R}^3$ and $e^{\hat{\xi}\theta_{co}} \in SO(3)$ be the position vector and the rotation matrix from the camera frame Σ_c to the object frame Σ_o. Then, the relative rigid body motion from Σ_c to Σ_o can be represented by $g_{co} = (p_{co}, e^{\hat{\xi}\theta_{co}}) \in SE(3)$. Similarly, $g_{wc} = (p_{wc}, e^{\hat{\xi}\theta_{wc}})$ and $g_{wo} = (p_{wo}, e^{\hat{\xi}\theta_{wo}})$ denote the rigid body motions from the world frame Σ_w to the camera frame Σ_c and from the world frame Σ_w to the object frame Σ_o, respectively, as shown in Fig. 1.

The objective of the visual feedback control is to bring the actual relative rigid body motion $g_{co} = (p_{co}, e^{\hat{\xi}\theta_{co}})$ to a given reference $g_d = (p_d, e^{\hat{\xi}\theta_d})$. Our goal is to determine the robot motion using the visual information for this purpose. The reference $g_d = (p_d, e^{\hat{\xi}\theta_d})$ for the rigid body motion $g_{co} = (p_{co}, e^{\hat{\xi}\theta_{co}})$ is assumed to be constant throughout this paper, because the camera can track the moving target object in this case.

In this subsection, let us derive a fundamental representation for the three coordinate frames of the visual feedback system. The rigid body motion $g_{wo} = (p_{wo}, e^{\hat{\xi}\theta_{wo}})$ of the target object, relative to the world frame Σ_w in Fig. 1, is given by

$$g_{wo} = g_{wc} g_{co} \tag{2}$$

which is obtained from the composition rule for rigid body transformations ([11], Chap. 2, pp. 37, eq. (2.24)). Using the notation g_{ab}^{-1} as the inverse of g_{ab}, the rigid body motion (2) can be rewritten as

$$g_{co} = g_{wc}^{-1} g_{wo}. \tag{3}$$

The relative rigid body motion involves the velocity of each rigid body. To this aid, let us consider the velocity of a rigid body as described in [11]. Now, we define the body velocity of the camera relative to the world frame Σ_w as

$$\hat{V}_{wc}^b = g_{wc}^{-1}\dot{g}_{wc} = \begin{bmatrix} \hat{\omega}_{wc} & v_{wc} \\ 0 & 0 \end{bmatrix} \quad V_{wc}^b = \begin{bmatrix} v_{wc} \\ \omega_{wc} \end{bmatrix} \tag{4}$$

where v_{wc} and ω_{wc} represent the velocity of the origin and the angular velocity from Σ_w to Σ_c, respectively ([11] Chap. 2, eq. (2.55)). Similarly, the body velocity of the target object relative to Σ_w will be denoted as

$$\hat{V}_{wo}^b = g_{wo}^{-1}\dot{g}_{wo} = \begin{bmatrix} \hat{\omega}_{wo} & v_{wo} \\ 0 & 0 \end{bmatrix} \quad V_{wo}^b = \begin{bmatrix} v_{wo} \\ \omega_{wo} \end{bmatrix} \tag{5}$$

where v_{wo} and ω_{wo} are the velocity of the origin and the angular velocity from Σ_w to Σ_o, respectively.

Differentiating (3) with respect to time, we have

$$\dot{g}_{co} = -g_{wc}^{-1}\dot{g}_{wc}g_{wc}^{-1}g_{wo} + g_{wc}^{-1}g_{wo}g_{wo}^{-1}\dot{g}_{wo}. \tag{6}$$

By substituting (4) and (5) into the above equation, we can obtain

$$\dot{g}_{co} = -\hat{V}_{wc}^b g_{co} + g_{co}\hat{V}_{wo}^b. \tag{7}$$

Here $g = (p, e^{\hat{\xi}\theta})$ denotes $g_{co} = (p_{co}, e^{\hat{\xi}\theta_{co}})$ for short. We multiply (7) by g^{-1} from left side to derive

$$g^{-1}\dot{g} = -g^{-1}\hat{V}_{wc}^b g + \hat{V}_{wo}^b. \tag{8}$$

Using the property of the adjoint transformation (see e.g. [11], Lemma 2.13), (8) can be rewritten as

$$V^b = -\mathrm{Ad}_{(g^{-1})}V_{wc}^b + V_{wo}^b, \tag{9}$$

Eq. (9) should be the fundamental representation for the three coordinate frames of the visual feedback system. Roughly speaking, the relative rigid body motion $g = (p, e^{\hat{\xi}\theta})$ will be derived from the difference between the camera velocity V_{wc}^b and the target object velocity V_{wo}^b. If $V_{wo}^b = 0$, then the fundamental representation for the visual feedback system (9) satisfies $\int_0^T (V_{wc}^b)^T(-e_r)d\tau \geq -\beta_r$ where e_r is defined as $e_r := [p^T \; e_R^T(e^{\hat{\xi}\theta})]^T$ and β_r is a positive scalar. The detail of this relation will be noticed and mentioned afterwards.

B. Camera Model

The relative rigid body motion $g = (p, e^{\hat{\xi}\theta})$ can not be immediately obtained in the visual feedback system, because the target object velocity V_{wo}^b is unknown and furthermore can not be measured directly. To control the relative rigid body motion using visual information provided by a computer vision system, we derive the model of a pinhole camera with a perspective projection as shown in Fig. 2.

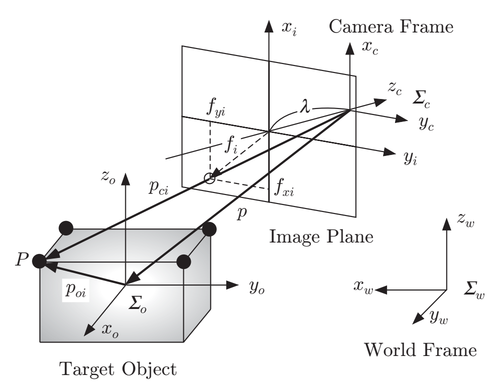

Fig. 2. Pinhole camera

Let λ be a focal length, $p_{oi} \in \mathcal{R}^3$ and $p_{ci} \in \mathcal{R}^3$ be coordinates of the target object's i-th feature point relative to Σ_o and Σ_c, respectively. Using a transformation of the coordinates, we have

$$p_{ci} = g p_{oi}, \tag{10}$$

where p_{ci} and p_{oi} should be regarded as $[p_{ci}^T\ 1]^T$ and $[p_{oi}^T\ 1]^T$ via the well-known representation in robotics, respectively (see, e.g. [11]).

The perspective projection of the i-th feature point onto the image plane gives us the image plane coordinate $f_i := [f_{xi}\ f_{yi}]^T \in \mathcal{R}^2$ as follows

$$f_i = \frac{\lambda}{z_{ci}} \begin{bmatrix} x_{ci} \\ y_{ci} \end{bmatrix} \tag{11}$$

where $p_{ci} := [x_{ci}\ y_{ci}\ z_{ci}]^T$. It is straightforward to extend this model to the m image points case by simply stacking the vectors of the image plane coordinate, i.e. $f := [f_1^T\ \cdots\ f_m^T]^T \in \mathcal{R}^{2m}$. We assume that multiple point features on a known object can be used.

The visual information f which includes the relative rigid body motion can be exploited, while the relative rigid body motion g can not be obtained directly in the visual feedback system. The relationship between the 3D workspace and the image plane will be discussed in the next session.

III. Nonlinear Observer and Estimation Error System

A. Nonlinear Observer

The visual feedback control task should require the information of the relative rigid body motion g. Since the measurable information is only the image information in the visual feedback systems, we consider a nonlinear observer in order to estimate the relative rigid body motion from the image information.

First, we shall consider the following model which is reproduced from the fundamental representation (9) just as Luenberger observer for linear systems.

$$\bar{V}^b = -\mathrm{Ad}_{(\bar{g}^{-1})} V_{wc}^b + u_e \tag{12}$$

where $\bar{g} = (\bar{p}, e^{\hat{\bar{\xi}}\bar{\theta}})$ and $\bar{V}^b$ are the estimated value of the relative rigid body motion and the estimated body velocity, respectively. The new input u_e is to be determined in order to converge the estimated value to the actual relative rigid body motion. Because the design of u_e needs a property of the whole visual feedback system, we will propose u_e in Section V-B.

Similarly to (10) and (11), the estimated image feature point $\bar{f}_i$ $(i = 1, \cdots, m)$ should be described as

$$\bar{p}_{ci} = \bar{g} p_{oi} \tag{13}$$

$$\bar{f}_i = \frac{\lambda}{\bar{z}_{ci}} \begin{bmatrix} \bar{x}_{ci} \\ \bar{y}_{ci} \end{bmatrix} \tag{14}$$

where $\bar{p}_{ci} := [\bar{x}_{ci}\ \bar{y}_{ci}\ \bar{z}_{ci}]^T$. $\bar{f} := [f_1^T\ \cdots\ f_m^T]^T \in \mathcal{R}^{2m}$ means the m image points case.

In order to establish the estimation error system, we define the estimation error between the estimated value $\bar{g}$ and the actual relative rigid body motion g as

$$g_{ee} = \bar{g}^{-1} g, \tag{15}$$

in other words, $p_{ee} = e^{-\hat{\bar{\xi}}\bar{\theta}}(p - \bar{p})$ and $e^{\hat{\xi}\theta_{ee}} = e^{-\hat{\bar{\xi}}\bar{\theta}} e^{\hat{\xi}\theta}$. Note that $p = \bar{p}$ and $e^{\hat{\xi}\theta} = e^{-\hat{\bar{\xi}}\bar{\theta}}$ iff $g_{ee} = I_4$, i.e. $p_{ee} = 0$ and $e^{\hat{\xi}\theta_{ee}} = I_3$. Using the notation $e_R(e^{\hat{\xi}\theta})$, the vector of the estimation error is given by $e_e := [p_{ee}^T\ e_R^T(e^{\hat{\xi}\theta_{ee}})]^T$. Hence, $e_e = 0$ iff $p_{ee} = 0$ and $e^{\hat{\xi}\theta_{ee}} = I_3$. Therefore, if the vector of the estimation error is equal to zero, then the estimated relative rigid body motion $\bar{g}$ equals the actual relative rigid body motion g.

From the above, we derive a relation between the actual image information and the estimated one. Suppose the estimation error is small enough that we can let $e^{\hat{\xi}\theta_{ee}} \simeq I + \mathrm{sk}(e^{\hat{\xi}\theta_{ee}})$, then the following relation between the actual feature point p_{ci} and the estimated one $\bar{p}_{ci}$ holds.

$$p_{ci} - \bar{p}_{ci} = e^{\hat{\bar{\xi}}\bar{\theta}} \begin{bmatrix} I & -\hat{p}_{oi} \end{bmatrix} \begin{bmatrix} p_{ee} \\ e_R(e^{\hat{\xi}\theta_{ee}}) \end{bmatrix}. \tag{16}$$

Using Taylor expansion with the first order approximation, the relation between the actual image information and the estimated one can be derived as

$$f_i - \bar{f}_i = \begin{bmatrix} \frac{\lambda}{\bar{z}_{ci}} & 0 & -\frac{\lambda \bar{x}_{ci}}{\bar{z}_{ci}^2} \\ 0 & \frac{\lambda}{\bar{z}_{ci}} & -\frac{\lambda \bar{y}_{ci}}{\bar{z}_{ci}^2} \end{bmatrix} (p_{ci} - \bar{p}_{ci}). \tag{17}$$

From the above equation, the relation between the actual image information and the estimated one can be given by

$$f - \bar{f} = J(\bar{g}) e_e, \tag{18}$$

where $J(\bar{g}) : SE(3) \to \mathcal{R}^{2m\times 6}$ is defined as

$$J(\bar{g}) := \begin{bmatrix} J_1^T(\bar{g}) & J_2^T(\bar{g}) & \cdots & J_m^T(\bar{g}) \end{bmatrix}^T \tag{19}$$

$$J_i(\bar{g}) := \begin{bmatrix} \frac{\lambda}{\bar{z}_{ci}} & 0 & -\frac{\lambda \bar{x}_{ci}}{\bar{z}_{ci}^2} \\ 0 & \frac{\lambda}{\bar{z}_{ci}} & -\frac{\lambda \bar{y}_{ci}}{\bar{z}_{ci}^2} \end{bmatrix} e^{\hat{\bar{\xi}}\bar{\theta}} \begin{bmatrix} I & -\hat{p}_{oi} \end{bmatrix} \tag{20}$$

$$i = 1, \cdots, m.$$

Note that the matrix $J(\bar{g})$ represents the relationship between the estimation error in the 3D workspace and in the image plane, while the well-known image Jacobian is the relationship between the velocity of the target object in the 3D workspace and in the image plane [1]. We assume that the matrix $J(\bar{g})$ is full column rank for all $\bar{g} \in SE(3)$. Then, the relative rigid body motion can be uniquely defined by the image feature vector. Because this may not hold in some cases when $n = 3$, it is known that $n \geq 4$ is desirable for the full column rank of the image Jacobian [12].

The above discussion shows that we can derive the vector of the estimation error e_e from image information f and the estimated value of the relative rigid body motion $(\bar{p}, e^{\hat{\bar{\xi}}\bar{\theta}})$,

$$e_e = J^{\dagger}(\bar{g})(f - \bar{f}) \tag{21}$$

where † denotes the pseudo-inverse. Therefore the estimation error e_e can be exploited in the 3D visual feedback control law using image information f obtained from the camera. Hence, the nonlinear observer is constructed by (12)–(14) and the estimation input u_e which can be determined from e_e in (21) with an estimation gain in Section V-B.

B. Estimation Error System

The estimation error system will be derived in the same way as the fundamental representation for the visual feedback system. Differentiating (15) and multiplying it by g_{ee}^{-1}, we can obtain

$$g_{ee}^{-1}\dot{g}_{ee} = g_{ee}^{-1}(\bar{g}^{-1}\hat{V}_{wc}^b g - \hat{u}_e g_{ee}) + (-g^{-1}\hat{V}_{wc}^b g + \hat{V}_{wo}^b) = -g_{ee}^{-1}\hat{u}_e g_{ee} + \hat{V}_{wo}^b. \quad (22)$$

Furthermore, using the property concerning the adjoint transformation, the above equation can be transformed into the following

$$V_{ee}^b = -\mathrm{Ad}_{(g_{ee}^{-1})} u_e + V_{wo}^b. \quad (23)$$

Eq. (23) represents the estimation error system. Similar to the fundamental representation (9), the estimation error system (23) satisfies $\int_0^T u_e^T(-e_e)d\tau \geq -\beta_e$ where β_e is a positive scalar. Hence, we consider that the estimation error system preserves the property of the fundamental representation.

IV. Passivity of Visual Feedback System

A. Control Error System

Let us derive the control error system in the same way as the estimation error system in order to establish the visual feedback system. First, we define the control error as follows.

$$g_{ec} = g_d^{-1}\bar{g} \quad (24)$$

which represents the error between the estimated value $\bar{g}$ and the reference of the relative rigid body motion g_d. It should be remarked that the estimated relative rigid body motion equals the reference one if and only if the control error is equal to the identity matrix in matrix form, i.e. $p_d = \bar{p}$ and $e^{\hat{\xi}\theta_d} = e^{\hat{\bar{\xi}}\bar{\theta}}$ iff $g_{ec} = I_4$. Using the notation $e_R(e^{\hat{\xi}\theta})$, the vector of the control error is defined as $e_c := [p_{ec}^T \ e_R^T(e^{\hat{\xi}\theta_{ec}})]^T$. Note that $e_c = 0$ iff $p_{ec} = 0$ and $e^{\hat{\xi}\theta_{ec}} = I_3$. Similarly to (23), the control error system can be obtained as

$$V_{ec}^b = -\mathrm{Ad}_{(\bar{g}^{-1})} V_{wc}^b + u_e. \quad (25)$$

This is dual to the estimation error system. Similar to the estimation error system, the control error system also preserves the property of the fundamental representation.

B. Property of Visual Feedback System

Combining (23) and (25), we construct the visual feedback system as follows.

$$\begin{bmatrix} V_{ec}^b \\ V_{ee}^b \end{bmatrix} = \begin{bmatrix} -\mathrm{Ad}_{(\bar{g}^{-1})} & I \\ 0 & -\mathrm{Ad}_{(g_{ee}^{-1})} \end{bmatrix} u_{ce} + \begin{bmatrix} 0 \\ I \end{bmatrix} V_{wo}^b \quad (26)$$

where $u_{ce} := \left[(V_{wc}^b)^T \ u_e^T\right]^T$ denotes the control input. Let us define the error vector of the visual feedback system as $e := \left[e_c^T \ e_e^T\right]^T$ which consists of the control error vector e_c and the estimation error vector e_e. It should be noted that if the vectors of the control error and the estimation error are equal to zero, then the estimated relative rigid body motion $\bar{g}$ equals the reference one g_d and the estimated one $\bar{g}$ equals the actual one g. Therefore, the actual relative rigid body motion g tends to the reference one g_d when $e \to 0$.

Now, we show an important lemma concerning a relation between the input and the output of the visual feedback system.

Lemma 1: If $V_{wo}^b = 0$, then the visual feedback system (26) satisfies

$$\int_0^T u_{ce}^T \nu_{ce} d\tau \geq -\beta_{ce}, \quad \forall T > 0 \quad (27)$$

where ν_{ce} is defined as

$$\nu_{ce} := \begin{bmatrix} -\mathrm{Ad}_{(g_d^{-1})}^T & 0 \\ \mathrm{Ad}_{(e^{-\hat{\xi}\theta_{ec}})} & -I \end{bmatrix} e \quad (28)$$

and β_{ce} is a positive scalar.

Proof: Consider the following positive definite function

$$V_{ce} = \frac{1}{2}\|p_{ec}\|^2 + \phi(e^{\hat{\xi}\theta_{ec}}) + \frac{1}{2}\|p_{ee}\|^2 + \phi(e^{\hat{\xi}\theta_{ee}}). \quad (29)$$

where $\phi(e^{\hat{\xi}\theta}) := \frac{1}{2}\mathrm{tr}(I - e^{\hat{\xi}\theta})$ is the error function of the rotation matrix and has the following properties (see e.g. [13]).

1) $\phi(e^{\hat{\xi}\theta}) = \phi(e^{-\hat{\xi}\theta}) \geq 0$ and $\phi(e^{\hat{\xi}\theta}) = 0$ iff $e^{\hat{\xi}\theta} = I_3$.
2) $\dot{\phi}(e^{\hat{\xi}\theta}) = e_R^T(e^{\hat{\xi}\theta})\omega = e_R^T(e^{\hat{\xi}\theta})e^{\hat{\xi}\theta}\omega$.

The positive definiteness of the function V_{ce} can be given by the property of the error function ϕ. Differentiating (29) with respect to time yields

$$\dot{V}_{ce} = e^T \begin{bmatrix} \mathrm{Ad}_{(e^{\hat{\xi}\theta_{ec}})} & 0 \\ 0 & \mathrm{Ad}_{(e^{\hat{\xi}\theta_{ee}})} \end{bmatrix} \begin{bmatrix} V_{ec}^b \\ V_{ee}^b \end{bmatrix}. \quad (30)$$

Observing the skew-symmetry of the matrices $\hat{p}_{ec}$ and $\hat{p}_{ee}$, i.e., $p_{ec}^T\hat{p}_{ec}\omega_{wc} = -p_{ec}^T\hat{\omega}_{wc}p_{ec} = 0$, $p_{ee}^T\hat{p}_{ee}\omega_{wc} = -p_{ee}^T\hat{\omega}_{wc}p_{ee} = 0$, the above equation along the trajectories of the system (26) can be transformed into

$$\dot{V}_{ce} = e^T \begin{bmatrix} -\mathrm{Ad}_{(g_d^{-1})} & \mathrm{Ad}_{(e^{\hat{\xi}\theta_{ec}})} \\ 0 & -I \end{bmatrix} u_{ce} = u_{ce}^T \nu_{ce}. \quad (31)$$

Integrating (31) from 0 to T, we can obtain

$$\int_0^T u_{ce}^T \nu_{ce} d\tau = V_{ce}(T) - V_{ce}(0) \geq -V_{ce}(0) := -\beta_{ce} \quad (32)$$

where β_{ce} is the positive scalar which only depends on the initial states of $g_{ec} = (p_{ec}, e^{\hat{\xi}\theta_{ec}})$ and $g_{ee} = (p_{ee}, e^{\hat{\xi}\theta_{ee}})$. ■

Remark 1: In the visual feedback system, $p_{ec}^T\hat{\omega}_{wc}p_{ec} = 0$, $p_{ee}^T\hat{\omega}_{ue}p_{ee} = 0$ holds. This skew-symmetric property is analogous to the one of the robot dynamics, i.e. $x^T(\dot{M} - 2C)x = 0$, $\forall x \in \mathcal{R}^n$ (where $M \in \mathcal{R}^{n\times n}$ is the manipulator inertia matrix and $C \in \mathcal{R}^{n\times n}$ is the Coriolis matrix [11]). Thus, Lemma 1 suggests that the visual feedback system (26) is *passive* from the input u_{ce} to the output ν_{ce} as in the definition in [14]. It should be noted that this property is triggered by the relation of the fundamental representation for the visual feedback system (9) in Section II-A.

V. Passivity-based Control of Dynamic Visual Feedback System

A. Property of Dynamic Visual Feedback System

The manipulator dynamics can be written as

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+g(q)=\tau+\tau_d \tag{33}$$

where q, $\dot{q}$ and $\ddot{q}$ are the joint angles, velocities and accelerations, respectively. τ is the vector of the input torques and τ_d represents a disturbance input.

Since the camera is mounted on the end-effector of the manipulator in the eye-in-hand configuration, the body velocity of the camera V_{wc}^b is given by

$$V_{wc}^b=J_b(q)\dot{q} \tag{34}$$

where $J_b(q)$ is the manipulator body Jacobian [11]. We define the reference of the joint velocities as $\dot{q}_d:=J_b^{\dagger}(q)u_d$ where u_d represents the desired body velocity of the camera.

Let us define the error vector with respect to the joint velocities of the manipulator dynamics as $\xi:=\dot{q}-\dot{q}_d$. Here, we define the weight matrices $W_c:=\mathrm{diag}\{w_{pc}I_3,\ w_{rc}I_3\}\in\mathcal{R}^{6\times 6}$ and $W_e:=\mathrm{diag}\{w_{pe}I_3,\ w_{re}I_3\}\in\mathcal{R}^{6\times 6}$ where $w_{pc},\ w_{rc},\ w_{pe},\ w_{re}\in\mathcal{R}$ are positive. Now, we consider the passivity-based dynamic visual feedback control law as follows

$$\begin{aligned}\tau=&M(q)\ddot{q}_d+C(q,\dot{q})\dot{q}_d+g(q)\\&+J_b^T(q)\mathrm{Ad}^T_{(g_d^{-1})}W_ce_c+u_\xi.\end{aligned} \tag{35}$$

The new input u_ξ is to be determined in order to achieve the control objectives.

Using (26), (33) and (35), the visual feedback system with manipulator dynamics (we call the dynamic visual feedback system) can be derived as follows

$$\begin{aligned}\begin{bmatrix}\dot{\xi}\\ V_{ec}^b\\ V_{ee}^b\end{bmatrix}=&\begin{bmatrix}-M^{-1}C\xi+M^{-1}J_b^T\mathrm{Ad}^T_{(g_d^{-1})}W_ce_c\\ -\mathrm{Ad}_{(\bar{g}^{-1})}J_b\xi\\ 0\end{bmatrix}\\&+\begin{bmatrix}M^{-1}&0&0\\0&-\mathrm{Ad}_{(\bar{g}^{-1})}&I\\0&0&-\mathrm{Ad}_{(g_{ee}^{-1})}\end{bmatrix}u+\begin{bmatrix}M^{-1}&0\\0&0\\0&I\end{bmatrix}\begin{bmatrix}\tau_d\\ V_{wo}^b\end{bmatrix}\end{aligned} \tag{36}$$

where $x:=[\xi^T\ \ e_c^T\ \ e_e^T]^T$ and $u:=[u_\xi^T\ \ u_d^T\ \ u_e^T]$. We define the disturbance of dynamic visual feedback system as $w:=\left[\tau_d^T\ (V_{wo}^b)^T\right]^T$. Before constructing the dynamic visual feedback control law, we derive an important lemma.

Lemma 2: If $w=0$, then the dynamic visual feedback system (36) satisfies

$$\int_0^T u^T\nu d\tau\geq-\beta,\quad\forall T>0 \tag{37}$$

where

$$\nu:=Nx,\ N:=\begin{bmatrix}I&0&0\\0&-\mathrm{Ad}^T_{(g_d^{-1})}W_c&0\\0&\mathrm{Ad}_{(e^{-\hat{\xi}\theta_{ec}})}W_c&-W_e\end{bmatrix}.$$

Proof: Consider the following positive definite function

$$\begin{aligned}V=&\frac{1}{2}\xi^TM\xi+\frac{1}{2}w_{pc}\|p_{ec}\|^2+w_{rc}\phi(e^{\hat{\xi}\theta_{ec}})\\&+\frac{1}{2}w_{pe}\|p_{ee}\|^2+w_{re}\phi(e^{\hat{\xi}\theta_{ee}}).\end{aligned} \tag{38}$$

Differentiating (38) with respect to time yields

$$\begin{aligned}\dot{V}=&\frac{1}{2}\xi^T\dot{M}\xi\\&+x^T\begin{bmatrix}M(q)&0&0\\0&W_c\mathrm{Ad}_{(e^{\hat{\xi}\theta_{ec}})}&0\\0&0&W_e\mathrm{Ad}_{(e^{\hat{\xi}\theta_{ee}})}\end{bmatrix}\begin{bmatrix}\dot{\xi}\\ V_{ec}^b\\ V_{ee}^b\end{bmatrix}.\end{aligned} \tag{39}$$

Observing the skew-symmetry of the matrices $\hat{p}_{ec}$ and $\hat{p}_{ee}$, i.e., $p_{ec}^T\hat{p}_{ec}e^{-\hat{\xi}\theta_d}\omega_{wc}=-p_{ec}^T(e^{-\hat{\xi}\theta_d}\omega_{wc})^{\wedge}p_{ec}=0$, $p_{ee}^T\hat{p}_{ee}\omega_{we}=-p_{ee}^T\hat{\omega}_{we}p_{ee}=0$, the above equation along the trajectories of the system (36) can be transformed into

$$\dot{V}=x^T\begin{bmatrix}I&0&0\\0&-W_c\mathrm{Ad}_{(g_d^{-1})}&W_c\mathrm{Ad}_{(e^{\hat{\xi}\theta_{ec}})}\\0&0&-W_e\end{bmatrix}u. \tag{40}$$

Integrating (40) from 0 to T, we can obtain

$$\int_0^T u^T\nu d\tau=V(T)-V(0)\geq-V(0):=-\beta \tag{41}$$

where β is the positive scalar which only depends on the initial states of ξ, g_{ec} and g_{ee}. ■

Remark 2: The visual feedback system (26) satisfies the passivity property as described in (27). It is well known that the manipulator dynamics (33) also has the passivity. These passivity properties are connected by the manipulator Jacobian (34). In Lemma 2, the inequality (37) would suggest that the dynamic visual feedback system (36) is *passive* from the input u to the output ν.

B. Stability Analysis for Dynamic Visual Feedback System

It is well known that there is a direct link between passivity and Lyapunov stability. Thus, we propose the following control input.

$$u=-K\nu=-KNx,\ K:=\begin{bmatrix}K_\xi&0&0\\0&K_c&0\\0&0&K_e\end{bmatrix} \tag{42}$$

where $K_\xi:=\mathrm{diag}\{k_{\xi 1},\cdots,k_{\xi n}\}$ denotes the positive gain matrix for each joint axis. $K_c:=\mathrm{diag}\{k_{c1},\cdots,k_{c6}\}$ and $K_e:=\mathrm{diag}\{k_{e1},\cdots,k_{e6}\}$ are the positive gain matrices of x, y and z axes of the translation and the rotation for the control error and the estimation error, respectively. The result with respect to asymptotic stability of the proposed control input (42) can be established as follows.

Theorem 1: If $w=0$, then the equilibrium point $x=0$ for the closed-loop system (36) and (42) is asymptotic stable.

Proof: In the proof of Lemma 2, we have already derived that the time derivative of V along the trajectory of the system (36) is formulated as (40). Using the control input (42), (40) can be transformed into

$$\dot{V} = -x^T N^T K N x. \tag{43}$$

This completes the proof. ■

Considering the manipulator dynamics, Theorem 1 shows the stability via Lyapunov method for the full 3D dynamic visual feedback system. It is interesting to note that stability analysis is based on the passivity as described in (37).

C. L_2-gain Performance Analysis for Dynamic Visual Feedback System

Based on the dissipative systems theory, we consider L_2-gain performance analysis for the dynamic visual feedback system (36) in one of the typical problems, i.e. the disturbance attenuation problem. Now, let us define

$$P := N^T K N - \frac{1}{2\gamma^2} W - \frac{1}{2} I$$

where $\gamma \in \mathcal{R}$ is positive and $W := \mathrm{diag}\{I, 0, W_e^2\}$. Then we have the following theorem.

Theorem 2: Given a positive scalar γ and consider the control input (42) with the weight matrices W_c and W_e and the gains K_ξ, K_c and K_e such that the matrix P is positive semi-definite, then the closed-loop system (36) and (42) has L_2-gain $\leq \gamma$.

Proof: By differentiating the positive definite function V defined in (38) along the trajectory of the closed-loop system and completing the squares, it holds that

$$\dot{V} + \frac{1}{2}\|x\|^2 - \frac{\gamma^2}{2}\|w\|^2 \leq -x^T P x \leq 0 \tag{44}$$

if P is positive semi-definite. Integrating (44) from 0 to T and noticing $V(T) \geq 0$, we have

$$\int_0^T \|x\|^2 dt \leq \gamma^2 \int_0^T \|w\|^2 dt + 2V(0), \ \forall T > 0. \tag{45}$$

This completes the proof. ■

The L_2-gain performance analysis of the dynamic visual feedback system is discussed via the dissipative systems theory. In H_∞-type control, we can consider some problems by establishing the adequate generalized plant. This paper has discussed L_2-gain performance analysis for the disturbance attenuation problem. The proposed strategy can be extended for the other-type of generalized plants of the dynamic visual feedback systems.

VI. Conclusions

This paper has investigated the dynamic visual feedback control for the three dimensional target tracking. Specifically, the stability via Lyapunov method for the full 3D dynamic visual feedback system has been discussed based on the passivity. Moreover, the L_2-gain performance analysis for the disturbance attenuation problem has been considered via the dissipative systems theory. The experimental testbed on the two degree-of-freedom manipulator as depicted in Fig. 3 are constructed in order to understand our proposed method simply, though it is valid for 3D visual feedback systems. Due to space limitations, the reader is referred to [15] for more details and experiment results. We expect to systematize the passivity based visual feedback control as well as the theory of the robot control based on the passivity approach.

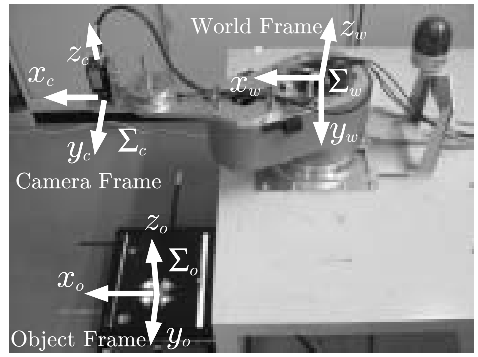

Fig. 3. Visual feedback system on 2DOF manipulator

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