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Adaptive Control of Linear Time Delay Systems

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***Abstract*— Two new output feedback adaptive control schemes based on Model Reference Adaptive Control (MRAC) and adaptive laws for updating the controller parameters are developed for a class of linear multi-input multi-output (MIMO) systems with state delay. A controller structure established on a new error equation parametrization is proposed to achieve tracking with the error tending to zero asymptotically. To achieve exact asymptotical tracking, we introduce, in the standard MRAC structure for plants without delay, a new additional adaptive feedforward control component as an output of a dynamical system driven by the reference signal. Adaptive laws are developed using the SPR-Lyapunov design approach and two assumptions regarding the prior knowledge of the high-frequency matrix K_p. This work is the first asymptotic exact zero tracking results for this class of systems in the framework of the certainty equivalence approach.**

I. Introduction

Many physical systems can be modeled by delay differential equations. In these models, time delays are often used to represent the effect of e.g. transmission, and transportation. Often time delays can be used as an approximation of complex models. Much effort has been devoted to providing a theory for the control of such systems. Interesting and important results in many directions are found, see, e.g. the 141 references in the recent survey paper [1]. However less attention has been given to the topic of output adaptive control of continuous-time state delay systems, and only a few results deal with *model reference adaptive output feedback control* of systems with state delays.

Adaptive stabilizing controllers were synthesized in [2], [3] for output feedback linear state delay systems, and in [4] for state feedback linear systems with state delays, subject to uncertainties with unknown bounds and known functional properties. All these stabilizing controllers guarantee that all closed loop solutions converge to a some bounded residual set. An adaptive discontinuous output feedback controller was considered in [5] to achieve exact asymptotic regulation for a class of single-input, single-output systems described by nonlinear functional differential equations. See also the recent paper for the MIMO case [6]. Subsequently, adaptive tracking control was considered for the same class of systems in [7], using a continuous feedback on one hand, and discontinuous feedback on the other hand. Using continuous feedback, [7] achieved only practical tracking , i.e. convergence to some bounded residual set. Discontinuous feedback enabled [7], as well as [8] to achieve exact asymptotic tracking, i.e. in the sense that the tracking error asymptotically approaches zero. State feedback MRAC was investigated in [9], [10].

Recently a new approach, [11], [12], was developed for the output model reference adaptive control of single input ($u(t) \in \mathbb{R}$) single output ($y(t) \in \mathbb{R}$) linear continuous-time plants with state delay described by equations, suitably initialized, of the form

$$\dot{x}(t) = Ax(t) + A_\tau x(t-\tau) + bu(t), \qquad y(t) = c^T x(t)$$

with unknown A, A_τ, b and c of appropriate dimensions and known time delay τ. The main idea is to treat the state delay element not as a part of the plant but rather as the input to the system $W_0(s) = c^T(Is-A)^{-1}b$ and then decompose the control law into two components. The first base component is designed by a standard MRAC procedure, [13]–[15], as for a plant without delay $W_0(s)$ but applied to the time-delay plant. The second component is formed by *a special adaptively adjusted dynamic system* $\mathrm{P}(s,\theta_{ff})$ as a function of the reference signal $r(t)$. This makes it possible to use the well-understood MRAC design technique.

The main contribution of the present paper is the design of a new adaptive control scheme which generalizes the results in [12] as follows:

- i the class of systems is enlarged to a class of multi-input $u(t) \in \mathbb{R}^m$ multi-output $y(t) \in \mathbb{R}^m$ systems.
- ii we construct two different types of prefilters $P(s,\theta_{ff})$, which issue the feedforward component u_{ff} whose function is to counteract the state delay.
- iii the adaptation algorithms are synthesized using the SPR-Lyapunov design approach for two cases of prior knowledge of the high-frequency matrix K_p.

The structure of the paper is as follows. In Section II we formulate the MIMO adaptive control problem. In Section III we suggest the new parametrization for the error equation, which leads to a new controller structure. It is developed in Section IV. In Sections V and VI we develop two adaptive designs for asymptotic output tracking when we use two different assumptions concerning the prior knowledge of the high-frequency matrix K_p: the symmetry assumption of [15], or the assumption on the signs of the leading principal minors of K_p [16], respectively. Some final remarks are found in Section VI.

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II. Problem Statement

In this section we formulate the control problem, including the state delay plant model and the reference model, assumptions and control objective. The uncertain multi-input ($u(t)$) multi-output ($y(t)$) linear continuous-time plant with state delay is of the form

$$\begin{aligned}\dot{x}(t) =& Ax(t)+A_\tau x(t-\tau)+Bu(t), \quad x(t)\in\mathbb{R}^n\\ y(t) =& Cx(t), \quad y(t)\in\mathbb{R}^m\end{aligned} \tag{1}$$

where $x(t)\in\mathbb{R}^n$, $y(t)\in\mathbb{R}^m$ and $u(t)\in\mathbb{R}^m$ are, respectively, the state, output and control input. The constant matrices A, A_τ and B of appropriate dimensions have unknown elements. The time-delay τ is assumed to be known. It is also assumed that the states are not accessible and only input-output measurements are available.

It is a specification that all signals of the closed loop system remain bounded and that the plant output $y(t)$ asymptotically exact follows the output $y_r(t)$ of a reference model with the transfer function

$$y_r(t) = W_r(s)r(t) \tag{2}$$

where $W_r(s)\in\mathbb{R}^{m\times m}$ is a stable rational transfer matrix, and $r(t)\in\mathbb{R}^m$ is a bounded reference input signal. Asymptotic tracking is demanded, i.e. $\lim_{t\to\infty}\|e(t)\|=0$.

The following assumptions are made on the plant (1) and the reference model (2): **(A1)** When there is no term with state delay, the plant (1) can be described by

$$y=W_0(s)u \quad W_0(s)=C(Is-A)^{-1}B\in\mathbb{R}^{m\times m} \tag{3}$$

where $W_0(s)$ is the transfer matrix associated with an undelayed plant; **(A2)** the observability index ν of $W_0(s)$ is known; **(A3)** the transmission zeros of $W_0(s)$ have negative real parts; **(A4)** $W_0(s)$ is strictly proper, full rank, and has vector relative degree 1; **(A5)** $A_\tau = BA_\tau^{*T}$; **(A6)** in view of the assumption (A4) and without loss of generality, a diagonal SPR reference model is defined, as in [16],

$$W_R(s)=diag\left[\frac{1}{s+a_{ri}}\right] \qquad a_{ri}>0,\ i=1,\dots,m. \tag{4}$$

For the high frequency gain matrix $K_p=\lim_{s\to\infty}sW_0(s)$ we consider the following two cases: **(A7.1)** there is a known matrix $S_p\in\mathbb{R}^{m\times m}$ such that $K_pS_p=(K_pS_p)^T$, or **(A7.2)** the signs of the leading principal minors of the high frequency gain matrix K_p are known.

III. Proposed Error Equation Parametrization

Let us assume that all the parameters of (1) are known, and let us define u_1^* as the standard matching control [13], [15] for the plant without delay (3)

$$u_1^*(t) = \theta_e^* y(t)+\theta_1^{*T}x_1(t)+\theta_2^{*T}x_2(t)+\theta_r^* r(t) \tag{5}$$

where

$$x_1 = H_m(s)[u_1^*] \quad x_1\in\mathbb{R}^{m(\nu-1)} \tag{6}$$

$$x_2 = H_m(s)[y] \quad x_2\in\mathbb{R}^{m(\nu-1)} \tag{7}$$

$$H_m(s) = \frac{[I_{m\times m}s^{\nu-2}\ \dots\ I_{m\times m}s\ I_{m\times m}]^T}{\Lambda(s)} \quad H_m(s)\in\mathbb{R}^{m(\nu-1)\times m} \tag{8}$$

$\theta_1^*,\theta_2^*\in\mathbb{R}^{m(\nu-1)\times m}$ $\quad\theta_e^*\in\mathbb{R}^{m\times m}$ $\quad\theta_r^*\in\mathbb{R}^{m\times m}$, $\Lambda(s)=s^{\nu-1}+\cdots+\lambda_m s+\lambda_0$ is a monic Hurwitz polynomial, and $I_{m\times m}\in\mathbb{R}^{m\times m}$ is the identity matrix.

With the definition of $\Lambda(s)$, $H_m(s)$ and $W_0(s)$ in (3), there exist $\theta_r^*=K_p^{-1}$, θ_e^*, θ_1^* and θ_2^* [13], [15] such that

$$\theta_r^* W_R^{-1}(s)W_0(s) = I_{m\times m}-\theta_e^{*T}H_m(s)-\theta_1^{*T}H_m(s)W_0(s)-\theta_2^{*T}W_0(s) \tag{9}$$

When applying (5) to the actual plant (1), then from (1) and (9) and for any u, the tracking error $e=y-y_r$ is given by

$$\begin{aligned}e =& W_r(s)K_p\Big[u-\theta_{ei}^* y-\theta_1^{*T}x_1-\theta_2^{*T}x_2-\theta_r^* r+A_\tau^{*T}x(t-\tau)\\ &-\theta_1^{*T}H_m(s)A_\tau^{*T}x(t-\tau)\Big].\end{aligned} \tag{10}$$

To find a suitable error equation parametrization, we manipulate the last term of (10). Firstly, we introduce a new dynamical system

$$z(t) = \theta_1^{*T}H_m(s)[A_\tau^{*T}x(t-\tau)] = \theta_z^{*T}z_x(t) \tag{11}$$

where $\theta_z^{*T}=[\theta_1^{*1T}A_\tau^{*T},\ \theta_1^{*2T}A_\tau^{*T},\ \dots,\ \theta_1^{*(\nu-1)T}A_\tau^{*T}]$ and

$$z_x(t) = H_n(s)[x(t-\tau)] \tag{12}$$

$$H_n(s) = \frac{[I_{n\times n}s^{\nu-2},\ \dots,\ I_{n\times n}s,\ I_{n\times n}]^T}{\Lambda(s)} \tag{13}$$

Here $A_z\in\mathbb{R}^{m\times n(\nu-1)}$, $z_x\in\mathbb{R}^{n(\nu-1)}$, $H_n(s)\in\mathbb{R}^{n(\nu-1)\times n}$ and $I_{n\times n}$ is the $n\times n$ identity matrix.

Remark 1: The transfer function matrix $H_n(s)$ from (13) has the same structure as the transfer matrix $H_m(s)$ from (8), only instead of the identity matrix $I_{m\times m}$ in the numerator of (8) we have the identity matrix $I_{n\times n}$.

Secondly we decompose the signals z_x in (12) into two components $z_x(t)=z_e(t)+z_r(t)$ where

$$\begin{aligned}z_e(t) =& H_n(s)\big[e_x(t-\tau)\big] \quad z_r(t)=H_n(s)\big[x_r(t-\tau)\big]\\ e_x(t-\tau) =& x(t-\tau)-x_r(t-\tau)\end{aligned} \tag{14}$$

where $x_r(t)\in\mathbb{R}^n$ is the state of the reference model (4) with the state space triple (A_r,B_r,C_r).

Then, using (11) and (14) from (10) we obtain the *basic error equation*

$$\begin{aligned}e(t) =& W_r(s)K_p\Big[u(t)-\theta_e^* e(t)-\theta_1^{*T}x_1(t)-\theta_2^{*T}x_2(t)-\theta_r^* r(t)\\ &-\theta_{x_r}^{*T}x_r(t)-\theta_\tau^{*T}x_r(t-\tau)-\theta_z^{*T}z_r(t)\Big]\\ &-W_r(s)K_p\Big[\theta_\tau^{*T}e_x(t-\tau)+\theta_z^{*T}z_e(t)\Big]\end{aligned} \tag{15}$$

where $\theta_\tau^*=-A_\tau^*$ and $\theta_{x_r}^*=C_r^T\theta_e^{*T}$.

Remark 2: Note that $e_x(t)$ and $z_e(t)$ are not available for measurement and we shall use them only for analysis.

IV. Proposed Adaptive Controller Structure

The error parametrization (15) motivates the following controller structure

$$\begin{aligned} u(t) =& \theta_e(t)e(t) + \theta_1^T(t)x_1(t) + \theta_2^T(t)x_2(t) + \theta_r(t)r(t) \\ &+ \theta_{x_r}^T(t)x_r(t) + \theta_\tau^T(t)x_r(t-\tau) + \theta_z^T(t)z_r(t) \end{aligned} \quad (16)$$

where $\theta_1,\ \theta_2 \in \mathbb{R}^{m(\nu-1)\times m}$, $\theta_e,\ \theta_r,\ \theta_y \in \mathbb{R}^{m\times m}$, $\theta_{x_r}(t), \theta_\tau(t) \in \mathbb{R}^{n\times m}$ and $\theta_z \in \mathbb{R}^{n(\nu-1)\times m}$ are the adaptation gain matrices, $x_1 = H_m(s)[u] \in \mathbb{R}^{m(\nu-1)}$, and x_2, $H_m(s)$ taken from (7)-(8).

For clarity, we shall decompose $u(t)$ as the sum of the two components $u_f(t)$ and $u_{ff}(t)$,

$$u(t) = u_f(t) + u_{ff}(t) \quad (17)$$

which will be defined in the next subsections IV-A and IV-B.

A. The standard control component with output feedback

The first component $u_f(t)$ contains the output feedback control component,

$$u_f(t) = \theta_e e(t) + \theta_1^T x_1(t) + \theta_2^T x_2(t) + \theta_r r(t) = \theta_f^T \widehat{\omega}_f \quad (18)$$

with $\theta_f = [\theta_e\ \theta_1^T\ \theta_2^T\ \theta_r]^T \in \mathbb{R}^{2m\nu\times m}$ and $\widehat{\omega}_f = [e^T\ x_1^T\ x_2^T\ r^T]^T \in \mathbb{R}^{2m\nu}$. $u_f(t)$ is the "classical" model matching adaptive control version of (5) which is widely used in MIMO MRAC for plants without time-delays, see e.g. the textbooks [13]–[15], with the modification that in (5), $e = y - y_r$ is used instead of y.

B. The additional dynamical feedforward control component

The second component defines additional feedforward.

$$u_{ff}(t) = \theta_{x_r}^T x_r(t) + \theta_\tau^T x_r(t-\tau) + \theta_z^T z_r(t) = \theta_{ff}^T \omega_{ff} \quad (19)$$

with $\theta_{ff}(t) = [\theta_{x_r}^T\ \theta_\tau^T\ \theta_z^T]^T \in \mathbb{R}^{n(\nu+1)\times m}$ and $\omega_{ff}(t) = [x_r^T(t)\ x_r^T(t-\tau)\ z_r^T(t)]^T \in \mathbb{R}^{n(\nu+1)}$ is the output of a dynamical system with the reference signal r as the input. In addition to the usual memoryless feedforward term $\theta_r(t)r(t)$ with the adjusted gain $\theta_r(t)$ contained in $u_f(t)$, see (18), $u_{ff}(t)$ includes terms with the adjusted matrix gains $\theta_{x_r}(t)$, $\theta_\tau(t)$ and $\theta_z(t)$. $u_{ff}(t)$ is hence formed by a *special adaptively adjusted dynamic system* as a function of the reference signal. This *dynamic feedforward system constitutes the main contribution of our approach*.

In the next two sections we will design adaptive laws for the two distinct assumptions, given in section II, about the high frequency gain matrix K_p. First we use the symmetry conditions of K_p [15] (Assumption A7.1), and then the assumption on the signs of the leading principal minors of K_p [16] (Assumption A7.2).

V. Adaptive controller: Assumption (A7.1)

Introducing the parameter errors $\tilde{\theta}_f(t)$ and $\tilde{\theta}_{ff}(t)$ and using the adaptive control (17) – (19), the basic tracking error equation (15) can be expressed as

$$\begin{aligned} e(t) =& W_r(s)K_p\Big[\tilde{\theta}_f^T(t)\widehat{\omega}_f(t) + \tilde{\theta}_{ff}^T(t)\omega_{ff}(t) \\ &- \theta_\tau^{*T} e_x(t-\tau) - \theta_z^{*T} z_e(t)\Big] \end{aligned} \quad (20)$$

where $\tilde{\theta}_f(t) = \theta_f(t) - \theta_f^*$, $\tilde{\theta}_{ff}(t) = \theta_{ff}(t) - \theta_{ff}^*$, $\theta_f^* = [\theta_e^*\ \theta_1^{*T}\ \theta_2^{*T}\ \theta_r^*]^T$ and $\theta_{ff}^* = [\theta_{x_r}^{*T}\ \theta_\tau^{*T}\ \theta_z^{*T}]^T$.

To design the mechanism of updating the controller matrices, the usual way of MRAC for the delay free systems is used, see, e.g. [16]. The augmented vector $\hat{x}(t) = [x\, x_1\, x_2]^T$ is introduced, and the state of the corresponding nonminimal realization $\hat{C}(sI-\hat{A})^{-1}\hat{B}$ of W_r is denoted by $\hat{x}_r(t)$. Then we can write the following state space representation for (20)

$$\begin{aligned} \frac{d\hat{e}(t)}{dt} =& \hat{A}\hat{e}(t) + \hat{B}K_p\Big[\tilde{\theta}_f^T(t)\widehat{\omega}_f(t) + \tilde{\theta}_{ff}^T(t)\omega_{ff}(t) \\ &- \theta_\tau^{*T}\hat{l}^T\hat{e}(t-\tau) - \theta_z^{*T}C_e\hat{z}_e(t)\Big] \\ \frac{d\hat{z}_e(t)}{dt} =& A_e\hat{z}_e(t) + B_e\hat{l}^T\hat{e}(t-\tau) \\ z_e(t) =& C_e\hat{z}_e(t), \quad \hat{l} = [I_{n\times n}\ 0_{n\times m(\nu-1)}\ 0_{n\times m(\nu-1)}]^T \\ e(t) =& y(t) - y_r(t) = \hat{C}\hat{e}(t) \end{aligned} \quad (21)$$

where the triple (A_e, B_e, C_e) is a minimal state space realization for the stable transfer matrix $H_n(s)$ from (14), and $0_{n\times m(\nu-1)}$ is a zero $n\times m(\nu-1)$ matrix.

Because $\hat{C}(sI-\hat{A})^{-1}\hat{B} = W_r(s)$ is SPR, the triple $(\hat{A}, \hat{B}, \hat{C})$ satisfies the following equations given by the matrix version of KY Lemma, see, e.g. [14], [16],

$$\hat{A}^T\hat{P} + \hat{P}\hat{A} + \hat{Q} = 0 \qquad \hat{P}\hat{B} = \hat{C}^T \quad (22)$$

where $\hat{P} = \hat{P}^T > 0$ and $\hat{Q} = \hat{Q}^T > 0$. Since A_e in (21) is stable, it also holds that

$$A_e^T P_z + P_z A_e + Q_z = 0 \quad (23)$$

where $P_z = P_z^T > 0$ and $Q_z = Q_z^T > 0$.

We are now ready to state the main result of this section.

Theorem 1: Consider system (1) and the reference model (4). Suppose that assumptions (A1) to (A7.1) hold. Then the adaptive control (17)–(19) with update laws

$$\begin{aligned} \dot{\theta}_f^T(t) &= -S_p e(t)\widehat{\omega}_f^T(t) \\ \dot{\theta}_{ff}^T(t) &= -S_p e(t)\omega_{ff}^T(t) \end{aligned} \quad (24)$$

guarantee that all the closed loop signals are bounded and the tracking error $e(t) = y(t) - y_r(t)$ converges to zero asymptotically, i.e. $\lim_{t\to\infty}\|e(t)\| = 0$.

To proof this theorem we will use the standard MRAC analysis technique for delay free plants, e.g. [13], [15], [16], but instead of using the standard quadratic Lyapunov function, an appropriate Lyapunov Krasovskii functional is added as in [12],

$$V = \hat{e}^T\hat{P}\hat{e} + \hat{z}_e^T P_z \hat{z}_e + \int_{t-\tau}^{t} \hat{e}^T(s) Q_e \hat{e}(s) ds \\ + \mathrm{tr}(\tilde{\theta}_f - \hat{K}_1)\Gamma_p(\tilde{\theta}_f - \hat{K}_1)^T + \mathrm{tr}(\tilde{\theta}_{ff}\Gamma_p\tilde{\theta}_{ff}^T), \quad (25)$$

where $Q_e = Q_e^T > 0$, $\Gamma_p = K_p^T S_p^{-1}$ wherein the known matrix S_p satisfies Assumption A7.1,

$$\hat{K}_1 = -\tfrac{r}{2}[K_p^{-1} \;\; 0 \;\; 0 \;\; 0], \quad (26)$$

and r is an as yet unspecified positive constant.

With this definition of $\hat{K}_1$, using (21), (22), (24) and Assumption (A7.1) we have $\mathrm{tr}[\hat{K}_1\Gamma_p\dot{\tilde{\theta}}_f^T] = -\frac{r}{2}\hat{e}(t)^T\hat{P}\hat{B}\hat{B}^T\hat{P}\hat{e}(t)$. Then some simple computations using (22), (24) and (26) with $\hat{Q} = Q + Q_e$, $Q = Q^T > 0$ lead to the derivative of V along the solution of (21),

$$\dot{V}|_{(21)} = -\hat{e}^T(t)Q\hat{e}(t) - r\hat{e}(t)^T\hat{P}\hat{B}\hat{B}^T\hat{P}\hat{e}(t) - \hat{z}_e^T(t)Q_z\hat{z}_e(t) \\ - \hat{e}^T(t-\tau)Q_e\hat{e}(t-\tau) - 2\hat{e}^T(t)\hat{P}\hat{B}K_p\theta_\tau^{*T}\hat{l}^T\hat{e}(t-\tau) \\ - 2\hat{e}^T(t)\hat{P}\hat{B}K_p\theta_z^{*T}C_e\hat{z}_e(t) \\ + 2\hat{z}_e^T(t)P_zB_e\hat{l}^T\hat{e}(t-\tau) \quad (27)$$

For convenience, let us define the matrices $Q_e = (q_{e1} + q_{e2} + q_{e3})I$, $Q_z = (q_{z1} + q_{z2})I$, and the scalar $r = r_1 + r_2$, where q_{ei}, q_{zi} and r_i, $(i = 1, \ldots)$ are positive constants. Note that these constants are only used in the process of the proof and not used in the control design, and hence we can suppose that they take arbitrary positive values.

Combining the second and fifth, second and sixth and third and seventh terms of (27), completing the squares and dropping negative terms we obtain

$$\dot{V}|_{(21)} \le -\hat{e}^T(t)Q\hat{e}(t) - q_{z2}\hat{z}_e^T(t)\hat{z}_e(t) - q_{e3}\hat{e}^T(t-\tau)\hat{e}(t-\tau) \\ - \hat{e}^T(t-\tau)\left[q_{e1}I - \tfrac{1}{r_1}\Psi_{\tau 1}\right]\hat{e}(t-\tau) \\ - \hat{e}^T(t-\tau)\left[q_{e2}I - \tfrac{1}{q_{z1}}\Psi_{\tau 2}\right]\hat{e}(t-\tau) \\ - \hat{z}_e^T(t)\left[q_{z2} - \tfrac{1}{r_2}\Psi_z\right]\hat{z}_e(t) \quad (28)$$

where

$$\Psi_{\tau 1} = \hat{l}\theta_\tau^* K_p^T K_p \theta_\tau^{*T}\hat{l}^T \quad \Psi_{\tau 2} = \hat{l}B_e^T P_z P_z B_e \hat{l}^T \\ \Psi_z = C_e^T\theta_z^* K_p^T K_p \theta_z^{*T} C_e \quad (29)$$

Let us select the values of r_1, r_2, q_{e2} and q_{z1} such that the following inequalities are satisfied,

$$r_1 > \tfrac{1}{q_{e1}}\lambda_{max}[\Psi_{\tau 1}] \;\; q_{z1} > \tfrac{1}{q_{e2}}\lambda_{max}[\Psi_{\tau 2}] \;\; r_2 > \tfrac{1}{q_{z2}}\lambda_{max}[\Psi_z]$$

where $\lambda_{max}(\Psi)$ is the maximum eigenvalue of Ψ. Then, we obtain from (28)

$$\dot{V}|_{(21)} \le -\hat{e}^T(t)Q\hat{e}(t) - q_{z2}\hat{z}_e^T(t)\hat{z}_e(t) \\ - q_{e3}\hat{e}^T(t-\tau)\hat{e}(t-\tau) \le 0 \quad (30)$$

This implies [17] that V and, therefore, $\hat{e}(t)$, $e(t)$, $\hat{z}_e(t)$, $\widetilde{\Theta}_f$, Θ_f, $\widetilde{\Theta}_{ff}$, $\Theta_{ff} \in \mathsf{L}_\infty$. This fact is central to the remainder of the stability analysis, which follows directly using the steps in [15].

Because $\hat{e}(t) = \hat{x}(t) - \hat{x}_r(t)$ and $\hat{x}_r(t) \in L_\infty$, it holds that $\hat{x}(t) = [x^T(t),\ x_1^T(t), x_2^T(t)]^T \in L_\infty$, which implies that $x(t), x_1(t), x_2(t)$ and $y(t) \in L_\infty$. Since $r(t)$ is uniformly bounded and the transfer matrix $H_n(s)$ from (14) is stable, $\widehat{\omega}_f = [e^T\ x_1^T\ x_2^T\ r^T]^T$ and $\omega_{ff}(t) = [x_r^T(t)\ x_r^T(t-\tau)\ z_r^T(t)]^T$ are bounded. Consequently $u(t) = u_f(t) + u_{ff}(t)$ is also bounded. Therefore, all the signals in the closed-loop system are bounded. From (25) and (30) we establish that $\hat{e}(t)$ and therefore $e(t) \in L_2$. Furthermore, using $\hat{e}(t), \hat{z}_e(t), \theta_f(t)$, $\theta_{ff}(t), \widehat{\omega}_f(t), \omega_{ff}(t) \in L_\infty$ in (21) we have that $\dot{\hat{e}}(t), \dot{e}(t) \in L_\infty$. Hence, $e \in L_2 \bigcap L_\infty$, and $\dot{e}(t) \in L_\infty$, which by Barbălat's Lemma [15] implies that $\|e(t)\| \to 0$ as $t \to \infty$.

VI. Adaptive controller: Assumption (A7.2)

To avoid the quite restrictive Assumption (A7.1), we will use in this section the recent results for multivariable MRAC design in [16] for plants without delays. The design in [16] is based on the *SDU* factorization [18] of the high frequency gain matrix K_p, with the assumption the signs of the leading principal minors of K_p are known. Such an assumptions is less restrictive than the symmetry condition in Assumption (A7.1). The following preliminary lemmas are needed.

Lemma 1 [16]: Every $m \times m$ matrix K_p with nonzero leading principal minors $\Delta_1, \ldots \Delta_m$ can be factored as $K_p = SDU$ where S is symmetric positive definite, D is diagonal, and U is unity upper triangular.

This factorization of K_p is convenient because of the distinct role played by each of its factors S, D and U. The role of S is to assure the $W_r(s)S$ is SPR. The rôle of D is to enable a straightforward extension of the SISO assumption about the sign of the high-frequency gain. The rôle of U is to allow its absorption by the controller parametrization [16].

Lemma 2 [16]: For any $W_r(s)$ from (4) a positive definite $S = S^T$ exists such that $W_r(s)S$ is SPR.

Substituting the *SDU* factorization of K_p in the basic error equation (15), and using the decomposition $Uu = u - (I_{m\times m} - U)$ as in [16], we obtain

$$e(t) = W_r(s)SD\Big[u(t) - (I-U)u(t) - U\theta_e^* e(t) - U\theta_1^{*T}x_1(t) \\ - U\theta_2^{*T}x_2(t) - U\theta_r^* r(t) - U\theta_{x_r}^{*T}x_r(t) - U\theta_\tau^{*T}x_r(t-\tau) \\ - U\theta_z^{*T}z_r(t)\Big] \\ - W_r(s)SD\Big[U\theta_\tau^{*T}e_x(t-\tau) + U\theta_z^{*T}z_e(t)\Big] \quad (31)$$

By defining $\hat{\theta}_e^* = U\hat{\theta}_e^*$, $\hat{\theta}_1^{*T} = U_i\theta_1^{*T}$, $\hat{\theta}_2^{*T} = U\theta_2^{*T}$, $\hat{\theta}_r^* = U\theta_r^*$, $\hat{\theta}_u^* = (I_{m\times m} - U)u$, $\hat{\theta}_{x_r}^{*T} = U\theta_{x_r}^{*T}$, $\hat{\theta}_\tau^{*T} = U\theta_\tau^{*T}$, $\hat{\theta}_z^{*T} = U\theta_z^{*T}$, and $\hat{\theta}_z^{*T} = U\theta_z^{*T}$, we obtain from (31)

$$e(t) = W_r(s)SD\Big[u(t) - \hat{\theta}_e^* e(t) - \hat{\theta}_1^{*T}x_1(t) - \hat{\theta}_2^{*T}x_2(t) \\ - \hat{\theta}_r^* r(t) - \hat{\theta}_u^* u - \hat{\theta}_{x_r}^{*T}x_r(t) - \hat{\theta}_\tau^{*T}x_r(t-\tau) - \hat{\theta}_z^{*T}z_r(t)\Big] \\ - W_r(s)SD\Big[\hat{\theta}_\tau^{*T}e_x(t-\tau) + \hat{\theta}_z^{*T}z_e(t)\Big]. \quad (32)$$

We can rewrite (32) as

$$
\begin{aligned}
e(t) =& W_r(s)SD\Big[u(t) - K_f^{*T}(t)\overline{\omega}_f(t) - \Theta_{ff}^{*T}(t)\omega_{ff}(t)\Big] \\
& - W_r(s)SD\Big[\hat{\theta}_\tau^{*T} e_x(t-\tau) + \hat{\theta}_z^{*T} z_e(t)\Big]
\end{aligned}
\tag{33}
$$

where

$$
\begin{aligned}
K_f^* =& [\hat{\theta}_e^* \; \hat{\theta}_1^{*T} \; \hat{\theta}_2^{*T} \; \hat{\theta}_r^* \; \hat{\theta}_u^*]^T, \; \Theta_{ff}^* = [\hat{\theta}_{x_r}^{*T} \; \hat{\theta}_\tau^{*T} \; \hat{\theta}_z^{*T}]^T, \\
\overline{\omega}_f =& [e^T \; x_1^T \; x_2^T \; r^T \; u^T]^T, \; \omega_{ff} = [x_r^T(t) \; x_r^T(t-\tau) \; z_r^T(t)]^T.
\end{aligned}
$$

In order to remove the zero entries from the above parametrization of K_f, we introduce, as in [16], the new parameter vectors Θ_f^k via the identity

$$
\left[\Theta_f^{*1T}\Omega_f^1 \cdots \Theta_f^{*kT}\Omega_f^k \cdots \Theta_f^{*mT}\Omega_f^m\right]^T = K_f^{*T}\overline{\omega}_f. \tag{34}
$$

In addition to the concatenated k-th rows of the matrices $\hat{\theta}_e^*$, $\hat{\theta}_1^*$, $\hat{\theta}_2^*$, $\hat{\theta}_r^*$, each row vector Θ_f^{*kT} includes the unknown entries of the k-th rows of $\hat{\theta}_u^*$. The strictly upper-trianglarity of $\hat{\theta}_u^*$ ensures that the control signal is implementable without singularity.

The corresponding regressor vectors are

$$
\begin{aligned}
\Omega_f^1(t) =& [\overline{\omega}_f^T \; u_2 \; u_3 \ldots u_{m-1} \; u_m]^T \\
\Omega_f^2(t) =& [\overline{\omega}_f^T \; u_3 \ldots u_{m-1} \; u_m]^T \\
& \vdots \\
\Omega_f^m(t) =& [\overline{\omega}_f^T \;]^T.
\end{aligned}
\tag{35}
$$

This new parametrization motives the following controller structure instead of (17)-(19)

$$
u(t) = \left[\Theta_f^{1T}\Omega_f^1 \cdots \Theta_f^{kT}\Omega_f^k \cdots \Theta_f^{mT}\Omega_f^m\right]^T + \theta_{ff}^T\omega_{ff}, \tag{36}
$$

and gives the following error equation instead of (33)

$$
\begin{aligned}
e(t) =& W_r(s)SD\Bigg[\begin{pmatrix} \Theta_f^{1T}(t)\Omega_f^1(t) \\ \vdots \\ \Theta_f^{mT}(t)\Omega_f^m(t) \end{pmatrix} - \begin{pmatrix} \Theta_f^{1*T}(t)\Omega_f^1(t) \\ \vdots \\ \Theta_f^{m*T}(t)\Omega_f^m(t) \end{pmatrix} \\
& + \theta_{ff}^T(t)\omega_{ff}(t) - \Theta_{ff}^{*T}(t)\omega_{ff}(t)\Bigg] \\
& - W_r(s)SD\Big[\hat{\theta}_\tau^{*T} e_x(t-\tau) + \hat{\theta}_z^{*T} z_e(t)\Big].
\end{aligned}
\tag{37}
$$

Introducing the parameter errors $\widetilde{\Theta}_f^k(t) = \Theta_f^k(t) - \Theta_f^{*k}$, $k = 1,\ldots,m$ and $\widetilde{\Theta}_{ff}(t) = \theta_{ff}(t) - \Theta_{ff}^*$, the equation for the tracking error follows from (37),

$$
\begin{aligned}
e(t) =& W_r(s)SD\Big[\Big(\widetilde{\Theta}_f^{1T}\Omega_f^1 \cdots \widetilde{\Theta}_f^{kT}\Omega_f^k \cdots \Theta_f^{mT}\Omega_f^m\Big)^T + \widetilde{\Theta}_{ff}^T\omega_{ff}\Big] \\
& - W_r(s)SD\Big[\hat{\theta}_\tau^{*T} e_x(t-\tau) + \hat{\theta}_z^{*T} z_e(t)\Big].
\end{aligned}
\tag{38}
$$

As in Section V, the augmented vector $\overline{x}(t) = [x \, x_1 \, x_2]^T$ is introduced, and the state of the corresponding non-minimal realization $\overline{C}(sI - \overline{A})^{-1}\overline{B}$ of $W_r(s)S$, $(\overline{C}\overline{B} = S)$ is denoted by $\overline{x}_r(t)$. Then we can write the following state space representation of (38)

$$
\begin{aligned}
\dot{\overline{e}}(t) =& \overline{B}D\Big[\Big(\widetilde{\Theta}_f^{1T}\Omega_f^1 \cdots \widetilde{\Theta}_f^{kT}\Omega_f^k \cdots \Theta_f^{mT}\Omega_f^m\Big)^T + \widetilde{\Theta}_{ff}^T(t)\omega_{ff}(t)\Big] \\
& - \overline{B}D\Big[\hat{\theta}_\tau^{*T}\hat{l}^T\overline{e}(t-\tau) + \hat{\theta}_z^{*T}C_e\overline{z}_e(t)\Big] \\
\dot{\overline{z}}_e(t) =& A_e\overline{z}_e(t) + B_e\hat{l}^T\overline{e}(t-\tau) \\
z_e(t) =& C_e\overline{z}_e(t) \\
e(t) =& y(t) - y_r(t) = \overline{C}\overline{e}(t)
\end{aligned}
\tag{39}
$$

Because $\overline{C}(sI - \overline{A})^{-1}\overline{B} = W_r(s)S$ is SPR [16], the triple $(\overline{A}, \overline{B}, \overline{C})$ satisfies the following equations, as in (22),

$$
\overline{A}^T\overline{P} + \overline{P}\overline{A} + \hat{Q} = 0 \qquad \overline{P}\overline{B} = \overline{C}^T \tag{40}
$$

where $\overline{P} = \overline{P}^T > 0$ and $\hat{Q} = Q_e + Q$.

To design the update laws, we use the functional

$$
\begin{aligned}
V =& \overline{e}^T\overline{P}\overline{e} + \overline{z}_e^T P_z\overline{z}_e + \int_{t-\tau}^t \overline{e}^T(s)Q_e\overline{e}(s)ds + \mathrm{tr}(\tilde{\Theta}_{ff}\Gamma^{-1}\overline{D}\tilde{\Theta}_{ff}^T) \\
& + \sum_{k=1}^m \left(\gamma_f^k\right)^{-1}|d^k|(\widetilde{\Theta}_f^k - \overline{K}_1^k)^T(\widetilde{\Theta}_f^k - \overline{K}_1^k)
\end{aligned}
\tag{41}
$$

where $\gamma_f^k > 0$, $\Gamma = \Gamma^T > 0$, $\overline{D} = \mathrm{diag}\{|d^1| \ldots |d^k| \ldots |d^m|\}$ wherein d^k are the entries of D, and

$$
\overline{K}_1^k = -r(d^k)^{-1}[I, \; 0, \ldots, \; 0]^T. \tag{42}
$$

The vectors $\overline{K}_1^k$ have the same dimension as Θ_f^k, and r is an "artificial" gain parameter whose value will be specified later.

Let the adaptation algorithm be

$$
\begin{aligned}
\dot{\tilde{\Theta}}_f^k =& -\gamma_f^k \mathrm{sign}(d^k)\Omega_f^k e^k, \quad k = 1,\ldots,m \\
\dot{\tilde{\Theta}}_{ff}^T =& -\mathrm{Sign}(D)\Gamma e(t)\omega_{ff}^T(t),
\end{aligned}
\tag{43}
$$

where $\mathrm{Sign}(D) = \mathrm{diag}\{\mathrm{sign}(d^1), \ldots, \mathrm{sign}(d^m)\}$

With this adaptation algorithm, the time derivative of (41) along the trajectories of the error system (39) becomes

$$
\begin{aligned}
\dot{V}|_{(39)} =& -\overline{e}^T(t)Q\overline{e}(t) - \overline{re}(t)^T\overline{P}\overline{B}\overline{B}^T\overline{P}\overline{e}(t) - \overline{z}_e^T(t)Q_z\overline{z}_e(t) \\
& - \overline{e}^T(t-\tau)Q_e\overline{e}(t-\tau) - 2\overline{e}^T(t)\overline{P}\overline{B}D\hat{\theta}_\tau^{*T}\hat{l}^T\overline{e}(t-\tau) \\
& - 2\overline{e}^T(t)\overline{P}\overline{B}D\hat{\theta}_z^{*T}C_e\overline{z}_e(t) + 2\overline{z}_e^T(t)P_zB_e\hat{l}^T\overline{e}(t-\tau)
\end{aligned}
$$

Using the same arguments as in Section V above, after the choice of r_1, r_2, q_{e2} and q_{z1} satisfying the inequalities

$$
r_1 > \tfrac{1}{q_{e1}}\lambda_{max}\left[\overline{\Psi}_{\tau 1}\right] \; q_{z1} > \tfrac{1}{q_{e2}}\lambda_{max}\left[\overline{\Psi}_{\tau 2}\right] \; r_2 > \tfrac{1}{q_{z2}}\lambda_{max}\left[\overline{\Psi}_z\right]
$$

where

$$
\overline{\Psi}_{\tau 1} = \hat{l}\hat{\theta}_\tau^* D^2\hat{\theta}_\tau^{*T}\hat{l}^T \; \overline{\Psi}_{\tau 2} = \hat{l}B_e^T P_z P_z B_e\hat{l}^T \; \overline{\Psi}_z = C_e^T\hat{\theta}_z^* D^2\hat{\theta}_z^{*T}C_e
$$

we obtain

$$
\begin{aligned}
\dot{V}|_{(39)} \leq& -\hat{e}^T(t)Q\hat{e}(t) - q_{z2}\hat{z}_e^T(t)\hat{z}_e(t) \\
& - q_{e3}\hat{e}^T(t-\tau)\hat{e}(t-\tau) \leq 0.
\end{aligned}
\tag{44}
$$

By applying the same arguments as in Theorem 1 it can be shown that $\lim_{t\to\infty}\|e(t)\| = 0$.

All this leads to the main result of this section:

Theorem 2: Consider the closed-loop system defined by the plant in (1), the controller in (36), and the updating algorithms in (43) with Assumption 7.2. Then the following two properties hold:

(i) all signals of the closed-loop system are bounded

(ii) $\lim_{t\to\infty} \|e(t)\| = 0$.

CONCLUSION

Two new output feedback adaptive control schemes based on Model Reference Adaptive Control (MRAC) and adaptive laws for updating the controller parameters are developed for a class of linear multi-input multi-output (MIMO) systems with state delay. An effective controller structure established on a new error equation parametrization is proposed to achieve *tracking with asymptotical zero error*. To achieve exact asymptotical tracking, we introduce, in the standard MRAC structure for the plants without delay, a new adaptive feedforward control component as an output of a dynamical system driven by the reference signal. The feedforward prefilter design procedure is developed to determine the necessary feedforward dynamic system which satisfies design conditions for two different assumptions about the prior knowledge of the high-frequency matrix K_p: the symmetry assumption of [15], and the assumption on the signs of the leading principal minors of K_p [16], respectively. The proposed adaptive control law constructions make economical use of known results of MIMO model reference adaptive control to the considered class of delayed system. Adaptive laws are developed using the SPR-Lyapunov design approach. This work is the first asymptotic exact zero tracking results for this class of systems in the framework of the certainty equivalence approach.

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REFERENCES

[1] K. Gu and S.-I. Niculescu, "Survey on recent results in the stability and control of time-delay systems," *Journal of Dynamic Systems, Measurement, and Control*, vol. 125, no. 2, pp. 158–165, 2003, (Special Issue: Time Delayed Systems).

[2] M. S. Mahmoud, "Adaptive control of a class of time-delay systems with uncertain parameters," *International Journal of Control*, vol. 63, no. 5, pp. 937–950, 1996.

[3] S. G. Foda and M. S. Mahmoud, "Adaptive stabilization of delay diferential systems with unknown uncertain bounds," *International Journal of Control*, vol. 71, no. 2, pp. 259–275, 1998.

[4] S. Oucheriah, "Adaptive robust control of class of dynamic delay systems with unknown uncertainty bounds," *International journal of adaptive control and signal processing*, vol. 15, pp. 53–63, 2001.

[5] E. P. Ryan and C. J. Sangwin, "Controlled functional differential equations and adaptive stabilization," *International Journal of Control*, vol. 74, no. 1, pp. 77–90, 2001.

[6] A. Ilchman and E. P. Ryan, "On gain adaptation in adaptive control," *IEEE Transactions on Automatic Control*, vol. 48, no. 5, pp. 895–899, 2003.

[7] E. P. Ryan and C. J. Sangwin, "Controlled functional diferential equations and adaptive tracking," *Systems & Control Letters*, vol. 47, pp. 365–374, 2002.

[8] F. Blanchini and E. P. Ryan, "A Razumikhin-type lemma for functional differential equations with application to adaptive control," *Automatica*, vol. 35, pp. 809–818, 1999.

[9] A. M. Tsykunov, *Adaptive control of the delayed plants*, Nauka, Moscow, 1984 (in Russian).

[10] B. M. Mirkin, "Combined direct and indirect adaptive control for plants with state time delay," *Vestnik MUK*, vol. 13, no. 1, pp. 45–48, 2001, (in Russian).

[11] B. M. Mirkin and P. O. Gutman, "Output adaptive model reference control of linear continuous state-delay plant," in *Proceedings of 15th International Symposium on the Mathematical Theory of Networks and Systems, August 12-16.*, Notre Dame, USA, 2002.

[12] B. M. Mirkin and P. O. Gutman, "Output-feedback model reference adaptive control for continuous state delay systems," *Journal of Dynamic Systems, Measurement, and Control*, vol. 125, no. 2, pp. 257–261, 2003, (Special Issue: Time Delayed Systems).

[13] S. Sastry and M. Bodson, *Adaptive control. Stability, Convergence, and Robustness*, Prentice-Hall, Englewood Cliffs, New Jersey, 1989.

[14] K. S. Narendra and A. M. Annaswamy, *Stable Adaptive Systems*, Prentice-Hall, New York, 1989.

[15] P. A. Ioannou and J. Sun, *Robust Adaptive Control*, Prentice-Hall, New Jersey, 1996.

[16] R. R. Costa, L. Hsu, A. K. Imai, and P. Kokotović, "Lapunov-based adaptive control of MIMO systems," *Automatica*, vol. 39, no. 7, pp. 1251–1257, 2003.

[17] J. K. Hale and S. M. V. Lunel, *Introduction to Functional Differential Equations*, Springer-Verlag, New York, 1993.

[18] A. S. Morse, "A gain matrix decomposition and some ot its applications," *Systems and Contr. Letters*, vol. 21, pp. 1–10, 1993.