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Adaptive Fuzzy Control of MMA Batch Polymerization Reactor Based on Fuzzy Trajectory Definition

Reza Solgi, Rasoul Vosough and Mehdi Rafizadeh

Abstract— **In this research an adaptive fuzzy controller was applied to temperature control of a MMA batch polymerization reactor which use jacket temperature error in additional reactor temperature error. But the desired jacket temperature is affected by noise and disturbance. Therefore there is uncertainty in desired value of this variable. Fuzzy numbers are applied to model this uncertainty and a fuzzy trajectory was achieved for jacket desired temperature. Then a special case of fuzzy controller called *Generalized Takagi-Sugeno-Kang fuzzy controller*, and an adaptation mechanism was designed. Experimental results present the fine performance of this controller in temperature control of solution polymerization of methyl methacrylate.**

Index Terms— **Adaptive fuzzy control, fuzzy trajectory, generalized Takagi-Sugeno-Kang controller, polymerization reactor control, methyl methacrylate.**

I. INTRODUCTION

IN the recent years, the importance of effective polymer reactor control has been studied. Polymerization kinetic is usually complex due to the nonlinearity of the process. Therefore, the control of the polymerization reactor has been staying a challenging task. Due to its great flexibility, a batch reactor is suitable to produce small amounts of special polymers and copolymers. The batch reactor is always dynamic by its nature. It is essential to have a suitable dynamic model of the process. Rafizadeh [1] presented a review on the proposed models and suggested an on-line estimation of some parameters. His model consists of the oil bath, electrical heaters, cooling water coil, and reactor. Peterson *et al* [2] presented a non-linear predictive strategy for semi batch polymerization of MMA. Soroush and Kravaris [3] applied a Global Linearization Control (GLC) method to control the reactor temperature. Performance of the GLC for tracking an optimum temperature trajectory was found to be suitable. DeSouza jr. *et al* [4] studied an expert neural network as an internal model in control of solution polymerization of vinyl estate. In their study, they compared their neural network control with a classic PID controller. Clarke-Pringle and MacGregor [5] studied the temperature control of a semi-batch industrial reactor. They suggested a coupled non-linear strategy and extended Kalman filter method. Mutha *et al* [6] suggested a non-linear model based control strategy, which includes a new estimator as well as Kalman filter. They conducted experiments in a small reactor for solution polymerization of MMA. Rho *et al* [7] assumed a first order model plus dead time to pursue the control studies and estimated the parameters of this model by on line ARMAX model. Rafizadeh [8] designed a sequential linearization adaptive controller for the solution polymerization of methyl methacrylate in a Batch Reactor.

After the innovation of fuzzy sets and fuzzy logic by Zadeh [8] and its successful implementation in boiler-control by Mamdani [9] , fuzzy control became a significant approach in the area of control engineering. In the recent decade, numerous applications of fuzzy theory have been reported in the field of chemical processes control [10].

Abony *et al.* [11] designed an adaptive fuzzy Sugeno controller by the COEM adaptive mechanism [12] to control the polymerization process and have shown this controller has better efficiency than the conventional PID controllers.

Asua used a fuzzy system to assign the optimal conditions of process and a model based controller to control the emulsion polymerization reactor in a simulation.

In this paper a new idea, *Fuzzy Trajectory Tracking* has been propounded. To track this fuzzy trajectory a controller called *Generalized Takagi-Sugeno-Kang (TSK) Fuzzy Controller* has been developed. Then to adapt this fuzzy trajectory during the course of polymerization, an adaptation mechanism has been designed. Finally the efficiency of this novel controller has been presented to track a predetermined reactor temperature trajectory.

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II. Experimental Setup

A schematic representation of the experimental batch reactor setup is shown in Fig. 1. The reactor is a Buchi type jacketed, cylindrical glass vessel. A multi-paddle agitator mixes the content. A PC is connected to the reactor via an ADVANTECH PCL-818L data acquisition card. The data acquisition software is developed in-house. The heating oil is circulated by a gear pump and its flow rate is about $2.5\times10^{-4}\ m^3 s^{-1}$. The heating-cooling system of the oil consisted of two 1500W electrical heaters and a coolant water coil. Two Resistance Temperature Detectors (RTD), were used with accuracy of $\pm 0.2\ ^oC$.

Methyl methacrylate and toluene were used as monomer and solvent, respectively. Benzoyl Peroxide (BPO) was used as the initiator.

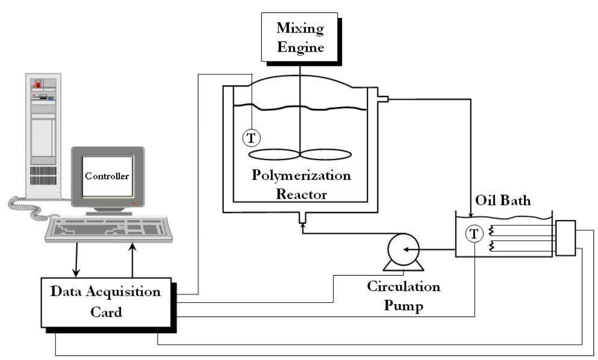

Fig. 1. Experimental setup

III. Fuzzy Trajectory

In this research a fuzzy controller is applied to control the temperature of the polymerization reactor. To avoid instability, in addition to reactor temperature error, fuzzy controller should use jacket temperature error too. As the desired temperature of the reactor is predetermined the reactor temperature error is defined as:

$$E_R = T_{d_R} - T_R \tag{1}$$

Therefore the reactor temperature error is a crisp number. On the other hand, to determine the jacket temperature error, its desired temperature must be known. Indeed desired temperature of jacket is a function of process dynamics and environmental disturbances, which make some sort of uncertainty in its value. The idea of fuzzy trajectory for jacket temperature is applied to determine the desired jacket temperature as a fuzzy number, illustrated in Fig. 2.

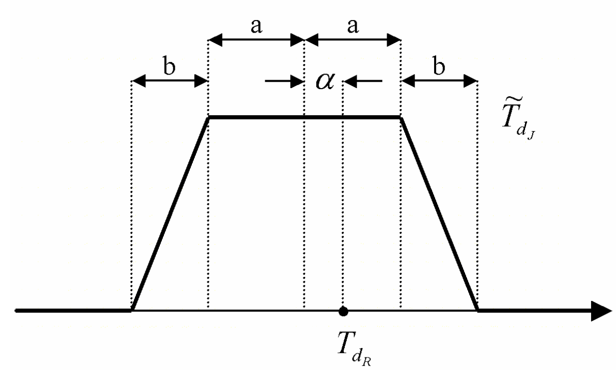

Fig. 2. Desired jacket temperature as a fuzzy number

In Fig. 2 a and b are parameters which represent uncertainty and α is the parameter determined by the environmental disturbances and exothermic rate of process.

By means of this definition, the desired jacket temperature could be defined as a fuzzy number in each sampling time. Since the jacket temperature is a crisp number, it should be first fuzzified to calculate the jacket temperature error. In this research singleton fuzzification is applied for this purpose:

$$\mu_{\widetilde{T}_J}(T) = \begin{cases} 1 & T = T_J \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Then to calculate the jacket temperature error a fuzzy subtraction should be used:

$$\widetilde{E}_J = \widetilde{T}_{d_J} - \widetilde{T}_J \tag{3}$$

There are two main methods to subtract two fuzzy numbers:

1. To utilize extension principle
2. α -cuts and intervals arithmetic.

If $\widetilde{A}$ and $\widetilde{B}$ are two fuzzy numbers, according to former, $\widetilde{A}-\widetilde{B}$ fuzzy number is defined as:

$$\mu_{\widetilde{A}-\widetilde{B}}(z) = sup\left\{ T(\mu_{\widetilde{A}}(x), \mu_{\widetilde{B}}(y)) \,\middle|\, z = x - y \right\} \tag{4}$$

where T is a T-Norm. In the latter, $^{\alpha}(\widetilde{A}-\widetilde{B})$ is defined as:

$$^{\alpha}(\widetilde{A}-\widetilde{B}) = {}^{\alpha}\widetilde{A} - {}^{\alpha}\widetilde{B} \tag{5}$$

where the difference between $[x, y]$ and $[x', y']$ is defined as:

$$[x, y] - [x', y'] = [x - x', y - y'] \tag{6}$$

It could be shown simply, that if singleton fuzzification of jacket temperature is applied, these two approaches lead to same results.

IV. Generalized TSK Fuzzy Controller

In this research E_R and $\tilde{E}_J$ are inputs of fuzzy controller. Rulebase of the proposed controller includes rules with the general form:

$$IF \;\; E_R \;\; is \;\; \tilde{A}^{(i)} \;\; AND \;\; \tilde{E}_J \;\; is \;\; \tilde{B}^{(i)} \;\; THEN \;\; u = c^{(i)} \tag{7}$$

Then the controller output is:

$$u = \frac{\sum_{i=1}^{m} w^{(i)} c^{(i)}}{\sum_{i=1}^{m} w^{(i)}} \tag{8}$$

where

$$\begin{aligned} w^{(i)} &= tv\left[\left(E_R \;\; is \;\; \tilde{A}^{(i)}\right) AND \left(\tilde{E}_J \;\; is \;\; \tilde{B}^{(i)}\right)\right] \\ &= T\left[tv\left(E_R \;\; is \;\; \tilde{A}^{(i)}\right), tv\left(\tilde{E}_J \;\; is \;\; \tilde{B}^{(j)}\right)\right] \end{aligned} \tag{9}$$

and $tv(\bullet)$ is the truth value operator. Therefore a method is needed, to determine truth value of linguistic terms e.g. $E_R \;\; is \;\; \tilde{A}$ and $\tilde{E}_J \;\; is \;\; \tilde{B}$.

The reactor temperature error E_R, is a crisp number, hence truth value of $E_R \;\; is \;\; \tilde{A}$ is equal to $\mu_{\tilde{A}}(E_R)$. On the other hand the jacket temperature error is a fuzzy number and the conventional methods could not be applied to determine the truth value of $\tilde{E}_J \;\; is \;\; \tilde{B}$.

Definition 1 - The fuzzy Set $\tilde{X}$ on $\Re$ is a *generalized fuzzy number*, *iff*: 1- $Core(\tilde{X}) \neq \varnothing$ and 2- For any α, ${}^{\alpha}\tilde{X}$ is a closed interval.

The advantage of this definition over the classic definition of fuzzy number [15], is inclusion of Gaussian fuzzification of crisp numbers.

Definition 2 - The fuzzy set $\tilde{A}$ on $\Re$ is a *fuzzy pseudo-number iff*: 1- $Core(\tilde{A}) \neq \varnothing$ and 2 - For any $0 < \alpha \le 1$, ${}^{\alpha}\tilde{A}$ is a closed interval or for any $0 < \alpha \le 1$, ${}^{\alpha}\tilde{A}$ is a semi-opened interval.

So fuzzy sets such as those is shown in Fig. 3, which are used as the linguistic variables in fuzzy rulebases, are fuzzy pseudo-numbers. We should determine the truth value of linguistic terms such as, $\tilde{X} \;\; is \;\; \tilde{A}$, in a generalized TSK fuzzy controller. In this research it is suggested that the truth value of $\tilde{X} \;\; is \;\; \tilde{A}$ be interpreted as $\tilde{X}$ *is approximately equal to* $\tilde{A}$. Therefore it's truth value is defined as follows:

$$\begin{aligned} tv(\tilde{X} \;\; is \;\; \tilde{A}) &= tv(\tilde{X} \approx \tilde{A}) \\ &= T\left(tv(\tilde{X} \le \tilde{A}), tv(\tilde{X} \ge \tilde{A})\right) \end{aligned} \tag{10}$$

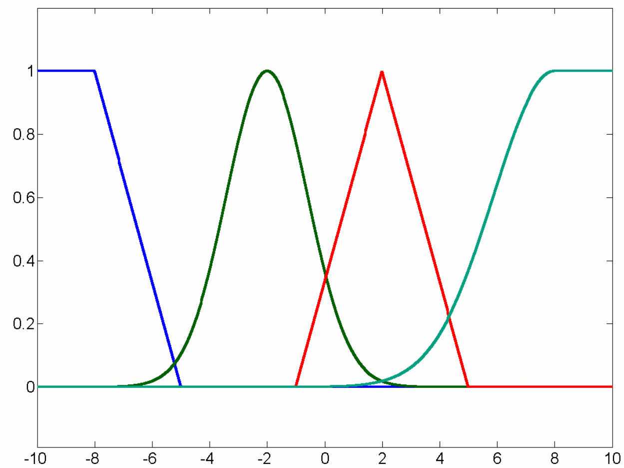

Fig. 3. Fuzzy pseudo-numbers

To evaluate the truth value of $\tilde{X} \ge \tilde{A}$ and $\tilde{X} \le \tilde{A}$, the following definitions[16] could be used alternatively :

$$tv(\tilde{X} \le \tilde{A}) = sup\left\{T\left(\mu_{\tilde{X}}(x), \mu_{\tilde{A}}(a)\right) \middle| x \le a\right\} \tag{11}$$

$$tv(\tilde{X} \ge \tilde{A}) = sup\left\{T\left(\mu_{\tilde{X}}(x), \mu_{\tilde{A}}(a)\right) \middle| x \ge a\right\} \tag{12}$$

By substituting (11) and (12) in (10) we have:

$$\begin{aligned} tv(\tilde{X} \;\; is \;\; \tilde{A}) = T\Big(& sup\left\{T\left(\mu_{\tilde{X}}(x), \mu_{\tilde{A}}(a)\right) \middle| x \le a\right\} \\ & , sup\left\{T\left(\mu_{\tilde{X}}(x), \mu_{\tilde{A}}(a)\right) \middle| x \ge a\right\}\Big) \end{aligned} \tag{13}$$

Theorem - If $\tilde{X}$ is a generalized fuzzy number and $\tilde{A}$ is a fuzzy pseudo-number, according to definition (13), the truth value of $\tilde{X} \;\; is \;\; \tilde{A}$ is equal to height of $\tilde{X} \cap \tilde{A}$. In the other words,

$$tv(\tilde{X} \;\; is \;\; \tilde{A}) = hgt\left(\tilde{X} \cap \tilde{A}\right) \tag{14}$$

Proposition- If $\widetilde{X}$ is a singleton fuzzy number,

$$\mu_{\widetilde{X}}(x) = \begin{cases} 1 & x = x^* \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

and $\widetilde{A}$ is a fuzzy pseudo-number, then

$$tv(\widetilde{X} \;\; is \;\; \widetilde{A}) = \mu_{\widetilde{A}}(x^*) \tag{16}$$

This proposition shows that if the inputs of generalized TSK fuzzy controller are singleton fuzzy sets, then the generalized TSK fuzzy controller will reduced to a TSK fuzzy controller.

Finally, according to the theorem mentioned above, antecedent satisfaction of the rules in generalized TSK fuzzy controller could be evaluated in this manner:

$$\begin{aligned} w^{(j)} &= T\left[tv\left(E_R \;\; is \;\; \widetilde{A}^{(j)}\right), tv\left(\widetilde{E}_J \;\; is \;\; \widetilde{B}^{(j)}\right)\right] \\ &= T\left[\mu_{\widetilde{A}^{(j)}}(E_R), hgt\left(\widetilde{E}_J \cap \widetilde{B}^{(j)}\right)\right] \end{aligned} \tag{17}$$

V. Adaptation Mechanism

As mentioned before, α is a parameter, determined by the exothermic nature of the reaction and environmental disturbances.

Because of the exothermic rate of the polymerization reaction, it is necessary to tune α along with the process. To tune α, we use the weighted average of the error criteria $\bar{e}$, defined as:

$$\bar{e}(k) = \frac{\tau}{T} \sum_{i=0}^{\infty} \exp(-iT/\tau) \; E_R(k-i) \tag{18}$$

(T is the sampling time and τ is the time constant) as the input of the adaptation mechanism fuzzy rulebase:

$$IF \;\; \bar{e} \;\; is \;\; \widetilde{N} \;\; THEN \;\; \Delta\alpha = \varepsilon$$
$$IF \;\; \bar{e} \;\; is \;\; \widetilde{Z} \;\; THEN \;\; \Delta\alpha = 0$$
$$IF \;\; \bar{e} \;\; is \;\; \widetilde{P} \;\; THEN \;\; \Delta\alpha = -\varepsilon$$

The fuzzy sets $\widetilde{N}$, $\widetilde{Z}$ and $\widetilde{P}$ are defined in Fig .4

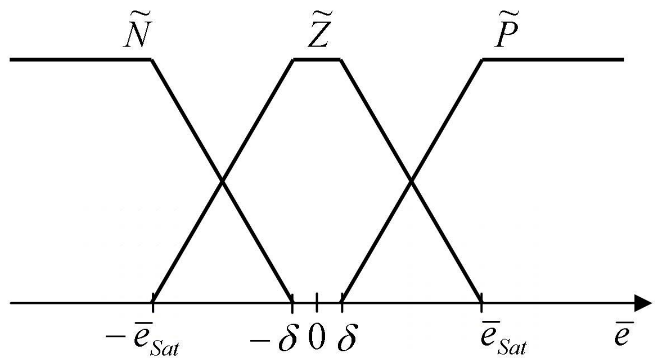

Fig. 4. Fuzzy Sets defined on $\bar{e}$

To avoid the violent oscillations, we should assign saturation limits for α. The block diagram of the system is shown in Fig .5.

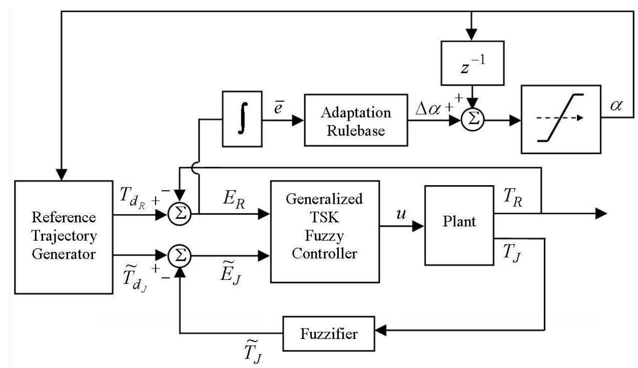

Fig. 5. Adaptive fuzzy controller block diagram

VI. Results and Discussions

In the experiment, sampling time is $T = 3_{Sec}$, and the parameters of desired jacket temperature fuzzy number(see Fig. 2) are:

$$a = 0^o C \;\;,\;\; b = 0.5^o C \;\;,\;\; \alpha_{Initial} = 0^o C$$

Also 3 and 5 rectangular fuzzy sets are defined on reactor temperature error and jacket temperature error respectively, as the linguistic variables (see Fig. 6 and Fig. 7). Therefore a rulebase with 15 rules was produced:

$$IF \;\; E_R \;\; is \;\; \widetilde{A}^{(i)} \;\; AND \;\; \widetilde{E}_J \;\; is \;\; \widetilde{B}^{(i)} \;\; THEN \;\; u = c^{(i)}$$

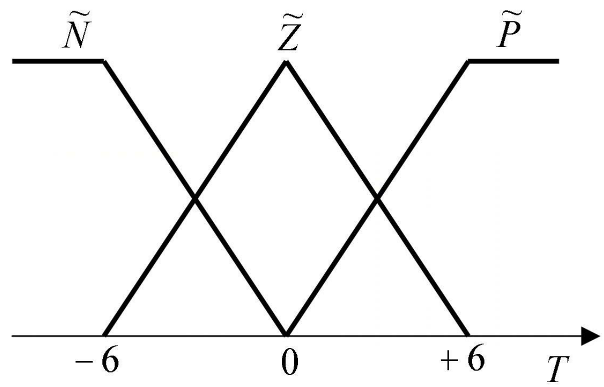

Fig. 7. Fuzzy Sets defined on jacket temperature error

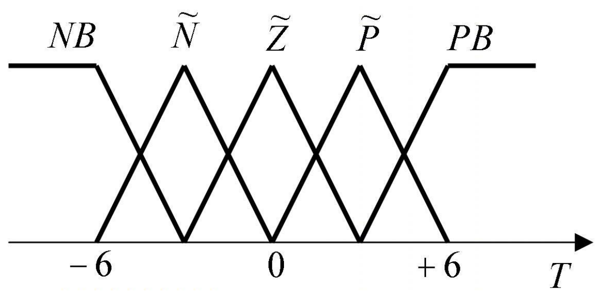

Fig. 6. Fuzzy Sets defined on reactor temperature error

Algebraic product is applied as T-Norm in the inference process of generalized TSK fuzzy controller, in the other words:

$$w^{(i)} = \mu_{\widetilde{A}^{(i)}}(E_R) hgt(\widetilde{E}_J \cap \widetilde{B}^{(i)}) \qquad (19)$$

The time constant in the $\overline{e}$ definition is $\tau = 10_{Sec}$, and 3 fuzzy sets were defined on $\overline{e}$ to form the adaptation rulebase, as shown in Fig. 4. In this definition $\overline{e}_{Sat} = 1^o C$ and $\delta = 0.05^o C$. Also ε parameter in the consequent part of latest rulebase was set as $0.0075^o C$.

Then the controller ability in the tracking of a predetermined reactor temperature trajectory was studied. The experimental conditions are listed in Table 1.

TABLE I
Experimental Conditions

Solvent	600^{cc}
Monomer(MMA)	200^{cc}
Initiator(BPO)	0.2 wt. percent of monomer

As shown in Fig. 8, a bell shaped trajectory is applied as the desired reactor temperature trajectory, and as the negligibility of MMA thermal initiation even in $100^o C$ [17], an hour preheating was added to this trajectory and the initiator was injected to reactant mixture in $t = 3600_{Sec}$. Mean absolute value of error is less than $0.13^o C$. It's an encouraging result, since it is about temperature sensor accuracy. Variations of α parameter of desired jacket temperature during the polymerization is shown in Fig. 9.

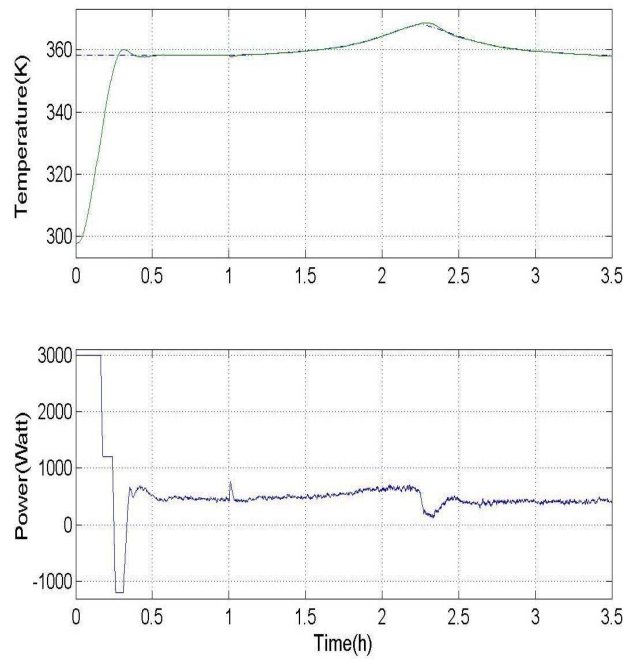

Fig. 8. Adaptive fuzzy controller ability in trajectory tracking

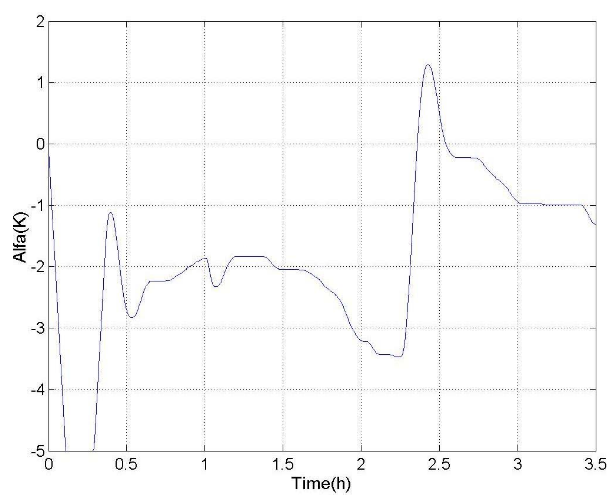

Fig. 9. Alfa parameter of desired jacket temperature fuzzy number during the polymerization

Also the defuzzified jacket temperature error vs. reactor temperature error is shown in Fig. 10.

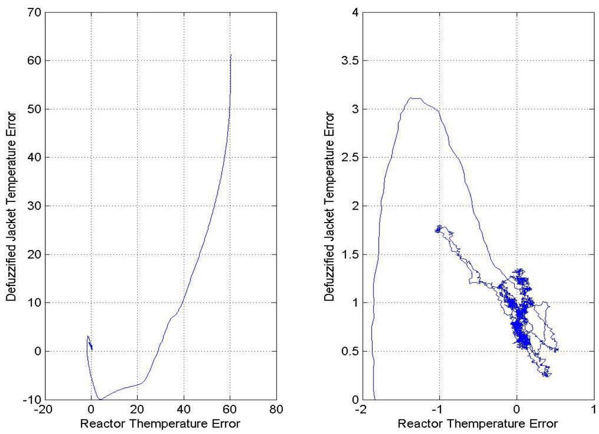

Fig. 9. Defuzzified jacket temperature error vs. reactor temperature error

VII. CONCLUSION

To control the MMA batch polymerization reactor an adaptive fuzzy controller was applied which use jacket temperature error in additional reactor temperature error. Fuzzy numbers are applied to model the uncertainty that exists in desired jacket temperature and a fuzzy trajectory was achieved for jacket desired temperature. Then a novel case of fuzzy controller called generalized TSK fuzzy controller and an adaptation mechanism was designed. Experimental results present the fine performance of this controller in temperature control of solution polymerization of methyl methacrylate.

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