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A new passivity property of linear RLC circuits with application to Power Shaping Stabilization

Eloísa García–Canseco and Romeo Ortega

Abstract— **In this paper we characterize the linear RLC networks for which passivity is preserved even if we take as port variables $(v_s, \frac{d}{dt}i_s)$ and/or $(\frac{d}{dt}v_s, i_s)$ instead of the classical variables (v_s, i_s) representing the external port voltage and current, respectively. This characterization is given in terms of an order relationship between the average electric and magnetic energies stored in the circuit. We apply this result to the problem of power shaping stabilization, a methodology recently proposed as an alternative to the more standard energy–shaping technique.**

Keywords: RLC circuits, passivity, stabilization, passivity–based control.

I. INTRODUCTION

Using *energy shaping* principles to design controllers for physical systems leads to powerful techniques that have their roots in the early work of [1]. The basic idea of these methodologies is to shape the energy of the system in order to satisfy a specific control goal. For instance, as extensively studied in [2] and references therein, assigning a minimum to the closed–loop energy function we can stabilize an equilibrium point. For physical systems a natural assignable candidate energy function is the difference between the energy of the plant and the energy supplied by the controller—leading to the so-called energy-balancing control, whose underlying stabilization mechanism is particularly appealing. Unfortunately, as shown in [3] energy–balancing stabilization is stymied by the existence of pervasive dissipation that appears in many engineering, particularly electrical, applications.

To overcome this obstacle for nonlinear RLC circuits, an alternative method was recently introduced in [4]. In this new method called *Power–Shaping* the storage function used to identify the passive maps is not the total energy but a function directly related with the power in the circuit. Furthermore, in contrast with the well known passivity property of the (conjugated variables) voltage and current, passivity is established now with respect to voltage and derivative of the current (or current and derivative of the voltage).

Instrumental for the application of this method is the identification of the RLC circuits that enjoy this new passivity properties. A class of nonlinear RLC circuits with convex energy functions and weak electromagnetic coupling, for which it is possible to "add differentiation" to the port terminals preserving passivity was identified in [4]. In this paper we focus our attention on *linear* RLC circuits. In this case, there is a clear interpretation in terms of the phase of the driving point impedance of the class of circuits for which adding differentiation at the port terminals preserves passivity. A complete characterization of this class is given here in terms of an order relationship between the average electric and magnetic energies stored in the circuit.

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Eloísa García–Canseco and Romeo Ortega are with the Laboratoire des Signaux et Systèmes, Supelec, Plateau du Moulon, 91192, Gif-sur-Yvette, cedex, France. (e–mail: {garcia,ortega}@lss.supelec.fr)

II. STEADY STATE ANALYSIS OF RLC CIRCUITS

In this section we study linear RLC circuits, using steady-state analysis and the conservation of complex power principle, to derive important properties of the driving point impedance. For the sake of simplicity, we are considering one–port networks (Fig. 1), that is, circuits containing either an independent voltage source or an independent current source. Also we assume $L_k,\ C_k,\ R_k > 0$ the inductance, capacitance and/or resistance of the k–th branch respectively. For further details the reader is referred to standard circuit theory textbooks, e.g., [6].

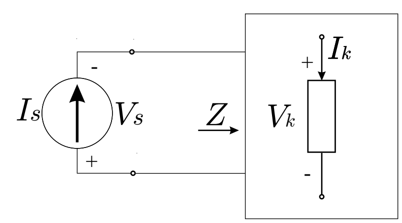

Fig. 1. RLC one–port network

A. Complex power

Tellegen's theorem asserts that the set of branch voltages that satisfy Kirchhoff's voltage law and the set of branch currents that satisfy Kirchhoff's current law, live in orthogonal linear spaces, consequently, $\sum_{k=1}^{b} v_k i_k = 0$, with b the number of branches. Suppose that the network is in sinusoidal steady state, then we can represent the branch voltage v_k by the complex number $V_k(j\omega)$ and the branch current i_k by $I_k(j\omega)$. Clearly, V_k and I_k also satisfy Tellegen's theorem and so does the conjugate $I_k^* = I_k(-j\omega)$ [6]. Therefore,

$$\sum_{k=1}^{b} \frac{1}{2} V_k I_k^* = 0, \tag{1}$$

where each term $\frac{1}{2}V_k I_k^*$ represents the complex power absorbed by the k–th branch. Equation (1) is the mathematical statement of the principle of *conservation of complex power*. This principle can be used to derive many important properties of driving–point impedance functions.

Proposition 1 *[6] Consider the RLC circuit of Fig. 1. If the network is in sinusoidal steady state, the complex power S delivered to the one–port by the source is given by*

$$S(j\omega) = S_{av}(\omega) + 2j\omega[\mathcal{E}_{L_{av}}(\omega) - \mathcal{E}_{C_{av}}(\omega)] \quad (2)$$

where $S_{av}(\omega)$ is the average power dissipated by the network and is defined as

$$S_{av}(\omega) = \sum_{k=2}^{b} \frac{1}{2} R_k |I_k(j\omega)|^2, \quad (3)$$

with $R_k > 0$ the resistance in the k–th branch, and for the average magnetic and electric energies stored in the circuit $\mathcal{E}_{L_{av}}$ and $\mathcal{E}_{C_{av}}$, we have respectively

$$\mathcal{E}_{L_{av}}(\omega) = \sum_{k=2}^{b} \frac{1}{4} L_k |I_k(j\omega)|^2, \quad (4)$$

$$\mathcal{E}_{C_{av}}(\omega) = \sum_{k=2}^{b} \frac{1}{4} \frac{1}{\omega^2 C_k} |I_k(j\omega)|^2 \quad (5)$$

with L_k, $C_k > 0$ the inductance and capacitance of the k–th branch.

Proof: Using equation (1), it is straightforward to show that the complex power can be written as

$$S(j\omega) = \sum_{k=2}^{b} \frac{1}{2} Z_k(j\omega) |I_k(j\omega)|^2 \quad (6)$$

where Z_k is the impedance of the k–th branch. In RLC networks with either resistances, inductances and/or capacitances Z_k is given by

$$Z_k(j\omega) = R_k + j\omega L_k + \frac{1}{j\omega C_k}$$

then the complex power $S(j\omega)$ can be expressed as

$$S(j\omega) = \sum_{k=2}^{b} \frac{1}{2} R_k |I_k(j\omega)|^2 + \sum_{k=2}^{b} \frac{1}{2} j\omega L_k |I_k(j\omega)|^2 + + \sum_{k=2}^{b} \frac{1}{2} \frac{1}{j\omega C_k} |I_k(j\omega)|^2 \quad (7)$$

Integrating each term of (7) over a period $T = 2\pi/\omega$ to obtain the average, we get that the average power dissipated in the resistors is given by (3), and the average energies stored in the inductors and capacitors are given by (4) and (5), respectively. Thus, (7) can be rewritten as (2). ■

Remark 1 The imaginary part of $S(j\omega)$, given by $\text{Im}\{S(j\omega)\} = 2\omega[\mathcal{E}_{L_{av}}(\omega) - \mathcal{E}_{C_{av}}(\omega)]$, is referred in the literature as the *reactive power* absorbed by the one–port network, and usually denoted by Q.

B. Driving point impedance

The driving–point impedance $Z(j\omega)$ of a one–port network with linear time invariant elements is the ratio of the current source input to the voltage response. The following proposition shows that $Z(j\omega)$ can be expressed in terms of the average power and the energy stored by the network.

Proposition 2 *[6] Consider the linear time–invariant RLC one–port network driven by a sinusoidal current source (Fig. 1), and operating in steady state. The driving point impedance seen by the source can be expressed as*

$$Z(j\omega) = \frac{1}{|I_s|^2} \{2S_{av}(\omega) + 4j\omega [\mathcal{E}_{L_{av}}(\omega) - \mathcal{E}_{C_{av}}(\omega)]\} \quad (8)$$

Proof: It follows immediately from the complex power equation (2) and (6). ■

Remark 2 It can be easily verified that $Z(j\omega)$ (8) is a positive real function. That is, $Z(j\omega)$ satisfies

$$\text{Re}\{Z(j\omega)\} = \frac{2}{|I_s|^2} S_{av}(\omega) \geq 0$$

where we have used (3).

III. PROBLEM FORMULATION AND NEW PASSIVITY PROPERTIES OF LINEAR RLC CIRCUITS

Our objective is to characterize the linear RLC circuits for which it is possible to add differentiation to the port terminals preserving passivity. More precisely, we consider two scenarios, in the first case we add a derivative action to the current source variable, and in the second case we take the derivative of the voltage source variable.

In the first scenario the port variables of interest are $(\frac{d}{dt}i_s, v_s)$. (Fig. 2)

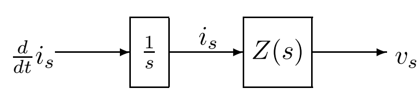

Fig. 2. Transfer function of interest for the port variables $(\frac{d}{dt}i_s, v_s)$

From Kalman–Yakubovich–Popov Lemma we know that passivity of linear systems is equivalent to positive realness of the transfer function. Then, according to Fig. 2 our objective is to find the driving point impedance functions $Z(s)$ such that the transfer function

$$H(s) = \frac{Z(s)}{s}$$

is positive real. From a graphical point of view, this means that the Nyquist plot of $H(j\omega)$ must remain in the fourth quadrant of the complex plane for all $\omega > 0$ (Fig. 3).

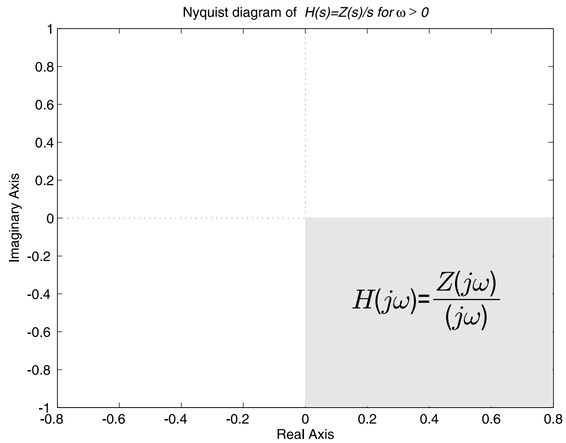

Fig. 3. Nyquist locus of $H(s) = \frac{Z(s)}{s}$.

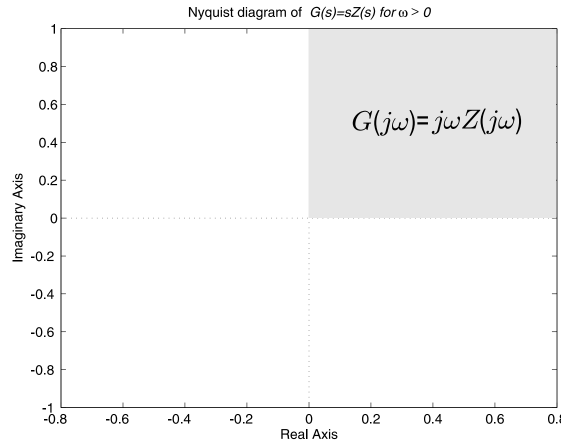

Fig. 5. Nyquist locus of $G(s) = sZ(s)$.

Proposition 3 *Consider a linear time–invariant RLC one–port network. If the average magnetic energy stored in the inductors is greater or equal than the average electric energy stored in the capacitors, i.e., if* $\mathcal{E}_{L_{av}}(\omega)-\mathcal{E}_{C_{av}}(\omega) \geq 0 \ \forall\omega \in \mathbb{R}$ *then, the circuit defines a passive system with port variables* $(\frac{di_S}{dt}, v_S)$*—see Fig. 2.*

Proof: If the network is passive with respect to $(\frac{di_S}{dt}, v_S)$, the transfer function $H(s) = \frac{1}{s}Z(s)$ (Fig. 2) must be a positive real function. That is, it must be stable and $H(j\omega)$ must satisfy Re$\{H(j\omega)\} \geq 0, \forall\omega$. Using equation (8), the transfer function $H(jw)$ is given by

$$H(jw) = \frac{2S_{av}(\omega)}{j\omega|I_s|^2} + \frac{4(\mathcal{E}_{L_{av}}(\omega) - \mathcal{E}_{C_{av}}(\omega))}{|I_s|^2} \quad (9)$$

After some calculations, we have that $H(j\omega)$ is positive real if $\mathcal{E}_{L_{av}}(\omega) - \mathcal{E}_{C_{av}}(\omega) \geq 0$, $\forall\omega \in \mathbb{R}$. ■

In the second scenario we are interested in the port variables $i_s, (\frac{d}{dt}v_s)$ (Fig. 4) .

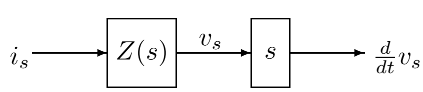

Fig. 4. Transfer function of interest for the port variables$(i_S, \frac{d}{dt}v_S)$

In this case, our goal is to look for driving point impedances $Z(s)$ such that the transfer function

$$G(s) = sZ(s)$$

is positive real. In other words, we want the Nyquist plot of $G(j\omega)$ remain in the first quadrant of the complex plane for all $\omega > 0$ (Fig. 5).

Proposition 4 *Consider a linear time–invariant RLC one–port network. If* $\mathcal{E}_{C_{av}}(\omega) - \mathcal{E}_{L_{av}}(\omega) \geq 0$, $\forall\omega \in \mathbb{R}$ *then, the circuit defines a passive systems with port variables* $(\frac{dv_S}{dt}, i_S)$*—see Fig. 4.*

Proof: Following similar arguments to those of proposition 3, we need the transfer function $G(s) = sZ(s)$ (Fig. 4) to be positive real. From (8) we obtain that the transfer function $G(j\omega)$ is given by

$$G(j\omega) = \frac{2j\omega S_{av}(\omega) - 4\omega^2(\mathcal{E}_{L_{av}}(\omega) - \mathcal{E}_{C_{av}}(\omega))}{|I_s|^2}, \quad (10)$$

which is positive real if $\mathcal{E}_{C_{av}}(\omega) - \mathcal{E}_{L_{av}}(\omega) \geq 0$, $\forall\omega \in \mathbb{R}$. ■

Remark 3 A one–port RLC network can satisfy either one of the properties or neither of them, the later happens when the difference between the average magnetic energy and the average electric energy is not sign definite.

Networks that satisfy Proposition 3 are those whose reactive power is non–negative, i.e., $Q = 2\omega(\mathcal{E}_{L_{av}}(\omega) - \mathcal{E}_{C_{av}}(\omega)) \geq 0$, $\forall\omega \geq 0$. Note that, since the average energy is always positive, all the RL networks satisfy this property because $\mathcal{E}_{C_{av}}(\omega) = 0$ in this case. On the other hand, networks that satisfy Proposition 4 are those whose reactive power is non–positive and we clearly have that all RC networks satisfy this property. These properties, which hold also in the nonlinear case, were first reported in [7].

Remark 4 We point out that the constraints in the propositions depend not only on the network topology but also on the numerical values of the R, L, C elements of the circuit—that obviously appear in the definitions of $\mathcal{E}_{L_{av}}(\omega)$ and $\mathcal{E}_{C_{av}}(\omega)$—see examples below.

IV. EXAMPLES

Let us illustrate with simple examples, the results raised above. To this end, consider the RLC circuit depicted in

Fig. 6. The driving-point impedance of the circuit is given by

$$Z(s) = \frac{R_L + R_C + (R_L R_C C + L)s + R_C L C s^2}{1 + R_C C s}$$

Making the analysis of this circuit in steady state, yields the following expression for the average magnetic energy stored in the inductor

$$\mathcal{E}_{L_{av}}(\omega) = \frac{LC^2\omega^2(1 + R_C^2 C^2)}{4D(\omega)} \tag{11}$$

with

$$\begin{aligned} D(\omega) \quad &= \quad R_C^2 L^2 C^4 \omega^6 + (L^2C^2 + R_L^2 R_C^2 C^4)\omega^4 + \\ &\quad +(R_L^2 C^2 + 2LC + 2R_L R_C C^2)\omega^2 + 1 \end{aligned}$$

and for the average electric energy stored in the capacitor

$$\mathcal{E}_{C_{av}}(\omega) = \frac{R_C^2 C^3 \omega^2}{4D(\omega)} \tag{12}$$

Evaluating the difference we get

$$\mathcal{E}_{L_{av}}(\omega) - \mathcal{E}_{C_{av}}(\omega) = \frac{(R_C^2 C^2 \omega^2 + L - R_C^2 C)C^2\omega^2}{4D(\omega)} \tag{13}$$

From the expression above we can deduce that the condition of Proposition 3 is fulfilled $\forall\omega$ only if

$$L > R_C^2 C.$$

In this case, the circuit defines a passive port with respect to the source variables $(\frac{di_S}{dt}, v_S)$. If the inequality is not satisfied nothing can be concluded regarding the new passivity property, and we should verify the condition of Proposition 4. If neither of propositions are satisfied, then we can conclude that the circuit does not preserve passivity if we add differentiation to the port variables.

In figure 7 is depicted the Nyquist locus of the circuit shown in Fig. 6 for different values of the elements. Notice that for $R1 = 1$, $R2 = 1$, $L = 1$, $C = 1.5$, the condition of proposition 3 is not satisfied, and then, the transfer function $H(s) = \frac{Z(s)}{s}$ is not positive real. Thus, its Nyquist diagram does not remain in the right–half plane, as it is shown by the dotted line.

Another interesting example is the series RLC circuit—see Fig. 8, whose transfer function is

$$Z(s) = \frac{1 + RCs + LCs^2}{Cs}$$

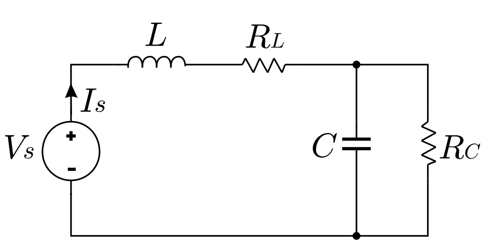

Fig. 6. Example of one–port RLC network.

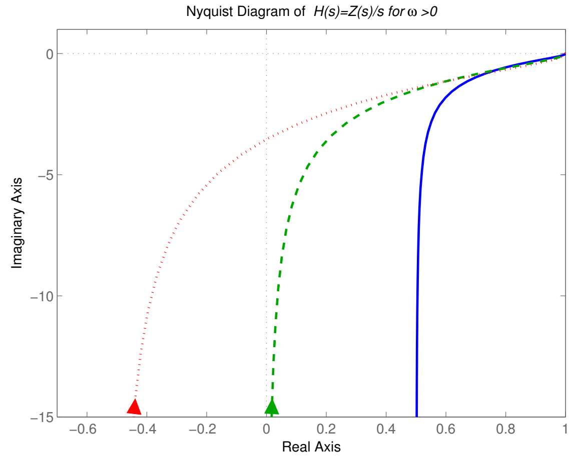

Fig. 7. Nyquist diagram of the RLC circuit of Fig. 6 for different values of the network elements: a) continuous line: $R1 = 1$, $R2 = 1$, $L = 1$, $C = 0.5$. b) dashed line: $R1 = 1$, $R2 = 1$, $L = 1$, $C = 1$. c) dotted line: $R1 = 1$, $R2 = 1$, $L = 1$, $C = 1.5$.

Using steady state analysis we get that the average magnetic energy stored in the inductor is given by

$$\mathcal{E}_{L_{av}}(\omega) = \frac{LC^2\omega^2}{4(1 - 2\,LC\omega^2 + L^2C^2\omega^4 + R^2C^2\omega^2)} \tag{14}$$

and for the average electric energy stored in the capacitor we obtain

$$\mathcal{E}_{C_{av}}(\omega) = \frac{C}{4(1 - 2\,LC\omega^2 + L^2C^2\omega^4 + R^2C^2\omega^2)} \tag{15}$$

Evaluating the difference we get

$$\mathcal{E}_{L_{av}}(\omega) - \mathcal{E}_{C_{av}}(\omega) = \frac{C(LC\omega^2 - 1)}{4(1 - 2\,LC\omega^2 + L^2C^2\omega^4 + R^2C^2\omega^2)} \tag{16}$$

From expression above, the condition to satisfy Proposition 3 is given by

$$LC\omega^2 > 1$$

which is not fulfilled $\forall\omega$. Testing the condition of Proposition 4 yields $LC\omega^2 < 1$ which it is neither fulfilled $\forall\omega$. Thus, we cannot add differentiation to the port terminals of a series RLC preserving passivity. A similar result is obtained for the parallel RLC circuit.

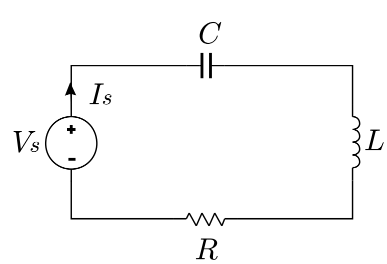

Fig. 8. Series RLC network.

Remark 5 If the network is purely resistive, then we have $\mathcal{E}_{L_{av}}(\omega) = \mathcal{E}_{C_{av}}(\omega) = 0$. From Propositions 3 and 4 we can infer that the circuit defines a passive systems either with respect to $(i_S, \frac{dv_S}{dt})$ or with respect to $(\frac{di_S}{dt}, v_S)$, because both transfer functions $H(j\omega) = \frac{2S_{av}(\omega)}{j\omega|I_s|^2}$ and $G(j\omega) = \frac{2j\omega S_{av}(\omega)}{|I_s|^2}$ are real positive functions . In other words, a purely resistive circuit satisfies both passivity properties established above.

V. APPLICATION FOR STABILIZATION

Let us illustrate with an (elementary) example how the limitations of energy balancing can be overcome via power balancing and motivate the interest for the new passivity property. Consider a voltage controlled linear one–dimensional series RL circuit. The dynamics of the circuit is obtained from Kirchhoff's voltage law as

$$L\frac{di_L}{dt} = -Ri_L + v_S, \tag{17}$$

where i_L is the inductor current, $L > 0,\ R > 0$ are the inductance and resistance, respectively, and v_S is the voltage at the port terminal, which is our control action. The energy stored in the inductor is $\mathcal{E}_L = \frac{1}{2}Li_L^2$. Of course, as the resistor and the inductor are passive, the circuit defines a passive system with port variables (v_S, i_S) and storage function $\mathcal{E}_L$, where $i_S = i_L$ is the source current.

The control objective is the stabilization of an equilibrium $i_L^\star$ of (17), whose corresponding equilibrium supply voltage is given by $v_S^\star = Ri_L^\star$. It is clear that, at any equilibrium $i_L^\star \neq 0$, the extracted power $R(i_L^\star)^2$ is nonzero, hence the circuit is not energy–balancing stabilizable[1].

To overcome this problem we have proposed in [4] to consider the function

$$G(i_R) = \int_0^{i_R} \hat{v}_R(i_R')di_R', \tag{18}$$

where i_R is the resistor current and $\hat{v}_R(i_R)$ the resistor characteristic. This function is well known in the circuits literature [5] as the resistors *content*, which has units of power—in particular, for linear resistors it is half the dissipated power, and in our case takes the form $G(i_L) = \frac{1}{2}Ri_L^2$.

With some simple calculations we can establish the power balance inequality

$$G[i_L(t)] - G[i_L(0)] \leq \int_0^t v_S^T(\tau)\frac{di_S}{dt}(\tau)d\tau, \tag{19}$$

which proves that the circuit is passive with port variables $(v_S, \frac{di_S}{dt})$ and storage function the resistor content. This property differs from the classical passivity property in two important respects: the presence of the derivative of i_S and the use of a new storage function. The dissipation inequality (19) suggests (similarly to energy–balancing) to shape the resistors content. That is, to look for functions $\hat{v}_S(i_L)$, $G_a(i_L)$ such that

$$\dot{G}_a \equiv -\hat{v}_S(i_L)\frac{di_L}{dt} \tag{20}$$

Applying then the control $v_S = \hat{v}_S(i_L) + w_S$ leads to the new dissipation inequality

$$G_d[i_L(t)] - G_d[i_L(0)] \leq \int_0^t w_S^T(\tau)\frac{di_S}{dt}(\tau)d\tau$$

where we defined $G_d(i_L) = G(i_L) + G_a(i_L)$. Moreover, if we ensure that $i_L^\star$=arg min$\{G_d(i_L)\}$, then (setting $w_S = 0$) $i_L^\star$ will be a stable equilibrium with Lyapunov function $G_d(i_L)$ and we say that the system is stabilized via power shaping.

Clearly, for any choice of $G_a(i_L)$, equation (20) is trivially solved with the control $v_S = \hat{v}_S(i_L) = -\frac{\partial G_a}{\partial i_L}$. For instance, if the resistance characteristic is exactly known we can take $G_a(i_L) = -G(i_L) + \frac{R_a}{2}(i_L - i_L^\star)^2$, with $R_a > 0$ some tuning parameter.

As seen from this elementary example, instrumental for the application of the power shaping method is the identification of a new passivity property of the circuit. More specifically, the technique is applicable to circuits for which it is possible to "add differentiation" to the port terminals preserving passivity.

A. Stabilization via power shaping

In this subsection we use the circuit of Fig. 6 to illustrate the applications of the results given above for stabilize the circuit via power shaping. The Brayton–Moser model of the circuit is given by[2]

$$\boldsymbol{A}\begin{bmatrix}\frac{di_L}{dt}\\ \dot{v}_C\end{bmatrix} = \begin{bmatrix}\frac{\partial P}{\partial i_L}(i_L, v_C)\\ \frac{\partial P}{\partial v_C}(i_L, v_C)\end{bmatrix} - \begin{bmatrix}v_S\\ 0\end{bmatrix} \tag{21}$$

with $\boldsymbol{A} = \text{diag}\{-L, C\}$ and the mixed potential function P given by

$$P = \frac{R_C}{2}(\frac{v_C}{R_C} - i_L)^2 - \frac{1}{2}(R_L + R_C)i_L^2.$$

The equilibrium points $(\bar{i}_L, \bar{v}_C)$ of (21) are given by

$$\bar{v}_C = R_C\bar{i}_L, \quad \bar{i}_L = \frac{\bar{v}_S}{R_C + R_L} \tag{22}$$

from which it is easy to see that for all (non–zero) equilibrium states, the power extracted from the controller is nonzero. Consequently, it is not possible to stabilize the circuit via energy–balancing.

We follow now the power–shaping procedure proposed in [4] to derive an alternative representation of the circuit that reveals the new passivity property. This is given by

$$\tilde{\boldsymbol{A}}\begin{bmatrix}\frac{di_L}{dt}\\ v_C\end{bmatrix} = \begin{bmatrix}\frac{\partial \tilde{P}}{\partial i_L}(i_L, v_C)\\ \frac{\partial \tilde{P}}{\partial v_C}(i_L, v_C)\end{bmatrix} - \begin{bmatrix}v_S\\ 0\end{bmatrix} \tag{23}$$

[1] As discussed in [3], energy–balancing stabilization is possible only for systems for which a finite amount of energy can be extracted from the source. This in particular implies that the supplied power evaluated at the equilibrium point should be equal to zero

[2] See [4], [8] for details on this model and the power–shaping procedure.

with the new admissible pair

$$\tilde{\boldsymbol{A}} = \begin{bmatrix} -L & 2R_C C \\ 0 & -C \end{bmatrix}, \tag{24}$$

$$\tilde{P} = \frac{R_C}{2}\left(\frac{v_C}{R_C} - i_L\right)^2 + \frac{1}{2}(R_L + R_C)i_L^2 \tag{25}$$

Notice the positive sign in the second right hand term of $\tilde{P}$ and that the symmetric part of the matrix $\tilde{\boldsymbol{A}}$ is negative definite precisely under the condition $L > R_C^2 C$ identified in Section IV. We thus obtain the desired dissipation inequality

$$\dot{\tilde{P}} = [\frac{di_L}{dt} \quad \dot{v}_C]\tilde{\boldsymbol{A}} \begin{bmatrix} \frac{di_L}{dt} \\ \\ \dot{v}_C \end{bmatrix} + \frac{di_S}{dt}v_S \leq \frac{di_S}{dt}v_S$$

Denoting with $(i_L^\star, v_C^\star)$ the desired equilibrium to be stabilized, we will shape the function $\tilde{P}$ to assign a minimum at this point. To this end, we propose to find functions $\tilde{P}_a(i_L), \hat{v}_S(i_L)$ such that

$$\dot{\tilde{P}}_a(i_L) = -\frac{di_S}{dt}\hat{v}_S(i_L), \tag{26}$$

yielding a new dissipation inequality for the desired potential function $\tilde{P}_d = \tilde{P} + \tilde{P}_a$. Since $i_L = i_S$, it is clear that, for any arbitrary (differentiable) function $\tilde{P}_a(i_L)$, the function $\hat{v}_s(i_L) = -\frac{\partial \tilde{P}_a}{\partial i_L}(i_L)$ solves (26). We propose for simplicity to complete the squares and add a quadratic term in the current errors with

$$\tilde{P}_a = -(R_L + R_C)i_L^\star i_L + \frac{1}{2}K(i_L - i_L^\star)^2 \tag{27}$$

with $K \geq 0$ a tuning parameter. This results in the controlled voltage

$$v_S = -K(i_L - i_L^\star) + (R_L + R_C)i_L^\star \tag{28}$$

which globally stabilizes the system with Lyapunov function

$$\tilde{P}_d = \frac{R_C}{2}\left(\frac{v_C}{R_C} - i_L\right)^2 + \frac{1}{2}(R_L + R_C + K)(i_L - i_L^\star)^2$$

VI. Conclusions and Future Work

The main contribution of this paper is the identification of the linear RLC networks that preserve passivity when a derivative action is added to the voltage or current source variables. In the case of n–ports networks, there exist a relation between these new passivity properties and the conditions that make the matrix $\tilde{\boldsymbol{A}}$ in the Brayton–Moser model be negative definite. We have also discussed the application of the *power shaping* methodology of [4] for the case of linear RLC circuits.

Some issues that remain open, and are currently being explored, are the following:

- Establish the connection between the characterizations of RLC circuits given here for the linear case and in [4] for general nonlinear circuits. Although the construction of the matrix $\tilde{\boldsymbol{A}}$ proposed in [4] is only a suficient condition, in the linear case, some calculations using Hamiltonian matrices and the LMI version of the positive real lemma [9] reveal that this construction fully characterizes the linear RLC circuits that satisfy the new passivity properties. Current research is under this way.
- Give a systematic procedure to complete the power shaping synthesis for general linear RLC circuits that satisfy these passivity properties.
- Applications of power shaping in power electronics: as we showed in Section III there is a clear relation between the expression $\mathbf{v}_S^T \frac{d\mathbf{i}_S}{dt}$ or $\dot{\mathbf{v}}_S^T \mathbf{i}_S$ and the notion of *reactive power*, henceforth power shaping ideas could be useful in the synthesis of reactive power compensators.
- Find mechanical and electromechanical analogs of these new properties.

References

[1] M. Takegaki and S. Arimoto. "A new feedback for dynamic control of manipulators", *Trans. of the ASME: Journal of Dynamic Systems, Measurement and Control*, 102, pp. 119-125. 1981

[2] R. Ortega, A. Loria, P.J. Nicklasson and H. Sira–Ramirez, *Passivity–based control of Euler–Lagrange Systems*, Springer–Verlag, Berlin, Communications and Control Engineering, Sept. 1998.

[3] R. Ortega, A. van der Schaft, I. Mareels and B. Maschke: "Putting energy back in control", *IEEE Control Syst. Magazine*, Vol. 21, No. 2, April 2001, pp. 18–33.

[4] R. Ortega, D. Jeltsema and J. Scherpen. "Power shaping: a new paradigm for stabilization of nonlinear RLC circuits". *IEEE Trans. Automatic Control 2003*, Vol 48, No 10, October 2003.

[5] P. Penfield, R. Spence and S. Duinker, *Tellegen's Theorem and Electrical Networks*, MIT Press, 1970.

[6] C.A.Desoer and E.S. Kuh. *Basic circuit theory*. Mc Graw Hill, 1969.

[7] R. Ortega and B.E. Shi, "A note on passivity of nonlinear RL and RC circuits", *IFAC World Conf.* Barcelona, Spain, 2002.

[8] R.K. Brayton and J. Moser, "A theory of nonlinear networks – I, II", *Quart. of App. Math.*, vol. 22, no. 1, pp. 1–33, respectively, vol. 22, no. 2, pp. 81–104, 1964.

[9] S. Boyd, L. El Ghaoui, E. Feron and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, SIAM studies in applied mathematics, Vol. 15. 1994.