Proceeding of the 2004 American Control Conference
Boston, Massachusetts June 30 - July 2, 2004

Active steering control with front wheel steering

Bing Zheng, Pahngroc Oh, and Barry Lenart
Chassis Advanced Technology
Visteon Corporation
6100 Mercury Dr., Dearborn, MI 48126

***Abstract* - The decoupling of the lateral and yaw motions of a car and car's yaw damping are achieved simultaneously by feedback of both yaw rate and front steering angle. A trade-off is made between the robust decoupling and yaw rate damping through the adjustment of the feedback gains with respect to vehicle speed. With this trade-off, the gain scheduled steering controller provides the desired yaw rate damping while keeping the yaw-lateral motion decoupled. The robustness of the decoupling can be achieved when arbitrary yaw damping is not desired. The developed control system is implemented in a steer-by-wire vehicle, and the test results are provided which illustrate the benefits of the control system.**

1. INTRODUCTION

Presently it is considered a task of the driver to learn the different steering responses of the car under different operating conditions. The driver is responsible for the judgment of physical limits. However, long before such limits are reached, there are significant differences between skilled and unskilled drivers and between different cars.

Paper [1], [2] introduced the integrating unit feedback of yaw rate error by the front wheels that makes the yaw mode unobservable from the front axle lateral acceleration and thereby takes uncertainty out of the steering transfer function. At the same time, the integrating unit feedback of yaw rate transforms the response of the front axle lateral motion to a steering input from a second order transfer function into a first order transfer function. The integrated yaw feedback considerably simplifies the driver's task. He or she plans the path and controls the lateral deviation of the front axle and is not concerned with the stabilization of the yaw motion, which is automatically compensated by integrating unit feedback.

The problem remains that with the integrated unit yaw feedback, yaw damping decreases with the increase of the vehicle speed. The papers [1], [2] meanwhile suggest using rear wheel steering to achieve the desired yaw damping. Though the rear wheel steering provides the capability of arbitrary pole placement for yaw control subsystem, it adds additional cost.

This paper revisits the results from [1], [2]. An extended control law is derived for linear tire characteristics and an ideal longitudinal mass distribution. By investigating the more general controller formation, a gain scheduled steering control system is provided. The control law is a compromise among the robust decoupling between the yaw rate and lateral acceleration and the yaw damping. It provides good yaw damping across the vehicle speed while maintaining the yaw-lateral decoupling with respect to the nominal model.

This paper is organized as follows. Section 2 discusses the steering dynamic model and assumptions. Section 3 describes the proposed steering control system. Section 4 discusses the selection of the control system parameters. Section 5 illustrates the implementation of the control law in an Audi A6 steer-by-wire vehicle as well as the test results. The summary of the paper is contained in section 6.

2. STEERING DYNAMICS

The dynamics of vehicle steering is described by the bicycle model as in paper [3]. It is obtained by lumping the right and left wheels together in the center of the front and rear axle as shown in the following figure. The model describes the yaw and lateral motions and neglects the roll motion.

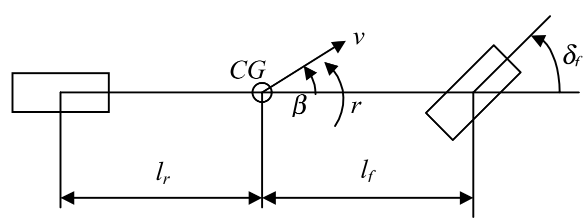

Figure 1. Bicycle model for steering dynamics

The corresponding linearized dynamic equation is

$$\begin{aligned} m_g v\dot{\beta} + \frac{1}{v}(m_g v^2 + c_f l_f - c_r l_r) r + (c_f + c_r)\beta - c_f \delta_f &= 0 \\ I_g \dot{r} + \frac{1}{v}(c_f l_f^2 + c_r l_r^2) r - (c_r l_r - c_f l_f)\beta - c_f l_f \delta_f &= 0 \end{aligned} \quad (1)$$

Where the parameters are:

β : side slip angle between the vehicle center and the velocity at the center of CG
r : yaw rate with respect to an inertial coordinate system
δ_f : front steering angle
$c_r (c_f)$: cornering stiffness for the rear (front) wheel
$l_r(l_f)$: distance from the center of gravity to the rear (front) axis's, l_r+l_f= wheel base

0-7803-8335-4/04/$17.00 ©2004 AACC

m_g : vehicle mass
I_g : moment of inertia with respect to vertical axis
v : vehicle longitudinal velocity which we assume always greater than zero

Rewriting the above equation into state space format, we have

$$\begin{bmatrix}\dot{\beta}\\ \dot{r}\end{bmatrix}=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}\begin{bmatrix}\beta\\ r\end{bmatrix}+\begin{bmatrix}b_1\\ b_2\end{bmatrix}\delta_f \quad (2)$$

Where

$$a_{11}=-\frac{C_r+C_f}{m_g v},\qquad a_{12}=-1-\frac{C_f l_f - C_r l_r}{m_g v^2}$$
$$a_{21}=-\frac{C_f l_f - C_r l_r}{I_g},\qquad a_{22}=-\frac{C_f l_f^2 + C_r l_r^2}{I_g v} \quad (3)$$
$$b_1=\frac{C_f}{m_g v},\qquad b_2=\frac{C_f l_f}{I_g}$$

Consider the following assumptions given in paper [2]:
The cornering stiffness has a common factor that describes the road surface condition or we can say

$$C_f=\mu c_f,\ C_r=\mu c_r \quad (4)$$

The longitudinal mass distribution is equivalent to two masses concentrated at the front and rear axles. Then,

$$I_g=\frac{m_g}{l_f l_r} \quad (5)$$

That is to say that the center of gravity is not shifted by changing the mass m_g, or we can say that l_f and l_r are constant.

Then, the coefficients of the state space mode become:

$$a_{11}=-\frac{c_r+c_f}{mv},\qquad a_{12}=-1-\frac{c_f l_f - c_r l_r}{mv^2}$$
$$a_{21}=-\frac{c_f l_f - c_r l_r}{m l_f l_r},\qquad a_{22}=-\frac{c_f l_f^2 + c_r l_r^2}{m l_f l_r v} \quad (6)$$
$$b_1=\frac{c_f}{mv},\qquad b_2=\frac{c_f}{m l_r}$$

3. STEERING CONTROL SYSTEM ANALYSIS

In this section we consider the steering control system design. We will first consider the controller having the following state space form:

$$\dot{x}=-ax+u$$
$$\delta_f=x+du \quad (7)$$

with

$$u=r_{ref}-k_1 r-k_2 x \quad (8)$$

The corresponding closed loop system can be represented by the following state space equations:

$$\begin{bmatrix}\dot{\beta}\\ \dot{r}\\ \dot{x}\end{bmatrix}=\begin{bmatrix}a_{11} & a_{12} & b_1\\ a_{21} & a_{22} & b_2\\ 0 & 0 & -a\end{bmatrix}\begin{bmatrix}\beta\\ r\\ x\end{bmatrix}+\begin{bmatrix}b_1 d\\ b_2 d\\ 1\end{bmatrix}u \quad (9)$$

with

$$u=r_{ref}-k_1 r-k_2 x \quad (10)$$

By selecting the new state vector as introduced in [1]

$$\begin{bmatrix}a_f\\ r\\ x\end{bmatrix}=\begin{bmatrix}-c & -\frac{c l_f}{v} & c\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\beta\\ r\\ x\end{bmatrix} \quad (11)$$

where a_f is the front axle lateral acceleration, and

$$c=\frac{c_f l}{l_r m} \quad (12)$$

then we have

$$\begin{bmatrix}\dot{a}_f\\ \dot{r}\\ \dot{x}\end{bmatrix}=\begin{bmatrix}d_{11} & c & -ca\\ d_{21} & d_{22} & d_{23}\\ 0 & 0 & -a\end{bmatrix}\begin{bmatrix}a_f\\ r\\ x\end{bmatrix}+\begin{bmatrix}c\hat{d}\\ b_2 d\\ 1\end{bmatrix}u \quad (13)$$

with

$$u=r_{ref}-k_1 r-k_2 x \quad (14)$$

where

$$d_{11}=-\frac{c_f l}{mvl_r},\qquad d_{21}=-\frac{c_f l_f - c_r l_r}{m l_f l_r}$$
$$d_{22}=-\frac{c_r l}{mvl_f},\qquad d_{23}=\frac{c_r}{m l_f} \quad (15)$$
$$\hat{d}=1-b_1 d(1+\frac{l_f}{l}\frac{1}{v})$$

Substitute (13) into (12), we have:

$$\begin{aligned}\begin{bmatrix}\dot{a}_f\\ \dot{r}\\ \dot{x}\end{bmatrix}&=\begin{bmatrix}d_{11} & c-c\hat{d}k_1 & -ca-c\hat{d}k_2\\ d_{21} & d_{22}-b_2 dk_1 & d_{23}-b_2 dk_2\\ 0 & -k_1 & -a-k_2\end{bmatrix}\begin{bmatrix}a_f\\ r\\ x\end{bmatrix}+\begin{bmatrix}c\hat{d}\\ b_2 d\\ 1\end{bmatrix}u_{ref}\\ &=\begin{bmatrix}d_{11} & c(1-\hat{d}k_1) & -c(a+\hat{d}k_2)\\ d_{21} & d_{22}-b_2 dk_1 & d_{23}-b_2 dk_2\\ 0 & -k_1 & -a-k_2\end{bmatrix}\begin{bmatrix}a_f\\ r\\ x\end{bmatrix}+\begin{bmatrix}c\hat{d}\\ b_2 d\\ 1\end{bmatrix}u_{ref}\end{aligned} \quad (16)$$

Let k_1, k_2, a, and $\hat{d}$ have the following relations:

$$1-\hat{d}k_1=0,\quad a+\hat{d}k_2=0 \quad (17)$$

or

$$\hat{d}=\frac{1}{k_1},\quad a=-\frac{k_2}{k1} \quad (18)$$

Then state space equation (15) becomes:

$$\begin{bmatrix} \dot{a}_f \\ \dot{r} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} d_{11} & 0 & 0 \\ d_{21} & d_{22} - b_2 dk_1 & d_{23} - b_2 dk_2 \\ 0 & -k_1 & -a - k_2 \end{bmatrix} \begin{bmatrix} a_f \\ r \\ x \end{bmatrix} + \begin{bmatrix} c\hat{d} \\ b_2 d \\ 1 \end{bmatrix} u_{ref}$$

(19)

State space equation (19) is in the canonical form [4], [5], and it shows that r and x are not observable from a_f. Thus the closed-loop control system (14) decouples a_f from r and x. Or we can say that the steering dynamics is split into the two subsystems by the closed-loop control. One subsystem is the lateral motion of the front axle represented by:

$$\dot{a}_f = d_{11} a_f + c\hat{d} u_{ref} \quad (20)$$

The other is the yaw motion represented by:

$$\begin{bmatrix} \dot{r} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} d_{22} - b_2 dk_1 & d_{23} - b_2 dk_2 \\ -k_1 & -a - k_2 \end{bmatrix} \begin{bmatrix} r \\ x \end{bmatrix} + \begin{bmatrix} b_2 d \\ 1 \end{bmatrix} u_{ref} \quad (21)$$

As stated in the paper [1], the driver has only to control lateral motion subsystem, keeping the car, as a mass point at the front axle, on top of the planned path by generating a lateral acceleration via the transfer function:

$$\frac{a_f}{u_{ref}} = \frac{c\hat{d}}{s - d_{11}} = \frac{v - \frac{c_f}{m} d (1 + \frac{l_f}{lv})}{1 + \frac{mvl_r}{c_f l} s} \quad (22)$$

The decoupled yaw motion will be compensated automatically by the control system which has the following characteristic polynomial:

$$p(s) = \begin{bmatrix} s - (d_{22} - b_2 dk_1) & -(d_{23} - b_2 dk_2) \\ k_1 & s + (a + k_2) \end{bmatrix} \quad (23)$$
$$= s^2 + 2\xi_c \omega_c s + \omega_c^2$$

where

$$\omega_c^2 = k_1 (d_{23} - b_2 dk_2) + (a + k_2)(b_2 dk_1 - d_{22})$$
$$= k_1 [\frac{c_r}{ml_f} + \frac{a(k_1 - 1)v}{l_r (1 + \frac{l_f}{l} \frac{1}{v})}] + a(1 - k_1)[\frac{c_r l}{mvl_f} + \frac{(k_1 - 1)v}{l_r (1 + \frac{l_f}{l} \frac{1}{v})}]$$
$$= k_1 \frac{c_r}{ml_f} + a(k_1 - 1)[\frac{v}{l_r (1 + \frac{l_f}{l} \frac{1}{v})} - \frac{c_r l}{mvl_f}]$$

(24)

$$\xi_c = \frac{(a + k_2) + (b_2 dk_1 - d_{22})}{2\omega}$$
$$= \frac{a(1 - k_1) + [\frac{c_r l}{mvl_f} + \frac{(k_1 - 1)v}{l_r (1 + \frac{l_f}{l} \frac{1}{v})}]}{2\sqrt{k_1 \frac{c_r}{ml_f} - ak_1 \frac{c_r l}{mvl_f} + a[\frac{c_r l}{mvl_f} + \frac{(k_1 - 1)v}{l_r (1 + \frac{l_f}{l} \frac{1}{v})}]}} \quad (25)$$

Here we are using the relations established from (9), (14) and (17) that:

$$\hat{d} = 1 - b_1 d (1 + \frac{l_f}{lv}) \quad (26)$$

$$d = v \frac{1 - \frac{1}{k_1}}{c_f m (1 + \frac{l_f}{lv})} \quad (27)$$

4. CONTROL SYSTEM PARAMETER SELECTION

For the controller in the forms of (7) and (8) there are two free parameters, two out of *a, d,* k_1, and k_2, that can be adjusted to meet the control system performance requirement. Without loss of generality, we will consider a, and k_1.

Recall from (22) that the lateral motion transfer function is:

$$\frac{a_f}{u_{ref}} = \frac{c\hat{d}}{s - d_{11}} = \frac{v - \frac{c_f}{m} d (1 + \frac{l_f}{lv})}{1 + \frac{mvl_r}{c_f l} s} \quad (28)$$

To make vehicle lateral motion controllable, we shall have:

$$c\hat{d} > 0 \quad (29)$$

from (18), this means that

$$k_1 > 0 \quad (30)$$

For the yaw damping, we have:

$$\xi_c == \frac{a(1 - k_1) + V}{2\sqrt{k_1 \frac{c_r}{ml_f} - ak_1 \frac{c_r l}{mvl_f} + aV}} \quad (31)$$

where

$$V = \frac{c_r l}{mvl_f} + \frac{(k_1 - 1)v}{l_r (1 + \frac{l_f}{l} \frac{1}{v})} \quad (32)$$

It can be easily seen that by selecting a properly, and $k_1 > 1$, ξ_c increases when v increases. Hence, by scheduling a and k_1 with respect to vehicle speed, we can adjust the yaw damping without satisfying the decoupling while having freedom to adjust the lateral motion transfer function.

For the case when a = 0, we have:

$$\xi_c = \frac{V}{2\sqrt{k_1 \frac{c_r}{ml_f}}} \quad (33)$$

Again the damping ratio can be adjusted by gain-scheduling k_1 with respect to vehicle speed. But the transfer function for the lateral motion will be influenced by adjusting the yaw damping.

For the case when k_1 is selected as 1, we have d = 0, and

$$\xi = \frac{\frac{c_r l}{mvl_f}}{2\sqrt{\frac{c_r}{ml_f}}} = \frac{l}{2v}\sqrt{\frac{c_r}{ml_f}} \quad (34)$$

This is the exact same as that in [1]. It can be easily verify that this is the exact same robust decoupling case as that in [1]. In this case, the damping ratio is fixed with given vehicle parameters.

More parameter adjust freedom can be obtained by increasing the order of the controller. Here we will consider only the second order case. Assume that controller has the following form:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_2 & -a_1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ b_c \end{bmatrix}$$
$$\delta_f = x_1 + du \quad (35)$$
$$u = r_{ref} - k_1 r - k_2 x_2$$

Then, corresponding augmented the state space model is following:=

$$\begin{bmatrix} \dot\beta \\ \dot r \\ \dot x_1 \\ \dot x_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & b_1 & 0 \\ a_{21} & a_{22} & b_2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -a_2 & -a_1 \end{bmatrix}\begin{bmatrix} \beta \\ r \\ x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_1 d \\ b_2 d \\ 1 \\ b_c \end{bmatrix} u \quad (36)$$

Introducing the new state vector:

$$\begin{bmatrix} a_f \\ r \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -c & \frac{cl_f}{v} & c & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \beta \\ r \\ x_1 \\ x_2 \end{bmatrix} \quad (37)$$

Then the state space equation takes the form:

$$\begin{bmatrix} \dot a_f \\ \dot r \\ \dot x_1 \\ \dot x_2 \end{bmatrix} = \begin{bmatrix} d_{11} & c & 0 & c \\ d_{21} & d_{22} & d_{23} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -a_2 & -a_1 \end{bmatrix}\begin{bmatrix} a_f \\ r \\ x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} c\hat d \\ db_2 \\ 1 \\ b_c \end{bmatrix} u \quad (38)$$

Substituting $u = r_{ref} - k_1 r - k_2 x_2$ into (38), we have:

$$\begin{bmatrix} \dot a_f \\ \dot r \\ \dot x_1 \\ \dot x_2 \end{bmatrix} = \begin{bmatrix} d_{11} & c(1-\hat d k_1) & 0 & c(1-\hat d k_2) \\ d_{21} & d_{22} - b_2 dk_1 & d_{23} & -b_2 dk_2 \\ 0 & -k_1 & 0 & 1-k_2 \\ 0 & -bk_1 & -a_2 & -(a_1 + b_c k_2) \end{bmatrix}\begin{bmatrix} a_f \\ r \\ x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} c\hat d \\ db_2 \\ 1 \\ b_c \end{bmatrix} u_{ref}$$

(39)

For

$$1 - \hat d k_1 = 0, \quad 1 - \hat d k_2 = 0 \quad (40)$$

We have

$$\begin{bmatrix} \dot a_f \\ \dot r \\ \dot\delta_1 \\ \dot\delta_2 \end{bmatrix} = \begin{bmatrix} d_{11} & 0 & 0 & 0 \\ d_{21} & d_{22} - b_2 dk_1 & d_{23} & -b_2 dk_2 \\ 0 & -k_1 & 0 & 1-k_2 \\ 0 & -b_c k_1 & -a_2 & -(a_1 + b_c k_2) \end{bmatrix}\begin{bmatrix} a_f \\ r \\ \delta_1 \\ \delta_2 \end{bmatrix} + \begin{bmatrix} c\hat d \\ db_2 \\ 1 \\ b_c \end{bmatrix} u_{ref}$$

(41)

The lateral motion is decoupled from the yaw motion with the second order controller. The transfer function for the lateral motion is the same as that with first order controller, while there is one more parameter here that can be used to adjust the yaw damping ratio.

5. VEHICLE IMPLEMENTATION AND TEST RESULTS

The steering control system developed was implemented in an Audi A6, modified with a Visteon steer-by-wire system. For the vehicle, a brushless electrical motor is used to drive the road wheel. The compensated motor servo control system has a time constant of 25 ms, which is negligible when compared with the vehicle yaw and lateral dynamics. Hence, we assume that motor control system has proportional input / output relation with a unit gain.

A pre-filter is designed for converting the driver input to the u_{ref} .This pre-filter has the following form:

$$u_{ref} = k_f \frac{v}{L + k_{us} v}\theta_{swa} \quad (42)$$

Where L is the wheelbase, k_f, k_{us} are constant determined by the vehicle, and θ_{swa} is the steering wheel angle.

The designed controller has the second order as in (35). The only control parameter d is scheduled with respect to the vehicle speed with the following relations:

$$d = k_c v \quad (43)$$

Where k_c is a positive constant. Values of k_1 and k_2 vary with respect to d as specified by (40). a_1 and a_2 are selected to assure the overall system stable.

Both objective and subjective single lane change evaluations were conducted on the vehicle with speeds of 40 mph, 50 mph, and 60 mph. The test was conducted at the Smithers Winter Test Facility in the Upper Peninsula of Michigan during winter.

The objective procedures were performed according to the Visteon Vehicle Dynamics Test Procedures [6]. The test data are shown in Figure 1-4.

The Single Lane Change test showed the benefits of the proposed control system. The first vehicle speed tested was 40 mph. Steering wheel input was less for this event with control system "on" as shown in Figure 2. Figure 3 displays the reduced yaw rate and yaw overshoot. Yaw rate overshoot at 40 mph was 7 degree per second with the control system off and 1.5 degrees per second with the system on.

At 50 mph, the vehicle behavior was very similar. The vehicle was stable with the control system on and off. However, yaw overshoot was reduced with the control system on. At 60 mph, the vehicle became unstable with the control system off. Figure 4 shows the steering wheel angle input required to complete the event. Figure 5 illustrates that the vehicle became unstable. The body slide slip angle increased to 90 degrees and vehicle recovery was not possible until speed was greatly reduced.

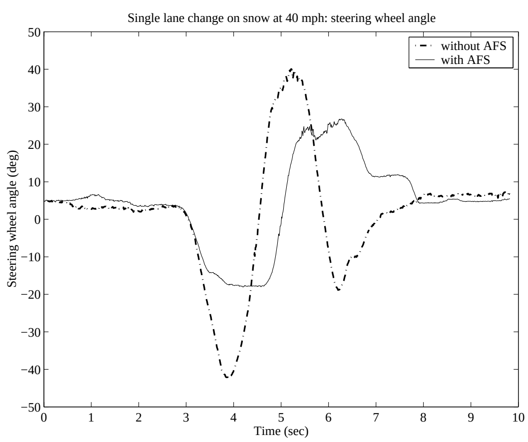

Figure 2: SWA Comparison – Single Lane Change on Snow (40mph)

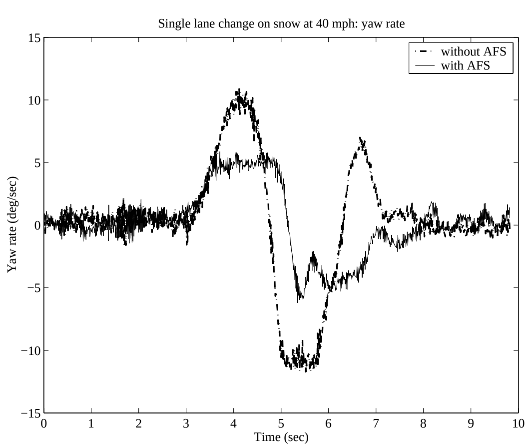

Figure 3: Yaw Rate Comparison – Single Lane Change on Snow (40mph)

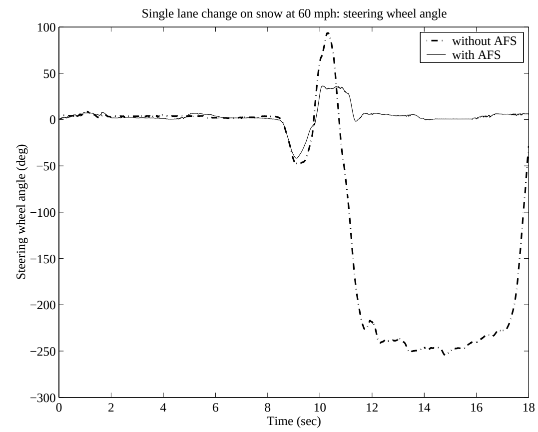

Figure 4: SWA Comparison – Single Lane Change on Snow (60mph)

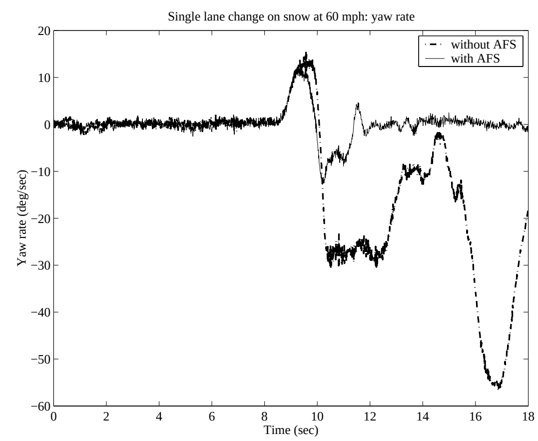

Figure 5: Yaw Rate Comparison – Single Lane Change on Snow (60mph)

6. SUMMARY

The decoupling of the lateral and yaw motions of a car and car's yaw damping are achieved simultaneously by feedback of both yaw rate and front steering angle with the scheduled gains. The test of the implemented control system on the real vehicle indicates the significant safety advantages in critical situations where the driver of the conventional car has to control an unexpected yaw motion.

7. ACKNOWLEDGEMENTS

The authors wish to acknowledge Mr. Matt Mikesell, senior manager of Chassis Advanced Technology at Visteon Corporation, for his review, comments and suggestions on the paper.

8. REFERENCES

[1] J. Ackermann, "Robust Car Steering by Yaw rate control," Proc. CDC, 1990.
[2] J. Ackermann, "Robust Yaw Damping of Cars with

Front and Rear Wheel Steering," IEEE Transactions On Control Systems Technology, Vol. 1, No. 1, March 1993, pp15-20.
[3] P. Riekert, and T. Schunck," Zur Fahrmechanik des gummibereiften Kraftfahrzeugs," Ingenieur Archive., Vol. 11, 1940. pp210-224.
[4] R. Kalman, "Mathematical description of linear dynamical systems," J.SIAM control., 1963. pp152-192.
[5] E. Gilbert, "Controllability and observability in multivariable control systems," J. SIAM control, 1963, pp128-151.
[6] VVDTP-009-02: Single Lane Change (Snow), Visteon Vehicle Dynamics Test Procedures, Visteon Corp., 2001.