

# Nonlinear Control Design for Implementation of Specific Pedal Feeling in Brake-By-Wire Car Design Concepts

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**Abstract** – Brake-By-Wire means that the direct mechanical link between brake pedal and braking cylinder is completely replaced by an electromechanical braking system. This concept, originally used for aircraft and military only, has massive advantages for implementing in new car designs. As a consequence, the pedal movement can be generated arbitrarily. Ideally, the pedal should behave like a conventional brake. In the paper a simplified model for the desired dynamical behavior of the pedal unit is presented, which can be tuned for specific requirements like pedal force, and pedal movement. Its numerical solution leads, due to the pedal force, to the desired time-indexed trajectory of the pedal. Secondly there is presented a closed loop control strategy for a hydraulic driven pedal consisting of an electric pump, servo valve and a differential cylinder in order to track the reference trajectory. Experimental results show the efficiency of the presented controller for the movement of brake pedals in brake-by-wire car design concepts as pedal simulator.

## I. INTRODUCTION

Brake-by-wire equipped cars open the door for many safety features [1]. Thereby the brake pedal and the main brake cylinders are merged into single activation units. This technology interprets the driver's braking intention electronically via wire, rather than mechanically or hydraulically, as has conventionally been the case [11]. Assistance function like Anti Blocking Systems, Brake Assistants and Electronic Stability Programs can be realized by using only software [2], [3].

The vibrations, which normally occur through the brake pedal when ABS intervenes, can be eliminated. As a consequence for brake by wire systems, there exist two control tasks. Firstly a break-power control facility has to be developed, which decelerates the vehicle due to the desired braking intention. Secondly the pedal has to be moved in respect of the braking force of the driver, which behaves ideally like a conventional brake system. This

paper focuses on the dynamic behavior of a conventional braking pedal [4]. As the driver of a car reacts very sensitive on the pedal feeling of the brake, there is a need for the development of brake by wire systems, to use a systematic design of the brake pedal dynamics. Therefore, for the design an active brake pedal has been chosen in order to optimize the man-machine-interface of the brake system. In the presented fully parameterized approach, the brake pedal dynamic behavior can be chosen nearly arbitrarily concerning to pedal force and pedal movement. The advantage is, that the car manufacturers can implement their typical braking feeling at the pedal independent to the installed wheel braking system. Solving the desired dynamic model for the brake pedal numerically leads then to the time indexed reference position of the pedal in respect of the braking force. Since fast and powerful movements occur, the pedal is driven by a hydraulic differential cylinder combined with a servo valve. The dynamic model of the hydraulic system [9], [10], which is presented here, shows dominant nonlinear effects. Therefore, it is derived a nonlinear control law in closed loop configuration, which base on the exact linearization via feedback [5]. The control has been developed on an experimental testbed, which consists of the hydraulically driven pedal, pressure sensors, force sensor and a dSpace-System. Enclosed measurement results show the efficiency of the nonlinear control design.

## II. EXPERIMENTAL SET-UP

The brake pedal of the testbed is driven by a hydraulic differential cylinder. A directional control servo valve in 5/3-way function supplies the cylinder with oil, the pedal angle is gauged by a potentiometer, two pressure sensors measure the pressure of each cylinder chamber. A strain gauge sensor, which is integrated in the pedal, determines the braking force affected by the test person. The control

algorithm is executed in real time condition by a dSpace system (fig. 1).

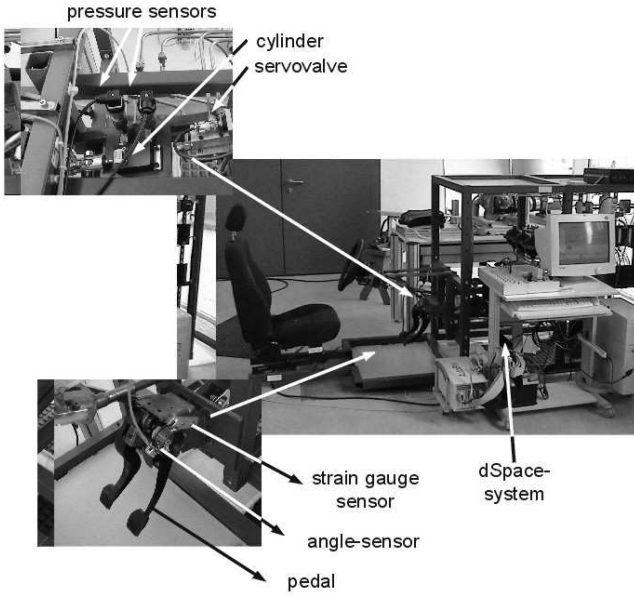


Fig. 1: Testbed consisting of pedal, hydraulic cylinder, servovalve, angle-sensor, pressure sensors and dSpace-System

### III. THE TRAJECORY GENERATION MODUL

A sketch view of the pedal can be seen in figure 2. The rod of the pedal is suspended on a hinge joint. The displacement angel  $\varphi$  causes the denoted position  $s$  of the pedal.  $F_p$  labels the braking force, which is vertically oriented to the rod. On conventional brakes, the brake feeling depends on used mechanical elements like springs, oil dampers, cylinders, length, diameter of the oil line, and the design of the wheel brake.

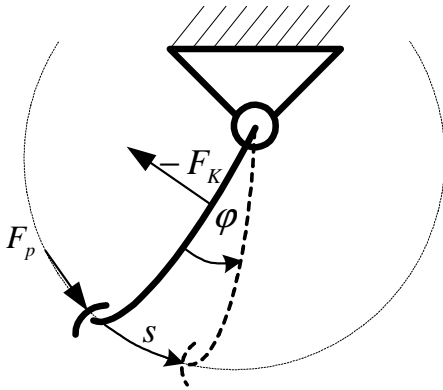


Fig. 2: Draft of the pedal,  $s$  denotes the circumstance position of the pedal caused by the displacement angle  $\varphi$ ,  $F_p$  is the effective force on the pedal.

The theoretical characterization of the pedal braking feeling is presented by a mathematical-physical behavior. In this approach it is modeled with a nonlinear delay system of first order [4] (fig. 2). Thereby, the dynamical behaviour of the pedal is described with a spring, viscous damping element and a coulomb friction element.

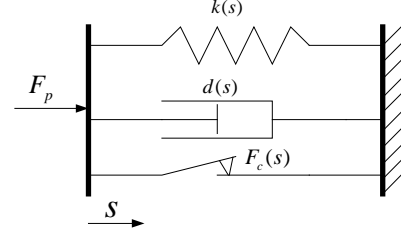


Fig. 3: Equivalent structure to describe the pedal braking feeling

The spring constant  $k(s)$  depends on the position representing the static behavior. The attenuation constant  $d(s)$  is a function of  $s$ , taking into account the dynamical feeling during the braking phase. The coulomb force  $F_c(s)$  models hysteresis effects and depends on  $s$  as well. The differential equation yields to a nonlinear delay system of first order:

$$F_p = k(s) \cdot s + d(s) \cdot \dot{s} + F_c(s). \quad (1)$$

The pedal force  $F_p$  represents the system input. A strain gauge sensor, which is integrated inside the pedal's rod, measures  $F_p$ . The numerically solved equation (1) yields to the time-indexed position  $s$  and velocity  $\dot{s}$ . Higher derivatives are computed by a filtered differentiation. These elements are combined to the trajectory generation modul (fig. 4). It calculates the position trajectory in respect of the measured pedal force reflecting the physical pedal braking behavior. The modul is fully parameterized by the nonlinear spring constant, the nonlinear viscous damping constant and the coulomb friction force and allows a nearly arbitrarily implementation of different pedal braking feelings.

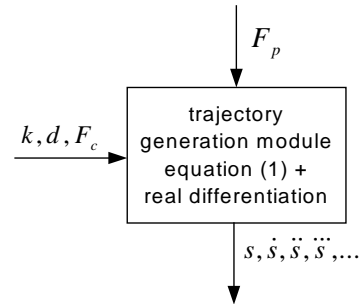


Fig. 4: Trajectory generation modul,  $s$  and higher derivatives are obtained by solving equation (1) numerically and a filtered differentiation.

Figure 5 shows some force-position trajectories computed by the planning modul. Thereby the parameters are adapted to meet the pedal braking behavior of a conventional car braking system. To meet the specific requirements of the conventional pedal brake feeling applying and releasing the brakes has to be distinguished by a different parameter set.

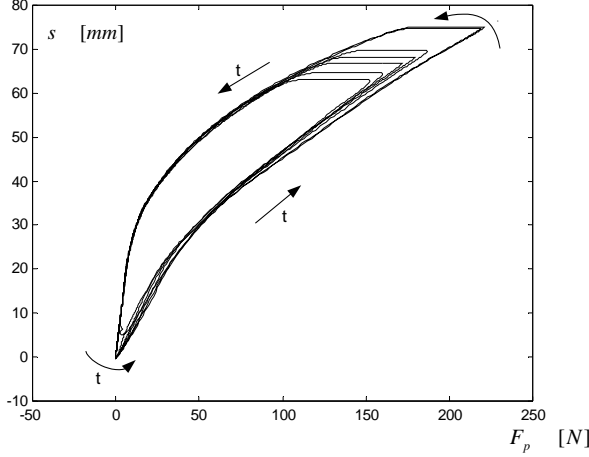


Fig. 5: Example of a force-position trajectory calculated by the path planning modul.

#### IV. TESTBED MODELING

The pedal rod, with a length of  $r_2$ , is fixed at the end on a hinge point. A linkage between the rod and the cylinder, which is fixed at a distance of  $r_1$ , connects actuator and pedal (see figure 6) mechanically. Two forces, the braking force  $F_p$  and the resulting force by the cylinder, move the pedal. Gravitational forces have just a small influence and are neglected in the following. The cylinder is inclined mounted at an angle of  $\alpha$ . To describe the motion dynamically, it is used Newton's second law of mechanics for linear motions. Rotatory effects are transformed into translatory one. Therefore the effective piston mass depends on the moment of inertia of the pedal, the force ratio between pedal and piston depends on the pedal kinematic. The used valve is an electronically controlled servo valve. Proportional to the set point voltage  $U$  a corresponding cross sectional area influence the amount of oil flow and pressure inside the chambers. The pressure difference in the cylinder is connected to the piston force by eq. (2).

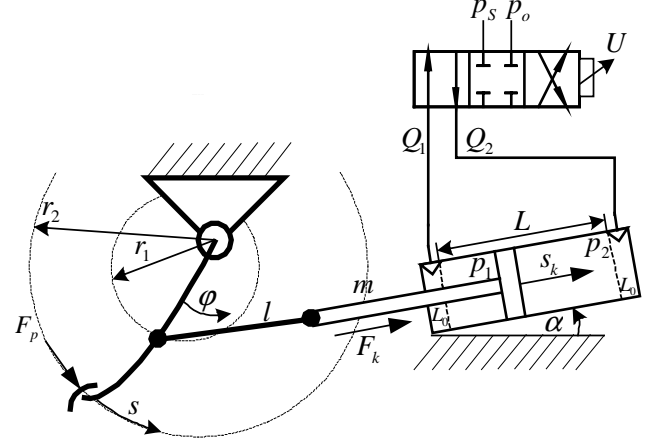


Fig. 6: Sketch of the testbed

#### Dynamic model of the movement

Due to Newton's law, the motion of the piston can be modeled as follows:

$$m(\varphi) \cdot \ddot{s}_k = p_1 A_1 - p_2 A_2 - F_f(\dot{s}_k) + F_k(\varphi), \quad (2)$$

where  $p_1, p_2$  indicate the pressure inside the left and right cylinder chamber respectively,  $A_1, A_2$  are the piston area seen from the left and right sight. The mass  $m(\varphi)$  denotes the effective total mass; any active moment of inertia is transformed into linear mass and depends on the time-varying angle  $\varphi$ . The force  $F_k(\varphi)$  is caused by the pedal force  $F_p$  depending on the geometrical projection. As the bearings of the lever is nearly frictionless, the variable  $F_f(\dot{s}_k)$  denotes just the friction force caused by the sealing material on the piston and is supposed to be a combination of viscous and coulomb friction:

$$F_f(\dot{s}_k) = a_f \cdot \dot{s}_k + b_f \cdot \text{sign}(\dot{s}_k). \quad (3)$$

where  $a_f, b_f$  denote the coefficients of the viscose and coulomb force.

#### Dynamic model of the pressure

Due to the compressibility of compressed oil, in a closed vessel with the inner volume  $V$  any displacement of the pressure  $\Delta p$  ends with a displacement of the volume  $\Delta V$ :

$$\Delta V = \beta \cdot V \cdot \Delta p \quad (4)$$

In this approach, this behavior is assumed to be linear. Therefore the compressibility factor  $\beta$  is constant. Dividing eq. (4) by  $\Delta t$ , it yields for infinite small displacements to:

$$\dot{p} = \frac{1}{\beta V} Q \quad (5)$$

In order to describe the derivative of the pressure inside the two chambers of the used cylinder, eq. (5) has to be expanded as the volume  $V$  depends on the piston position and the flow rate  $Q$  depends on the flow rate supplied by the valve and flow rate shift caused by the velocity of the piston:

$$\begin{aligned}\dot{p}_1 &= \frac{1}{\beta A_1(L_0 + s_k)}(Q_1 - A_1 \dot{s}_k) \\ \dot{p}_2 &= \frac{1}{\beta A_2(L + L_0 - s_k)}(Q_2 + A_2 \dot{s}_k),\end{aligned}\quad (6)$$

Thereby  $L_0$  labels the dead zone inside the cylinder and  $L$  indicates the action range of the piston. The volume flow rates  $Q_1, Q_2$  are those from the valve. The valve response is modeled due to turbulent flow behavior inside the valve [7]:

$$\begin{aligned}Q_1 &= U \cdot \alpha C_m \sqrt{p_S - p_1} \\ Q_2 &= -U \cdot \alpha C_m \sqrt{p_2 - p_0} \\ Q_1 &= -U \cdot \alpha C_m \sqrt{p_1 - p_0} \\ Q_2 &= U \cdot \alpha C_m \sqrt{p_S - p_2}\end{aligned}\quad \left. \begin{array}{l} \} \text{ for } U > 0 \\ \} \text{ for } U \leq 0 \end{array} \right\} \quad (7)$$

The flow rate depends on the square root of the pressure difference in this approach.  $p_S, p_0$  label the constant supply pressure and oil tank pressure respectively. The parameters  $\alpha, C_m$  denote the normalized delivery power and flow conductivity of the used valve.  $U$  stands for the set-point voltage. As the valve stroke is fast controlled, dynamic effects are neglected.

## V. CONTROL DESIGN

The objective of the control is to track the cylinder along the desired trajectory, which is calculated by the trajectory generation modul to meet the adjusted pedal braking feeling. States of the presented model are the pressures inside the cylinder chambers  $p_1, p_2$ , the position  $s_k$  and the velocity  $\dot{s}_k$  of the piston. System input is the set point voltage  $U$  of the valve. The position of the piston has to be controlled and presents therefore the output of the model. [6] and [7] show that a system consisting of a differential cylinder and one valve in 5/3-way function is not differentially flat [8]. That means, neither the pressures can be expressed by the position nor the set-point voltage can be calculated by the inverse system. By measuring the pressures, which is possible on the presented testbed, one can overcome that problem with the exact linearization method via feedback [5], [6]. This technique is in the following used to develop a stabilized tracking of the position. Starting point is the differential equation of

motion (2), which is expressed to the acceleration and is than differentiated to the jerk:

$$\ddot{s}_k = \frac{1}{m(\varphi)}(\dot{p}_1 A_1 - \dot{p}_2 A_2 - \dot{F}_f(\dot{s}_k) + \dot{F}_k(\varphi)) \quad (8)$$

The derivative of the pressure is substituted with eq. (6) yielding to:

$$\ddot{s}_k = \frac{1}{m(\varphi)} \left( \frac{Q_1 - A_1 \dot{s}_k}{\beta A_1(L_0 + s_k)} A_1 - \frac{Q_2 + A_2 \dot{s}_k}{\beta A_2(L + L_0 - s_k)} A_2 - \dot{F}_f(\dot{s}_k) + \dot{F}_k(\varphi) \right). \quad (9)$$

To suppress the singularity of the friction-derivative, in this approach the coulomb force function is approximated by  $F_c = b_f \cdot \tanh(\dot{s}_k / \varepsilon)$ . The flow rates  $Q_1$  and  $Q_2$  of eq. (9) are obtained by eq. (7). The corresponding two functions are then transposed to the set point voltage, representing the inverse system:

$$U = \frac{\ddot{s}_k m(\varphi) D + A_1^2 D_2 \dot{s}_k + A_2^2 D_1 \dot{s}_k + \dot{F}_f(\dot{s}_k) D - \dot{F}_k(\varphi) D}{\alpha C_m (A_1 D_2 \sqrt{p_S - p_1} + A_2 D_1 \sqrt{p_2 - p_0})} \quad (10)$$

for  $U > 0$ .

$$U = -\frac{\ddot{s}_k m(\varphi) D + A_1^2 D_2 \dot{s}_k + A_2^2 D_1 \dot{s}_k + \dot{F}_f(\dot{s}_k) D - \dot{F}_k(\varphi) D}{\alpha C_m (A_1 D_2 \sqrt{p_1 - p_0} + A_2 D_1 \sqrt{p_S - p_2})} \quad (11)$$

for  $U \leq 0$ .

The variables  $D_2, D_1, D$  are abbreviations, standing for:

$$D_1 \stackrel{\text{Def}}{=} \beta A_1(L_0 + s_k), \quad D_2 \stackrel{\text{Def}}{=} \beta A_2(L + L_0 - s_k), \quad D \stackrel{\text{Def}}{=} D_1 D_2. \quad (12)$$

As the jerk  $\ddot{s}_k$  is not part of the system states, it can be chosen arbitrarily:

$$v \stackrel{\text{Def}}{=} \ddot{s}_k. \quad (13)$$

The inverse system eq. (10), (11) (fig. 7) compensate the dominant nonlinear effects. Using  $v$  as new reference input, it assures a linear input/output behavior of a third order integrator. As the piston position and velocity are not directly measured, it is calculated by using the pedal angle  $\varphi$  and velocity  $\dot{\varphi}$ . The parameters  $m(\varphi), F_k(\varphi)$  are adapted by the angle  $\varphi$ , while the forces are further numerically differentiated.

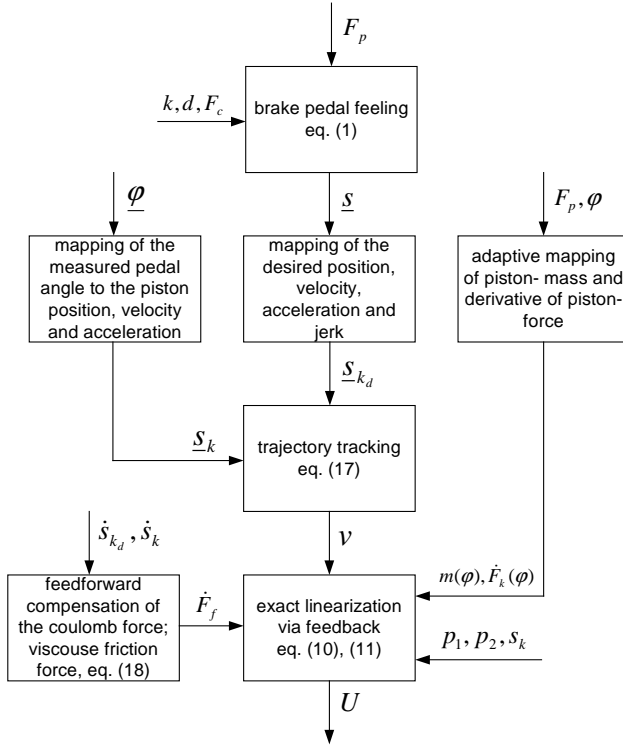


Fig. 7: Block diagram of the control concept including the trajectory generation module.

The trajectory-tracking concept bases on the differential equation of the tracking error:

$$0 = \ddot{e} + K_2 \dot{e} + K_1 e + K_0 e, \quad (14)$$

where

$$e = s_{k_d} - s_k \quad (15)$$

is the error in position,  $K_i$  are computed due to pole assignment  $P_i$ :

$$\prod_{i=1}^3 (s - P_i) = s^3 + \sum_{j=0}^2 K_j s^j. \quad (16)$$

Combining eq. (15) and (14) lead to the trajectory control:

$$v = \ddot{s}_{k_d} + \sum_{j=0}^2 K_j (s_{k_d} - s_k)^{(j)}. \quad (17)$$

The desired position and its derivatives are calculated by using the presented trajectory generation modul (eq. (1)), whereas the output signals have to be transferred into the corresponding piston coordinates. An advantage seems to be to separate the derivative of the coulomb friction force

and viscous friction force. That means, the derivative of the coulomb friction force is calculated by the desired piston velocity, while the derivative of the viscous force effect is computed by the measured one:

$$\dot{F}_f(\dot{s}_{k_d}, \dot{s}_k) = a_f \ddot{s}_k + b_f / \varepsilon \cdot (1 - \tanh(\dot{s}_{k_d} / \varepsilon)) \cdot \ddot{s}_{k_d} \quad (18)$$

The whole control strategy can be seen in figure 7, underlined variables denote vectors consisting of the corresponding states and limited number of differentiations.

## VI. MEASUREMENT RESULTS

The following results show a typical braking intention. The pedal is pushed there in a fast way, to simulate a quick deceleration of the vehicle. The releasing process is much slower assuming the vehicle is well decelerated. The trajectory generation modul calculates thereby the reference path  $\underline{s}$  due to a chosen pedal braking feeling. Depending on the geometry of the levers, it is calculated the desired

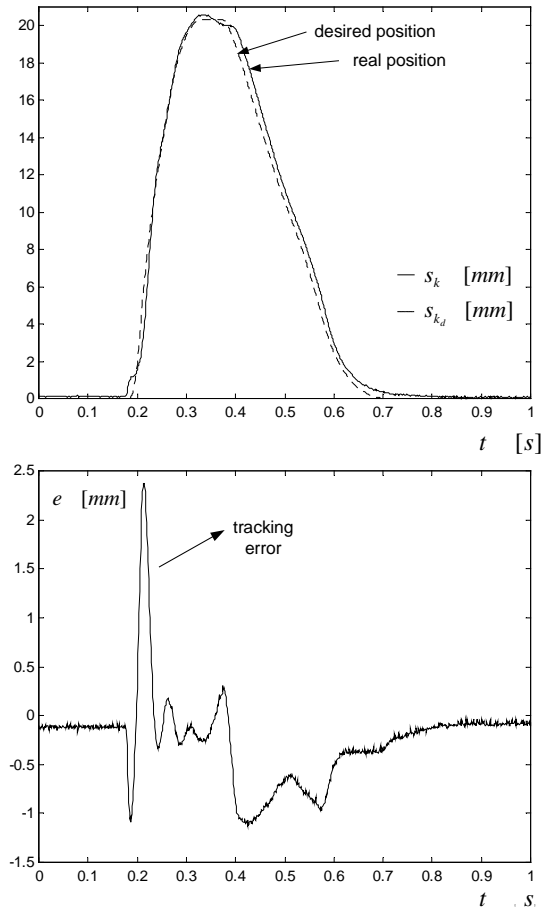


Fig. 8: Position response of a braking intention, upper: desired and real position of the piston, lower: corresponding error in position.

trajectory of the piston  $\underline{s}_{k,d}$ , which can be seen in the upper of figure 8. The presented control concept consists of a compensation of the nonlinearity by state feedback and a closed loop tracking of the desired position, velocity and acceleration. The output signal is that for the valve. In the upper of figure 8 the real position is confronted with the desired position. One can see in the lower of figure 8 the resulting error in position, which amounts at the very beginning of the movement about 2.5 mm. Figure 9 shows the resulting real and desired velocity. It can be seen, that the real velocity is higher than the desired one. This effect can physically not be recognized as the braking intention is fast. Slower braking ambitions result in a maximum error in position of 0.5mm.

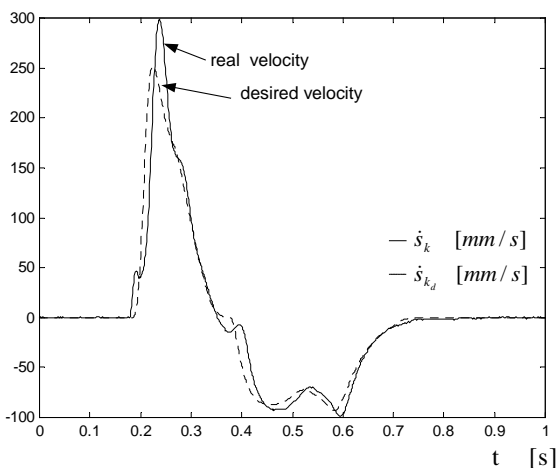


Fig. 9: Desired and real velocity of the piston.

## VII. CONCLUSION

Brake by wire systems play an important role for the automobile industry, as new safety features can be implemented. Thereby it doesn't exist a direct mechanical link between the pedal and the brake units anymore, but an electronically one.

As the human operator is accustomed to operate conventional brakes, the dynamic behavior of the brake pedal feeling was investigated.

As a result the dynamic behavior of the pedal can be modeled with a viscous damped spring element and a coulomb friction force element. Thereby, all parameters vary due to the position of the pedal representing the pedal braking feeling. By tuning these parameters the brake pedal behavior can be changed nearly arbitrarily. The trajectory generation module bases on the numerical solution of this nonlinear differential equation in respect of the time indexed pedal force. Profitable is the possibility to use different pedal braking feeling by just changing the parameters of the planning module.

Since the pedal has to be moved fast and powerful, a differential cylinder drives the pedal hydraulically. As

control unit it is used a directional control servo valve in 5/3 way function. The presented dynamic model of this configuration is highly nonlinear. Therefore, it is used a nonlinear control concept, which cancels the nonlinear effects by state feedback of the two pressures inside the cylinder, the position and the velocity of the piston. Based on this, a trajectory controller tracks the position of the reference position calculated by the trajectory generation module.

These strategies are verified on an experimental set-up, and show the efficiency of the presented control concept.

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