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Intuitive Representation of Gain Schedulers To Facilitate Their Design and Tuning

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***Abstract*—A method of intuitive continuous gain scheduler design has been developed with a visual representation of the schedulers. Inspection of the unscheduled response allows for development of an initial gain scheduler. Comparison between the gain scheduler plot and the resulting system response enhances insight into the closed-loop system behavior, and one can adjust the gain scheduler to modify the system response. Several iterations of each gain scheduler design are given, along with the resulting system responses to a step command. Using this method, gain schedulers were developed for the precision control of a linear motor with distinct preexisting controllers in two dissimilar operational environments. In each operational environment, the addition of gain scheduling substantially improved the closed-loop transient response. In the first case, the dead-band region was reduced by 90%. In the second, the rise time of the system was reduced by a factor of 7~14. The successful application of this design methodology to these distinct operational environments suggests that it would also be applicable to the development of gain schedulers for many other precision-motion-control applications where dead band is prevalent in the system response.**

I. INTRODUCTION

There have been many contributions to gain scheduling, with renewed interest in recent years. Rugh and Shamma [1] gave a survey of gain scheduling in which they summarize the current state of the art, and give classifications and examples. Gain scheduling was used in the control of motors [2–4] and positioning applications [5–6]. A common theme in recent work is the pursuit of more analytical approaches to gain scheduling [7–13] as opposed to ad-hoc methods of the past. However, the work presented in this paper is more empirical in nature.

Gain scheduling is commonly used in designing controllers for time-varying and/or nonlinear systems [7, 9, 14]. For nonlinear systems, gain scheduling is usually implemented by designing controllers for discrete operating points along the system operation range. A switching or interpolation scheme is implemented to select the controller behavior between these points [15–17]. In contrast, the method described in this paper uses only a single preexisting controller. Scheduling is incorporated by a continuous function, which alters the feedback error to the controller.

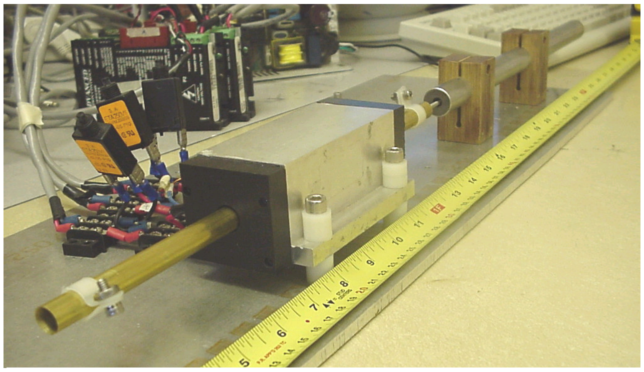

Fig. 1: Linear motor (front) connected to position sensor (back).

This paper introduces this scheduling method, as well as a novel visual representation of the scheduler, facilitating intuitive design and tuning. This development methodology is applied to design two gain schedulers for implementation to two distinct operational environments. In each case, a pre-existing lead-lag controller we designed is used to drive a linear motor, shown in Fig. 1 [18].

The first operational environment has significant noise disturbance present, and the primary purpose of the control system is to achieve a fast response as required for many positioning applications. The second environment has less disturbance as the motor is placed atop a vibration isolation table. The primary purpose of the second controller scheme is to have little or no overshoot to step responses. The performance requirement is for minimal overshoot as in robotics applications, where overshoot generated with a long-range motion command can be very dangerous. Demonstrating the feasibility of the gain schedulers developed for these distinct applications validates the methodology for other design environments. The following sections detail the development and evaluation of the gain schedulers used in this paper.

In the following section, the experimental setup is

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discussed wherein the linear motor is operated in these two distinct operational environments. Also, the preexisting controller for the first environment is discussed. Section III details the gain scheduler design methodology and incorporation of the scheduler into the control loop. The novel visual representation of the gain scheduler and its advantages are also discussed. In Section IV, experimental results are given demonstrating the effect of incorporating gain schedulers for the two operational environments. The paper is concluded with a summary of the work.

II. EXPERIMENTAL SETUP

The linear motor used for this experimentation, given in Fig. 1, was designed and constructed by the authors [18]. The motor is comprised of nine cylindrical current-carrying coils arranged in three-phase flow. A brass tube free to slide through the coils contains permanent magnets fixed in an array corresponding to the pitch of the coils. When the coils are powered, they exert a Lorentz force against the magnets in the brass tube, causing translation. The plant is modeled as a simple mass, without damping or a spring constant. With a mass of 175 grams, the plant transfer functions is

$$\frac{Y(s)}{f(s)} = \frac{1}{0.175s^2}. \tag{1}$$

Position feedback is provided by an LVDT (linear variable differential transformer), which outputs an analog signal. This signal is filtered and then sampled by an A/D channel of a controller board installed in a computer. The computer then outputs commands to three PWM (pulse-width modulation) amplifiers, which power the coils. The system controller is implemented using Matlab/Simulink. Refer to [18] for the detailed motor design and its instrumentation structure.

Before implementing gain scheduling, a controller was designed for each of the two operational environments discussed in Section I. The controller transfer function for the first operational environment with a 5-kHz sampling rate was

$$1.7\times10^5\,\frac{(z-0.996)(z-0.9608)}{(z-1)(z-0.67032)}. \tag{2}$$

The control bandwidth was 40 Hz, with a phase margin of 73.6°.

III. GAIN SCHEDULER DEVELOPMENT

A. Design Methodology

In order to facilitate intuitive creation and tuning of gain schedulers, a visual representation has been developed in this paper. In this way, the gain scheduler can be compared with the response to determine what changes to the scheduling function could be made to improve results. The method discussed in this paper schedules the gain continuously through the use of a look-up table included in the forward loop of the controller. Discrete points are assigned amplification values in the table, and linear-interpolation provides for values between the defined points.

The gain scheduler is implemented into the control loop as given in Fig. 2. The input to the table is the error value, which is the summation of the desired position and the negative feedback from the position sensor. Based on the error value, the gain scheduler outputs a scaling factor (from 1 to 10) which is multiplied by the original error value and input to the lead-lag controller. This modified value can be conceptualized as either an amplified controller gain or magnified position error. In either case the result is to boost the absolute value of the output of the controller when within a defined range of position error. A desired scaling output as a function of error value can be achieved by appropriate selection of values of the table.

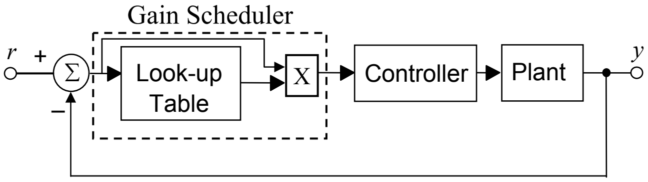


Fig. 2: System control loop with gain scheduler implemented.

The general shape of the gain scheduler was determined by investigating unscheduled system responses to various step inputs. Fig. 3 gives the response of the linear motor shown in Fig. 1 to an input step command of minus 5 mm. The spikes given in the plot are not actual position displacements, but electrical noise picked up by the analog position sensor.

From the step response in Fig. 3 it is clear that there is a substantial dead-band region present, where the system has moved to a point near the desired point, but it takes significant time to move again to reach the desired position. This delay is caused by the integrator windup in our controller (2) due to friction between the motor shaft and bearings. To mitigate this delay in response, the controller output must be increased faster, to cause the current to the motor coils to increase more quickly. The controller gain cannot simply be kept increased increasing because this would cause the system to compromise the stability and damped response for which it was designed. Gain scheduling provides a sufficient solution, whereby the gain of the system can be increased by some factor when the system is within a predefined range around the desired position, allowing the system to overcome the dead band.

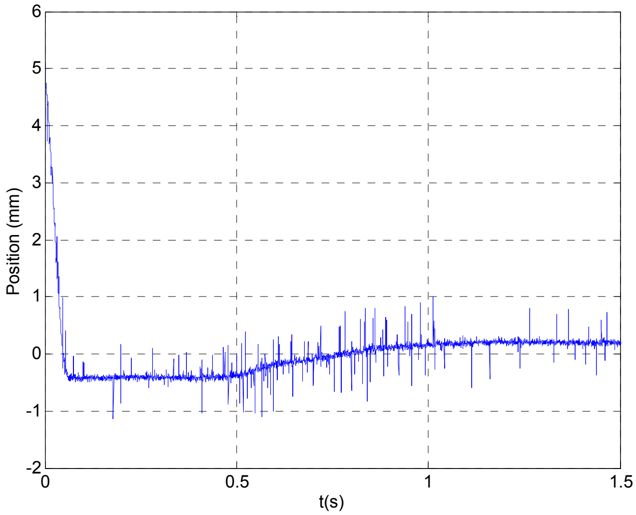


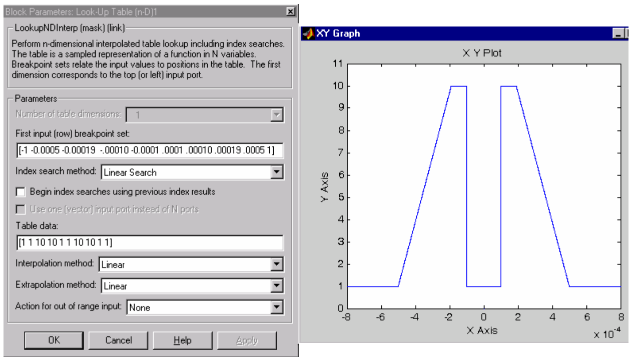
Fig. 3: Linear motor response to –5 mm input command with controller (2) without gain scheduling.

When selecting values for the scaling factor and range of influence, there were several considerations that were addressed. For each scheduler, an unaltered position region was allowed to remain when the error was very small, the magnitude of which corresponds to the amplitude of the positioning noise present. Amplifying the controller gain in this region would exacerbate the noise and would not significantly improve the positioning speed.

From the step response, an idea of where (with respect to position error) static friction begins to dominate the positioning force can be determined. This is the beginning of the dead-band region. No matter how small the step size, the friction-induced pause in translation consistently appeared when the position error was within a certain range of zero error, so a general relationship can be made. The point of application of the gain scheduler is selected to begin when the error is just larger than this range, so that the actuator will not pause in its translation to the desired position. Based on the repeated step response experiments, the gain scheduler must take some action when the system error is less than 0.5 mm, because this is where the first pause in translation occurs. The magnitude of the amplification factor requires compromising between the amount of mitigation of the dead-band region and maintaining system stability and minimal noise. The amplification was limited to a factor of 20, which was found to be sufficient to eliminate the dead-band region without compromising system stability or significantly increasing overshoot, even with added external load [18].

B. Visual Representation of Gain Scheduler

A visual representation of the gain scheduler behavior with respect to the input variable facilitates better intuitive design. To that end, a gain scheduler is represented with a continuous function that signifies the relation between actual position error and modified position error, which corresponds to input and output of the gain scheduler. Presenting the scheduler in this manner allows clear perception of the behavior of the gain scheduler with respect to position error. Upon testing, one can compare the new step response with the gain scheduler, and see what modification to the scheduler needs to be made to further improve performance. Some distinct scheduling schemes can be implemented, and the results evaluated to determine which gain scheduler or combination of schedulers give the best result. In each case, the schedulers are skew-symmetric about the zero error point, as the sign of the error is inconsequential to the gain scheduler for this motor because the dead band is identical regardless of step direction. Fig. 4 depicts a screen capture of the look-up table entries and corresponding plot. Note that in this plot the x-axis is error and the y-axis is the amplification factor.


Block Parameters: Look-Up Table (n-D)1

LookupNDInterp (mask) (link)

Perform n-dimensional interpolated table lookup including index searches. The table is a sampled representation of a function in N variables. Breakpoint sets relate the input values to positions in the table. The first dimension corresponds to the top (or left) input port.

Parameters

Number of table dimensions: 1

First input (row) breakpoint set: [-1 -0.0005 -0.00019 -.00010 -0.0001 .0001 .00010 .00019 .0005 1]

Index search method: Linear Search

☐ Begin index searches using previous index results

☐ Use one (vector) input port instead of N ports

Table data: [1 1 10 10 1 1 10 10 1 1]

Interpolation method: Linear

Extrapolation method: Linear

Action for out of range input: None

Fig. 4: Look-up table and corresponding plot, where the x-axis is error and y-axis is amplification factor.

For the system discussed in this paper, several different gain schedulers were implemented and tuned to find one yielding the best result for each of the two operational environments discussed in Section I; one with a fast controller, the other with a controller designed for minimal overshoot. For the fast controller operational environment, a sample of five gain schedulers is portrayed in Fig. 5, which were designed based on the unscheduled system response in Fig. 3. In each case, the actual position error value (the input to the gain scheduler) is given as the horizontal axis, and the modified position error (the output from the gain scheduler to the controller) is the vertical axis. For the case of no action by the gain scheduler, the plot of actual position vs. modified position is a straight line with unity slope, as given in Fig. 5(a). Deviations from this line indicate where the scheduler is taking action. These schedulers and resulting step responses are discussed in the following section.

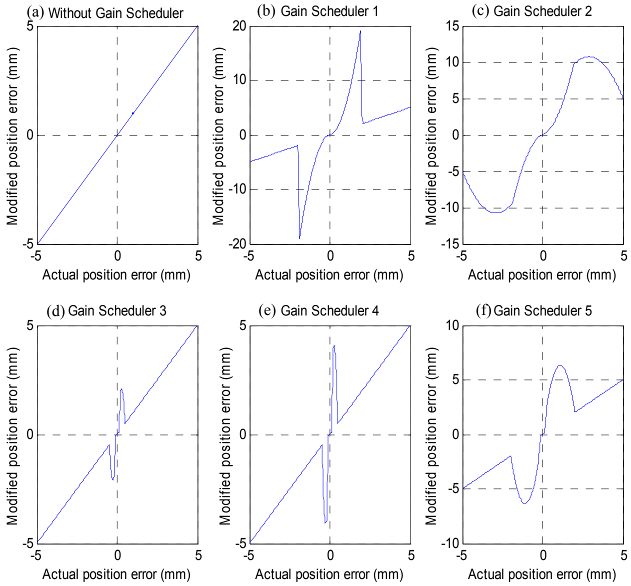


Fig. 5: Multiple gain schedulers are plotted with output vs. input. The gain is unity outside the shown regions. Different scales are used for the vertical axis for clarity.

IV. EXPERIMENTAL RESULTS AND GAIN SCHEDULER MODIFICATIONS

A. Controller for Fast Response

To test the effect of gain schedulers, they were incorporated into the control loop and the system response was investigated. Fig. 6 gives the corresponding system step responses with the gain schedulers from Fig. 5 in place. The apparent high-amplitude noise present is due to mechanical vibration present in the laboratory due to other equipment in the building and electrical noise picked up by the LVDT position sensor.

With no gain scheduler in place, the rise time is less than 100 ms, but the system would not correct the initial overshoot to move toward the desired position for another 400 ms due to the dead band. The output magnification of gain scheduler 1 increases sharply when the position error is at 2 mm. It then tapers back down to unity when near zero error. The resulting step response has a similar rise time to the unscheduled response, but takes action to correct the position error more quickly because the scheduler modifies the position error, causing the integrator to accumulate it more quickly. This response has error amplitude of about 400 μm as it oscillates about the desired position. To mitigate this oscillation amplitude, more magnification is required in this error region. There is also more noise present in this response than in the unscheduled response because of the large amplification of the error signal by this gain scheduler.

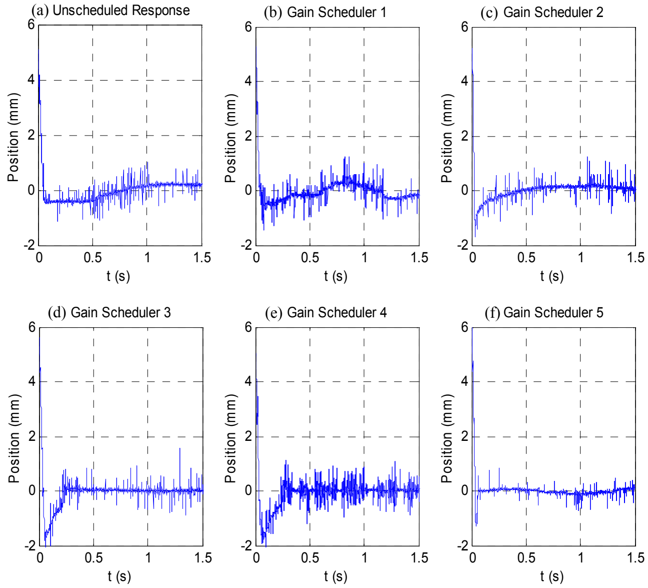


Fig. 6: System responses to minus 5-mm input commands with gain schedulers from Fig. 3 incorporated into controller.

In gain scheduler 2, the maximum amplitude of the gain scheduler magnification was reduced to about 11 in order to reduce the amplitude of the noise. The range of application was also broadened, and the location of peak magnification was shifted to a position error of 3 to 4 mm. The rise time of the corresponding system step response shown in Fig. 6 (c), with this scheduler in place was faster, had less noise, and was also smoother. However the result still has significant fluctuation about the desired position. Gain schedulers 3 and 4 have similar form, with the point of peak magnification at 0.5 mm and magnification exclusively within 1 mm of error. The maximum magnification factor for scheduler 4 is about twice that of scheduler 3. The step responses (Fig. 6 (d) and (e)) also have similar form, with the response with scheduler 4 contributing significantly more noise to the response, because it has twice the amplification of gain scheduler 3.

Evaluation of the system response with the implementation of each of the four schedulers gives diverse perspective of how the step response is affected by scheduling. Gain scheduler 5 was designed as a combination of schedulers 2 and 3, with a wider application range and smoother curve throughout that range. The part of the curve near the zero point was left less modified, so as to prevent adding noise to the system. The drawback of this design choice is that the position response is a little slower when the error is within this region near the zero point. With the scheduler in this form, the noise was reduced and the transient-response times were improved as clearly shown in Fig. 6 (f). However, the overshoot was increased by a factor of three. This cost is acceptable considering the purpose of the controller scheme is fast response without amplifying noise.

B. Controller for Minimal Overshoot

In this operational environment, the motor was moved to a vibration isolation table to reduce mechanical disturbance, and the shaft bearings were modified to reduce friction. The gain scheduler design method was also implemented to develop a scheduler used with the second controller for the same linear motor. Multiple gain schedulers were developed according to the aforementioned methodology, and were each incorporated into the control loop. Fig. 7 gives the system response to a larger 20-mm step with different gain schedulers incorporated, and also for the unscheduled response. For the unscheduled response, given as the dash-dot right-most line, the rise time is over 7 s. For no-overshoot applications, where the position cannot cross the desired point, implementing gain scheduling reduced the rise time to about 1 s, as given by the critically damped solid line (the second line from the left). For applications where a few percent of overshoot are permissible, a different gain scheduler reduced the rise time to 0.5 s, given as the left-most line.

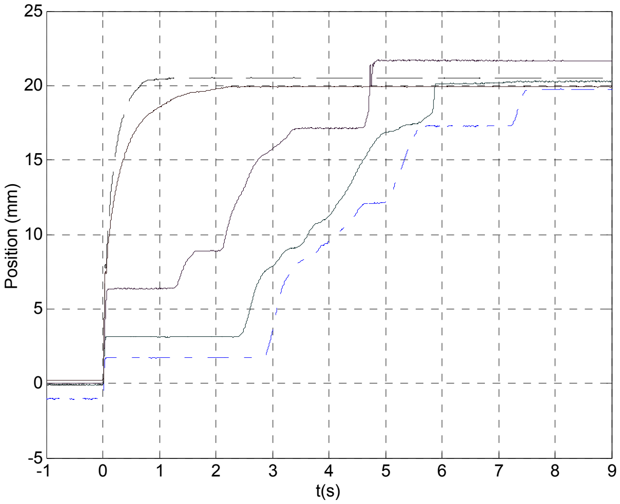


Fig. 7: System responses to a long-range step command with various gain schedulers in place.

V. CONCLUSION

A method of continuous gain scheduler design has been introduced with a visual representation of the scheduler. The position error input to the controller is modified by the gain scheduler, which alters the system performance. The gain scheduler is a continuous function implemented as a linearly-interpolating lookup table. The visual representation of the easily-modified lookup table allows for intuitive gain scheduler design, and discernment of necessary modifications based on the scheduled step response and desired transient response characteristics.

This method was used to design gain schedulers which were implemented into a control loop for the position control of a linear motor beset with friction-induced dead band, in two distinct operational environments, with dissimilar desired performance. In the first case, the purpose of the controller was to achieve fast positioning. Several iterations of the gain scheduler design were given, along with the resulting system response to a step command. The final gain scheduler in the first case reduced the dead band present by over 90%.

For the second case, a gain scheduler was developed for implementation with a controller designed for minimal overshoot applications. In this case, for 5% overshoot or no overshoot, the rise times were 1 s and 0.5 s, respectively, to a 20-mm step command. This was a significant reduction in transient response time considering a 7-s rise time before any gain scheduling. For each case, the addition of gain scheduling substantially improved the system response.

The visual representation used to develop appropriate gain schedulers allowed intuitive design and modification based on the closed-loop system response. The successful application of this design methodology to the two distinct cases suggests that it would also be applicable to the development of gain schedulers for numerous other systems, particularly where undesirable dead band is prevalent in the system response.

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