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Design and experimental validation of a linear robust controller for an active suspension of a quarter car

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***Abstract*— This paper presents the design and experimental validation of a robust linear controller for an active suspension mounted in a quarter car test-rig. The presented method is based on linear techniques well supported by CACSD-software tools, yielding a fast control design approach, applicable to almost any active suspension system. Linear black box models are identified using frequency domain identification techniques while robust linear control design techniques account for the model uncertainties introduced by the linear model approximation of the nonlinear dynamics. Although this linear approach introduces some conservatism, it is shown that the desired performance is achieved in simulation as well as on the experimental test-rig.**

I. INTRODUCTION

Comfort and road handling performance of a passenger car are mainly determined by the damping characteristic of the shock absorbers. Passive shock absorbers have a fixed damping characteristic determined by their design. Semi-active and active suspension systems offer the possibility to vary the damper characteristics along with the road profile e.g. by changing the restriction of one or two current controlled valves. An active shock absorber has the additional advantages that negative damping can be provided and that a larger range of forces can be generated at low velocities.

Literature describes several linear and nonlinear techniques to control a car using an active suspension. [1],[2] and [3] apply **linear** control strategies based on linear physical car models consisting of lumped masses, linear springs and dampers, and an active shock absorber modelled as an ideal force source. Real car dynamics are much more complex and therefore these linear control approaches are not appropriate for practical applications.

Nonlinear control strategies such as linear parameter varying gain scheduling [4],[5], backstepping [6] and adaptive control [7],[8] , have been applied to active suspension systems. These controllers are based on a nonlinear physical car and damper model which have a large number of parameters. The experimental identification of these model parameters is a complex problem. In addition, the design and tuning of the above mentioned nonlinear controller is not straightforward. Basically the use of nonlinear models and controllers lead to very time-consuming designs, since no standard techniques or software tools are available.

This paper discusses the application, implementation and validation of an integrated experimental linear robust control design approach on a quarter car test-rig equipped with an active suspension system. This approach is guided by practical considerations with respect to controller complexity and ease of design and tuning. It is based on linear experimental identification and robust linear control design techniques, supported by CACSD-tools, available e.g. in Matlab. The system is excited, a linear model is identified based on the measured frequency response. The modelling error caused by noise, nonlinearities and neglected dynamics are used to estimate the model uncertainty. A robust linear controller is designed using μ-synthesis.

The conservativeness of the resulting controller depends on the linearity of the system. The nonlinearity is mainly introduced by the highly nonlinear dynamic characteristic of the active shock-absorber. The experimental results discussed in this paper show, that the dynamic behavior of the considered quarter car test-rig is sufficiently linear to achieve the required performance using a linear controller. This aspect is validated by comparing the measured and predicted closed loop performance.

Section II gives the description of the quarter car test setup and the active suspension. Section III discusses the experimental identification of the system and estimation of the model uncertainty. The design and experimental validation of the robust controller are discussed in sections IV and V, respectively. Final conclusions are given in section VI.

II. DESCRIPTION OF THE QUARTER CAR TEST SETUP

The quarter car test-rig consists of an active shock absorber, described in section II-A, and the quarter car structure, described in section II-B.

A. Description of the active shock absorber

The active shock absorber hardware is designed such that its open loop (uncontrolled) dynamic characteristic is comparable to that of a passive shock absorber tuned for

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the same type of car. The performance is then further improved by means of a controller that regulates the damping characteristic of the active shock absorber according to the road.

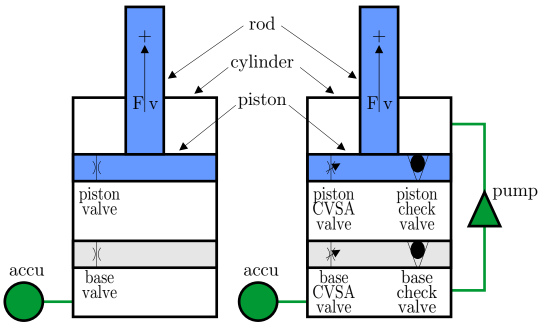

Fig. 1. Passive (left) and active (right) shock absorber schemes.

The active shock absorber hardware (see fig. 1 right) corresponds to that of a passive shock absorber (see fig. 1 left) in which the piston and base valve are each replaced by a check valve and a current controlled CVSA [1] valve. The active performance of the system is realized by adding to the shock-absorber a hydraulic pump generating constant flow of 15 liter per minute resulting in an RMS power consumption of approximately 500W.

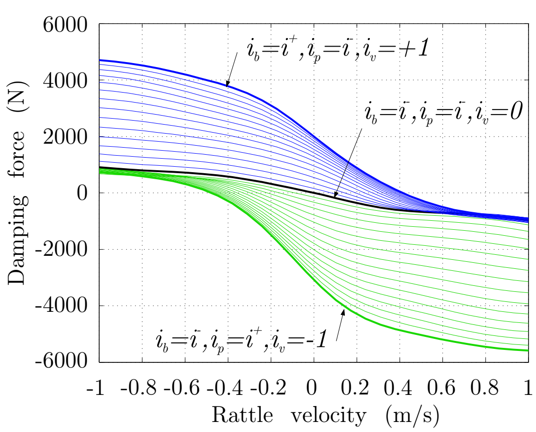

Fig. 2. Feasible working range of the active shock absorber. By varying only one control current at the time, the entire working range is covered.

Figure 2 shows the simulated feasible working range of the active shock absorber. The damping force is plotted as a function of the rattle velocity [2] for several combinations of currents to the base and piston valve, respectively i_b and i_p. The fact that the entire envelope is covered by varying only one current signal at the time is exploited by introducing a new, artificial signal i_v, a *virtual* current, which varies from -1 to +1 and from which i_b and i_p can be calculated unambiguously. This single virtual current replaces the two original currents to the base valve and the piston valve as the system input signal in the control design. This substitution reduces the controller from a multiple-output to a single-output system which is clearly more straightforward to design.

[1]CVSA: continually variable semi-active

[2]rattle velocity: relative velocity of the rod with respect to the cylinder; positive values corresponding to the rod moving out of the cylinder

B. Description of the quarter car structure

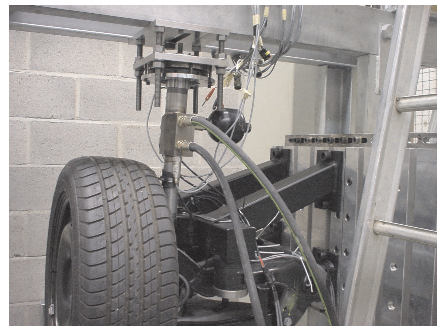

Fig. 3. Picture of the quarter car test-rig equipped with the active suspension.

The quarter car structure (see fig. 3) is a test-rig built to simulate the vertical dynamics of a single corner of a car. It consists of a wheel, a sliding cantilever representing one quarter of a car body mass, and in between these two, a spring and an active damper with bushing. The wheel is excited at the bottom by an hydraulic actuator thereby simulating a vertical road displacement. A guideway imposes a vertical motion on the beam and the wheel.

From a control design point of view, this is a multi-input multi-output (MIMO) system. The acceleration of the body and the wheel a_b and a_w are the outputs of the system available to the controller. The virtual current i_v to the active shock absorber is the input of the system calculated by the controller. The imposed road displacement x_a acts on the system as an external disturbance.

III. System Identification

The system dynamics and uncertainties are identified using a frequency domain approach, consisting of three steps: **(A)** system excitation and FRF matrix estimation, **(B)** identification of a parametric nominal model, and **(C)** estimation of the model uncertainty.

A. System excitation and FRF matrix estimation

The quarter car test-rig is excited by applying a band-limited gaussian random signal to the control input of the active shock-absorber i_v and integrated gaussian white noise to the hydraulic actuator that imposes a road displacement x_a. The measured outputs are the body and wheel acceleration a_b and a_w.

The excitation signals are applied simultaneously to the system for 300 seconds sampled at 1kHz. The first resonance of the system is the well damped body mode and is expected at 1.5Hz. The measurement data is divided into

145 blocks of 4096 samples, with 50% overlap. An H_1-estimator [9] is used to estimate the MIMO FRF matrix (see fig. 5, dotted lines).

B. Identification of parametric transfer function matrix

A parametric transfer function (TF) matrix is fitted on the measured multiple-input multiple-output frequency response function (MIMO FRF) matrix, using the nonlinear least squares frequency domain identification method [10]. The aim is to fit a simple yet accurate linear MIMO model on the measured MIMO FRF matrix. The convergence of this optimization is improved if a good estimate of the number of model parameters (number of system poles and transfer function zeros) is available, and if a priori system knowledge based on physical insight can be included.

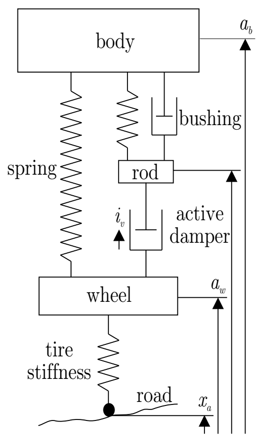

Fig. 4. Schematic representation of the system.

This information is obtained by representing the system as a lumped parameter model consisting of 3 masses connected by springs and dampers (see fig. 4). The dynamics of the active shock absorber are for this purpose neglected by assuming that the shock absorber force is proportional to the control current. These assumptions suggests that a linear model of order 6 exists which describes the 3 system modes: the body mode, the wheelhop mode and a bushing mode. Analysis of this lumped parameter model also shows that a double differentiator (double zero at 0Hz) is included in all 4 transfer functions.

The system also contains a delay of $6ms$, originating from o.a. the hydraulic tubing and the electrodynamic valves, etc. It was estimated based on the linear phase lag present in all FRFs. A 1st-order Pade approximation is used to convert this delay to a pair of complex poles and right-half-plane zeros. These poles and zeros were added to the reduced order model, yielding the parametric nominal model $\mathbf{G}$ (see fig. 5).

C. Estimation of linear multiplicative uncertainty models

The nominal model $\mathbf{G}$ is a linear approximation of the quarter car test-rig dynamics. Model uncertainties are caused by sensor noise, nonlinearities and unmodelled high-frequency dynamics. These uncertainties are estimated by comparing measured FRFs with the FRF matrix of $\mathbf{G}$. The relative difference between these FRFs are averaged (see fig. 6, dotted line), yielding the multiplicative uncertainty $\mathbf{W_T^{ab}}$ and $\mathbf{W_T^{aw}}$. Then linear models are fitted on the uncertainty estimates (see fig. 6, solid line).

A 12th-order model is fitted on the multiplicative uncertainty for the body acceleration output (see fig. 6 top). The uncertainty is well below 100% (0dB) between 0.5 and 10Hz. The lightly damped zero at 12 Hz in the model $\mathbf{G}$ causes the relative uncertainty to peak at this frequency.

A 4th order model is fitted on the multiplicative uncertainty for the wheel acceleration output (see fig. 6 bottom). The uncertainty is below 100% (0dB) between 1 and 15Hz.

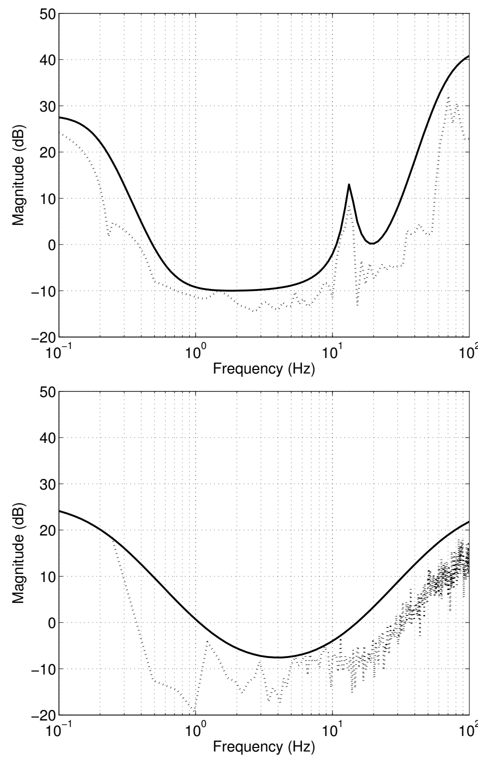

Fig. 6. FRFs of the estimated multiplicative uncertainty (dotted) and the fitted linear models (solid).

IV. Control design

The aim is to design a linear controller that attenuates the body acceleration in the frequency region around the body mode (1.5Hz), without amplifying it in other frequency regions. These specifications correspond to improving the comfort characteristics of the system. Robust performance is accounted for by taking into account estimated model uncertainties.

A. Design of weighting functions

The *generalized plant* used in classic robust control design consists of 2 types of inputs and outputs [11]: exoge-

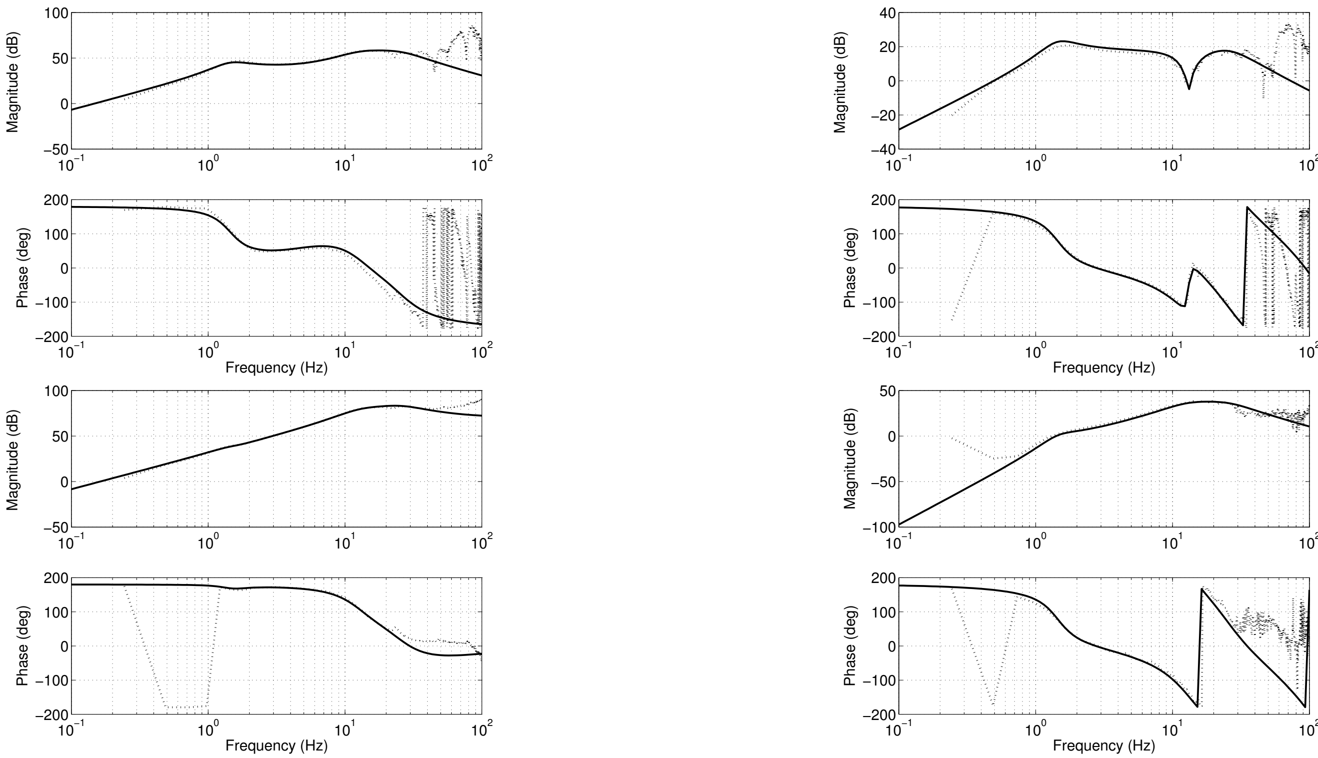

Fig. 5. FRFs of $\mathbf{G}$ measured (dotted) and calculated from the reduced order MIMO model (solid): from road displacement x_a (left) and virtual current i_v (right) to body (top) and wheel (bottom) acceleration a_b and a_w

nous inputs and outputs, and controller inputs and outputs. The frequency content of the exogenous inputs signals and the desired frequency content of the exogenous output signals is expressed by frequency domain weighting functions, such that the desired performance can be expressed as a bound on the H-infinity norm of the augmented plant. This section explains how this augmented plant is obtained for the quarter car system with the active suspension (see fig. 7).

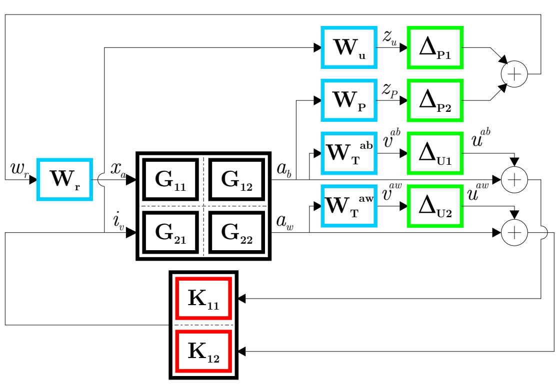

Fig. 7. Generalized plant for the quarter car system with active suspension.

The system has one exogenous input, the simulated road displacement x_a, which acts as an unmeasured disturbance. The frequency content of a typical stochastic road is integrated white noise, which is modelled by the weighting function $\mathbf{W_r}$. It has a lag-compensator characteristic with a -20dB/decade slope in the frequency region of interest, and a zero slope at low and high frequencies to satisfy the necessary conditions for a sound state-space H-infinity control design problem formulation [11].

A first exogenous output signal z_u is defined as the weighted control input i_v, using a weighting function $\mathbf{W_u}$. $\mathbf{W_u}$ is set equal to 1, expressing that the 2-norm of the control input should be smaller then 1 for all frequencies to avoid excessive saturation of the control current i_v.

A second exogenous output signal z_P is defined as the weighted body acceleration signal a_b, using a weighting function $\mathbf{W_P}$. $\mathbf{W_P}$ is a band-pass filter with cut-off frequencies around the body mode resonance, such that the amplitude of the system from w_r to z_P is larger then one in the frequency region of this resonance. This expresses that the controller should attenuate the body acceleration at the body mode resonance. The exact locations of the cut-off frequencies of this band-pass filter weight were the tuning parameters of this control design. The desired width of the frequency region of attenuation was increased until the control design problem became infeasible.

The system is also augmented with the multiplicative output uncertainty models $\mathbf{W_T^{ab}}$ and $\mathbf{W_T^{aw}}$. This creates the uncertainty inputs u_Δ^{ab} and u_Δ^{aw} and outputs v_Δ^{ab} and u_Δ^{aw}.

The generalized plant $\mathbf{P}$ is obtained by absorbing the weights $\mathbf{W_r}$, $\mathbf{W_u}$, $\mathbf{W_P}$, $\mathbf{W_T^{ab}}$ and $\mathbf{W_T^{aw}}$ in the model $\mathbf{G}$ yielding the block diagram representation of fig. 8.

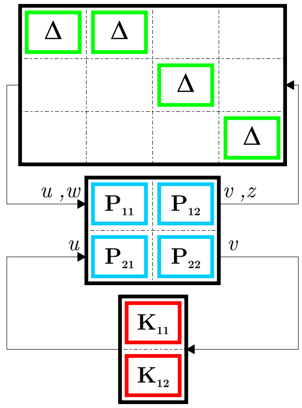

Fig. 8. Generalized plant with absorbed weights.

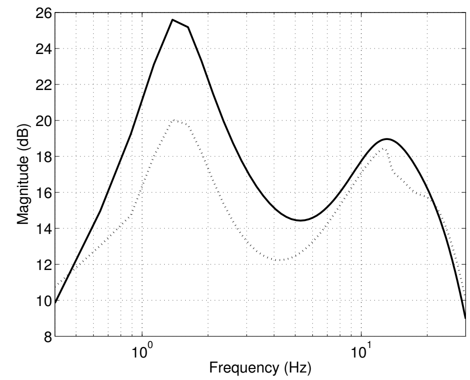

Fig. 10. Comparison of the simulated disturbance FRFs from the road to the body acceleration for the uncontrolled (solid) and controlled (dotted) active suspension.

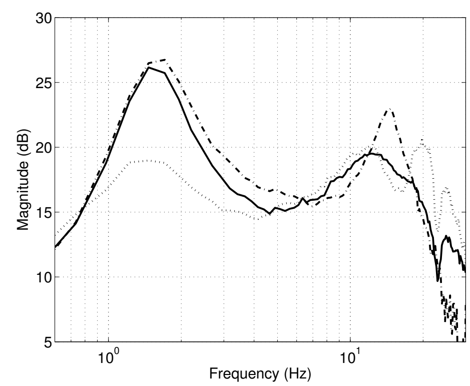

Fig. 11. Comparison of the experimental disturbance FRFs from the road to the body acceleration for the uncontrolled (solid) and controlled (dotted) active suspension, and for the tuned passive suspension (dashdot).

B. μ-synthesis

A controller $\mathbf{K}$ achieves robust performance if it stabilizes the closed loop system shown in fig. 8. The μ-synthesis control design framework is capable of taking into account the structure of the Δ-blocks since it is based on the structured singular value of the closed loop system. The controller synthesis is based on the DK-iteration scheme [11] in which alternatingly D-scaling functions are fit to approximate the structured singular value (D-step) and an H-infinity controller is designed (K-step).

Figure 9 shows the synthesized controller obtained with μ-synthesis. The inputs of the controller are the measured body and wheel acceleration a_b and a_w, the output is the virtual current to the active shock absorber i_v. The order of the controller is 60, which is reduced to 14 using an optimal Hankel approximation [11].

V. CONTROLLER VALIDATION

The performance of the controller is validated by comparing the open loop (uncontrolled - $i_v = 0$) with the closed loop (controlled) disturbance transfer functions from the road to the body acceleration. This validation is performed both in simulation based on the identified linear models, and experimentally on the quarter car test-rig.

A. Simulated closed loop performance

The open loop and closed loop disturbance transfer functions from the road to the body acceleration are calculated analytically based on the identified linear models. Figure 10 shows that the controller attenuates the body acceleration around the body mode, from 1 up to 5 Hz, by 6dB or 50% of its uncontrolled value. At higher frequencies (above 20Hz) there is only a slight amplification w.r.t. the uncontrolled disturbance transfer function.

B. Experimental closed loop performance

A typical stochastic road displacement profile (integrated white noise) is applied to the hydraulic shaker which imposes a desired road profile to the quarter car, while the body and wheel acceleration are measured. Three suspensions were compared: a standard passive shock absorber tuned for this setup, the active shock absorber used in open loop, and the active shock absorber with the μ-synthesis controller.

The performance with respect to comfort is compared by considering the disturbance transfer functions from the road to the body acceleration (see fig. 11). Comparing performance for the uncontrolled and controlled active shock absorber shows that the controller attenuates the body mode resonance with 6dB as was expected from the simulation results. An additional resonance occurs at 20Hz but its amplitude remains below that of the other resonances.

Comparing the tuned passive shock absorber with the controlled active shock absorber shows that the body acceleration is attenuated with 6dB around the body mode resonance, while it remains attenuated up to 6Hz.

The nominal model and the uncertainty models are identified sufficiently accurately since the simulated results correspond closely to the experimental results.

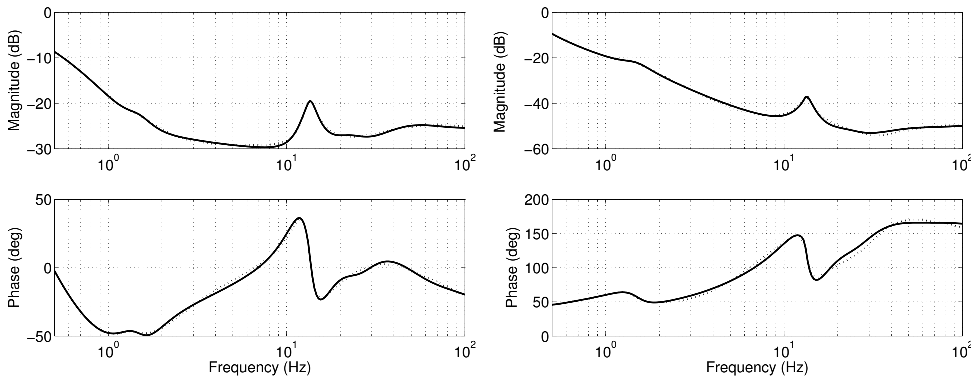

Fig. 9. Controller obtained with μ-synthesis (solid) and reduced order controller (dotted). The controller inputs are the body acceleration a_b (left) and the wheel acceleration a_w (right). The controller output is the virtual current i_v.

VI. CONCLUSIONS

The application of linear robust control design techniques on a quarter car test-rig equipped with an active suspension resulted in a controller that is able to significantly increase comfort compared to the comfort obtained with a tuned passive shock-absorber: the attenuation of the body acceleration around the body mode frequency is increased with 50%. The controller is robustly performant with respect to model uncertainties introduced by the approximation of the nonlinear system dynamics with a linear model. However parameter variations caused by load variations or wear of suspension parts is not taken into account.

The presented approach includes the experimental frequency domain identification of linear models of the quarter car system, including the active shock absorber, and the estimation of the model uncertainties introduced by this linearization, based on data obtained from appropriately designed experiments. μ-synthesis based on the DK-iteration scheme was used to design the controller, taking into account the estimated model uncertainties and constraints on the input current to the valves. The results obtained on the test-rig and by means of simulations are comparable, showing that the nominal model and the uncertainty models are identified correctly.

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