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Control of Passive Systems using the Satis£cing Paradigm

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***Abstract*— This paper extends the application of the recently proposed satis£cing control strategy to passive systems. The satis£cing control is based on notions of satis£cing decision theory. In this framework a set of asymptotically stabilizing control laws can be parameterized in order to obey an instantaneous cost-bene£t inequality. Using the nonlinear version of the Kalman-Yacubovitch-Popov property this technique can be extended to passive systems. To illustrate the novelty of this approach, the paper considers the attitude control problem of a rigid body.**

I. Introduction

This paper develops a version of satis£cing control [1] for passive systems. The proposed approach has the advantage of obtaining a globally asymptotically stabilizing control law as an output feedback. The satis£cing approach can be seen as a formal application of cost-bene£t analysis to decision and control problems [2], [3], [4]. The approach is based on the Epistemic Utility Theory introduced in [5].

The basic idea is to de£ne two utility functions that quantify the bene£ts and costs of an action. At a state x, the bene£ts of choosing a control u are given by the "selectability" function $p_s(u,x)$. Similarly, the costs associated with choosing u are given by the "rejectability" function $p_r(u,x)$. The satis£cing set is constituted by the relationship $p_s \geq bp_r$: i.e, $S(x,b) = \{u : p_s(u,x) \geq bp_r(u,x)\}$ where the selectivity index b is a positive parameter.

The satis£cing control developed in [1] is a nonlinear state feedback which is based on Control Lyapunov Function (CLF). In general, £nding a CLF is an open problem. In this paper, a satis£cing control for passive systems is proposed. For this class of systems, using the nonlinear version of the Kalman-Yacubovitch-Popov lemma, the satis£cing set can be reformulated resulting in an output feedback which does not depend on CLFs.

This paper is organized as follows. In Section 2, we £x our notation and nomenclature and reviewing some of the basic de£nitions and concepts from passive systems theory. In Section 3, is developed the passive satis£cing control. In Section 4, the proposed controller is £rst applied to linear systems. Then the rigid body orientation control problem is considered. Global asymptotic stability under linear error quaternion feedback and nonlinear angular velocity feedback is shown. Simulation results are presented in the Section 5, where attitude control of a satellite model is used. Section 6 offers some conclusions.

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II. Passive Systems

In this paper, we consider nonlinear systems described by equations of the form

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^m$. The set $\mathcal{U}$ of admissible inputs consists of all piecewise continuous functions de£ned on $\mathbb{R}^m$. The vector £elds $f(x)$ and the m columns of $g(x)$ are smooth (i.e., C^∞) and $h(x)$ is a smooth mapping. We suppose that the vector £eld $f(\cdot)$ has at least one equilibrium; thus, without loss of generality, after possibly a coordinate shift, we can assume $f(0) = 0$ and $h(0) = 0$.

We review in this section a number of basic concepts related to the dissipativity and passivity notions (see [6], [7] for additional details).

Let $w(u,y)$ be a real-valued function, called the supply rate. We assume that for any control signal $u \in \mathcal{U}$ and for any initial condition x^0, for $t = 0$, the output $y(t) = h(\phi(t,x^0,u))$ of (1), where $\phi(t,x^0,u)$ is the system solution at time t, is such that $w(u(s),y(s))$ satis£es

$$\int_0^t |w(s)|ds < \infty \tag{2}$$

for all $t \geq 0$.

De£nition 1: A system (1) with supply rate w is said to be *dissipative if there exists a C^0 nonnegative function $V : \mathbb{R}^n \to \mathbb{R}$, called the storage function, such that for all $u \in \mathcal{U}$, initial condition x^0 and $t \geq 0$*

$$V(x) - V(x^0) \leq \int_0^t w(s)ds \tag{3}$$

where $x = \phi(t,x^0,u)$.

Through the paper, we shall be interested in studying dissipative systems with supply rate given by inner product, i.e.,

$$w = < u, y > = y^T u. \tag{4}$$

De£nition 2: The system (1) is passive if it is dissipative with supply rate $w = < u, y >$, and the storage function V satis£es $V(0) = 0$.

In other words, a system (1) is passive if there exists a C^0 nonnegative function $V : \mathbb{R}^n \to \mathbb{R}$, which satis£es $V(0) = 0$, such that

$$V(x) - V(x^0) \leq \int_0^t y^T(s)u(s)ds \tag{5}$$

We now present a fundamental property of passive systems which is a nonlinear enhancement of the Kalman-Yacubovitch-Popov (KYP) lemma for positive real linear systems [8].

De£nition 3: A system (1) has the KYP property if there exists a C^1 nonnegative function $V : \mathbb{R}^n \to \mathbb{R}$, with $V(0) = 0$, such that

$$V_x^T f \leq 0 \tag{6}$$
$$V_x^T g = h^T(x) \tag{7}$$

for each system state $x \in \mathbb{R}^n$.

Lemma 1: A system (1) which has the KYP property is passive, with storage function V. Conversely, a passive system having a C^1 storage function has the KYP property.

Lemma 1 will be useful to develop a satis£cing control version for passive systems.

III. Passive Satisficing Control

In this section we will extend to passive systems the control strategy based on the satis£cing paradigm presented in [1].

In [1], Stirling's satis£cing decision approach (see [4]) was used to derive a new family of universal formulas based on Control Lyapunov Functions (CLFs). In this approach the selectivity is de£ned as:

$$p_s(u, x) = -V_x^T(f + gu)$$

where V is a CLF for the system (1). A rejectability function is de£ned by

$$p_r(u, x) = l(x) + u^T Ru$$

where $R = R^T \geq 0$ and $l(x)$ is a locally Lipschitz nonnegative function. The satis£cing set is therefore de£ned as:

$$S(x, b) = \{u \in \mathbb{R}^m : -V_x^T(f + gu) \geq b(l(x) + u^T Ru)\}$$

In the case of passive systems we are able to de£ne the rejectability function in term of $l(y)$ instead of $l(x)$. Since, for passive systems, from (6), $V_x^T f \leq 0$, in order to make the rejectability function more positive and to increase the system stability margins we embedded $-V_x^T f$ into p_r. Therefore, we can to rede£ne the rejectability function as

$$p_r(x) = b(l(x) + u^T Ru) - V_x^T f$$

Therefore, in order to guarantee that the satis£cing set $S(y, b)$ is nonempty the following inequality has to be satis£ed:

$$-V_x^T f - V_x^T gu \geq b(l(y) + u^T Ru) - V_x^T f$$

Considering that system (1) is passive, by Lemma 1 and De£nition 3, using the KYP property we have that:

$$-h^T(x)u \geq bl(y) + bu^T Ru \tag{8}$$

Thus, since $y = h(x)$, it follows that a suf£cient condition for $S(y, b)$ be a nonempty set is given by:

$$u^T bRu + y^T u + bl(y) \leq 0 \tag{9}$$

Inequality (9) has a quadratic form, which can be solved applying the following lemma [1].

Lemma 2: If $A = A^T > 0$, then the set of solutions to the quadratic inequality

$$\xi^T A\xi + d^T \xi + c \leq 0 \tag{10}$$

where $\xi \in \mathbb{R}^s$, is nonempty if only if

$$\frac{1}{4} d^T A^{-1} d - c \geq 0, \tag{11}$$

and is given by

$$\xi = -\frac{1}{2} A^{-1} d + \sqrt{\frac{1}{4} d^T A^{-1} d - c} A^{-1/2} \nu, \tag{12}$$

where $\nu \in \{\xi \in \mathbb{R}^s : \| \xi \| < 1\}$. ■

Considering $A = bR$, $d = y$, $c = bl(y)$, inequality (9) can be solved from Lemma 2. Thus, the satis£cing set is nonempty if and only if

$$\frac{1}{4} y^T (bR)^{-1} y - bl(y) \geq 0, \tag{13}$$

Furthermore, if $S(y, b)$ is nonempty it is given by:

$$S(y, b) = \Big\{ u = -\frac{1}{2}(bR)^{-1} y + \tag{14}$$
$$+ \sqrt{\frac{1}{4} y^T (bR)^{-1} y - bl(y)} (bR)^{-1/2} \nu \Big\}$$

with $\|\nu\| < 1$.

The satis£cing control is characterized by the following output feedback:

$$k(y) = -\frac{1}{2}(bR)^{-1} y + \sqrt{\frac{1}{4} y^T (bR)^{-1} y - bl(y)} (bR)^{-1/2} \nu \tag{15}$$

From (13), it follows that the condition for the satis£cing set be nonempty depends on b. Thus, we will determine the values of b (in terms of y) which satisfy condition (13).

From (13), we have that

$$\frac{1}{4b} y^T R^{-1} y \geq bl(y) \Longrightarrow b^2 \geq \frac{1}{4l(y)} y^T R^{-1} y$$

Finally

$$-\sqrt{\frac{1}{4l(y)} y^T R^{-1} y} \leq b \leq \sqrt{\frac{1}{4l(y)} y^T R^{-1} y}$$

Since $b \geq 0$, we have that

$$0 \leq b \leq \bar{b}(y) \tag{16}$$

where

$$\bar{b}(y) = \sqrt{\frac{1}{4l(y)} y^T R^{-1} y} \quad (17)$$

If $0 \leq b(y) \leq \bar{b}(y)$ the satis£cing set given by (14) is nonempty.

De£ning $b(y) = \eta\bar{b}(y)$, where $0 < \eta < 1$, and substituting it in (15), we obtain

$$k(y) = -\frac{1}{2}(\eta\bar{b}R)^{-1}y + + \sqrt{\frac{1}{4}y^T(\eta\bar{b}R)^{-1}y - \eta\bar{b}l(y)}(\eta\bar{b}R)^{-1/2}\nu, \quad (18)$$

which represent the satis£cing control law for passive systems.

Remark 1: The proposed satis£cing controller could be extended to £nite dimensional nonlinear dynamical system which can be made passive via state feedback. In [6] are given necessary and suf£cient conditions for weakly minimum-phase systems to obtain passivity feedback equivalence.

Remark 2: Performance objectives can be used to select $\eta(x)$ and $\nu(x)$. A straightforward approach is to make a prediction of the system state at the next time step and then to optimize over the selection parameter to minimize a predicted given cost [9].

IV. Case Studies

In this section, we illustrate the implementation of the satis£cing control for passive systems given by (18).

A. Linear System

Consider the following £rst order plant:

$$\dot{y} = -y + u \quad (19)$$

De£ning $l(y) = y^2$ and $R = r \in \mathbb{R}$ the satis£cing control law is given by:

$$k(y) = y\left[\frac{-1}{2\eta\bar{b}r} + \frac{1}{\sqrt{\eta\bar{b}r}}\sqrt{\frac{1}{4\eta\bar{b}r} - \eta\bar{b}\nu}\right] \quad (20)$$

where

$$\bar{b} = \frac{1}{2\sqrt{r}} \quad (21)$$

Substituting (21) in (20) the control law can be rewritten as:

$$k(y) = -\frac{y}{\eta\sqrt{r}}\left[1 - \sqrt{1-\eta^2\nu}\right] \quad (22)$$

From (22), we can observe how the η, ν and r parameters affect the proportional gain of the controller.

Figure 1 presents the simulations results for initial condition $y(0) = 20$ and $r = 1, \eta = \nu = 0.5$.

In the case one considers a non-quadratic cost $l(y) = y^2 + y^4$, the satis£cing control law is given by:

$$k(y) = -\frac{y}{r\eta\bar{b}} + \frac{\sqrt{\frac{x^2}{r\eta\bar{b}} - \eta\bar{b}(y^2+y^4) + 2 * y}}{\sqrt{\eta\bar{b}r}}\nu$$

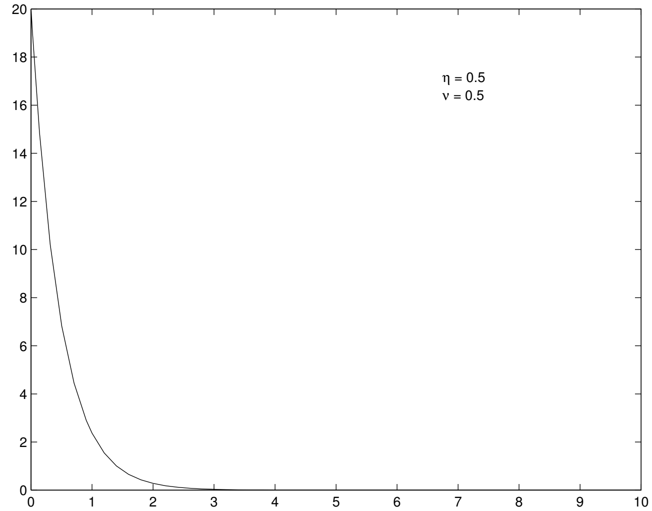

Fig. 1. Linear Systems Response for quadratic $l(y)$

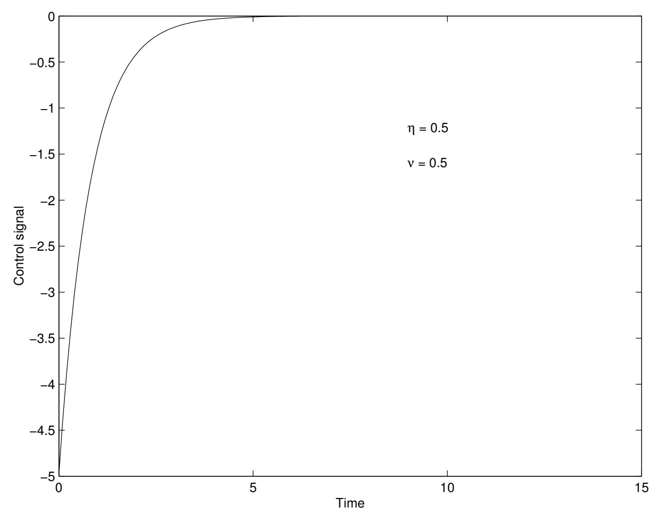

Fig. 2. Linear System: Signal control for quadratic $l(y)$

where

$$\bar{b} = \frac{\sqrt{r} + \sqrt{r + y^2 + y^4}}{\sqrt{r}y(1+y^2)}$$

Figure 3 presents the simulations results for initial condition $y(0) = 20$, $r = 1$ and $\eta = \nu = 0.5$.

Note that for this case an optimal control with cost function given by

$$J = \int_0^\infty (y^2 + y^4 + u^2)dt$$

can be calculated solving the Hamilton-Jacobi-Bellaman equation. The resulting optimal control is given by

$$u^* = y - y\sqrt{2 + y^2}$$

Comparison between both approach will be addressed in future works.

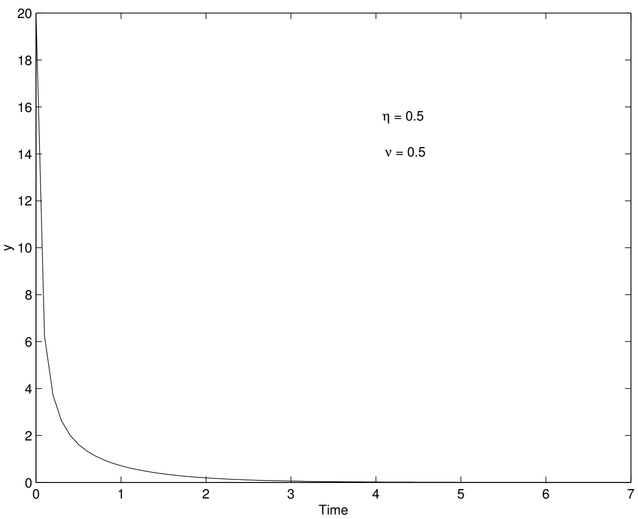

Fig. 3. Linear Systems Response with non-quadratic $l(y) = y^2 + y^4$

B. Attitude Control

Consider two orthonormal right-handed coordinate frames: the inertial (world) coordinate, E^0, and body coordinate (attached to the rigid body), E^B. Deﬁne the 3×3 attitude matrix as A. The rigid body differential can be written in the inertial coordinate as:

$$\begin{aligned} \dot{\omega} &= -H^{-1}\omega \times H\omega + \tau \\ \dot{q} &= \frac{1}{2}E(q)\omega \end{aligned} \tag{23}$$

where

$$E(q) = \begin{pmatrix} -q_1 & -q_2 & -q_3 \\ q_0 & q_3 & -q_2 \\ -q_3 & q_0 & q_1 \\ q_2 & -q_1 & q_0 \end{pmatrix},$$

and $H \in \mathbb{R}^{3\times 3}$ is the inertia matrix, $\omega \in \mathbb{R}^3$ is the angular velocity vector, $\tau \in \mathbb{R}^3$ is the control torque vector and $q = [q_0 \quad q_v^T]$, which $q_v = [q_1 \quad q_2 \quad q_3]^T$, is the unit quaternion representation of the attitude matrix A (see [10] for a more detailed discussion).

In this section we apply the passive satisﬁcing control to the rigid body orientation control [14]. We consider the set-point control problem of driving the attitude matrix A to a steady-state target attitude A_d. Deﬁne the error attitude matrix as $A_e = A_d A^T$, and let $e^T = [e_0 \quad e_v^T]$ be the unit quaternion representation for A_e. The error kinematic is then given by

$$\dot{e} = \frac{1}{2}E(e)\omega.$$

Using the result from [11], which shows that the map from v to ω is passive, i.e. $\int_0^T \omega^T v dt \geq -\gamma_0^2$ for some γ_0 $(\forall T)$, the next theorem shows that the linear feedback of the vector error quaternion, e_v, and the nonlinear feedback of the ω, called passive satisﬁcing control, is globally asymptotically stabilizing.

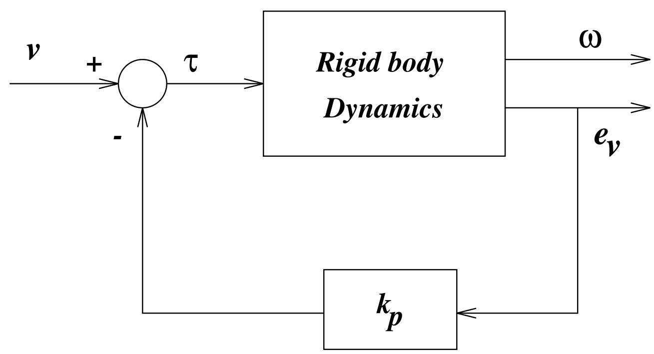

Fig. 4. Proportional Only Feedback

Theorem 1: Consider the proportional only feedback system of Figure 4. If $k(\omega)$ is a passive satisﬁcing control, then the control input $\tau = -k_p e_v + k(\omega)$, where k_p is a positive scalar, asymptotically stabilizes the close loop system.

Proof: Consider the following Lyapunov function candidate motivated by the total energy of the system (23):

$$V = k_p[(e_0 - 1)^2 + e_v^T e_v] + \frac{1}{2}\omega^T H\omega \tag{24}$$

Using (23), the derivative along the solution can be computed

$$\dot{V} = \omega^T(\tau + k_p e_v) \tag{25}$$

With the control law

$$\tau = -k_p e_v - k(\omega) \tag{26}$$

the derivative of V is given by

$$\dot{V} = \omega^T k(\omega) \tag{27}$$

Using (18), $\dot{V}$ is written by:

$$\dot{V} = -\frac{1}{2\eta\bar{b}}\omega^T R^{-1}\omega + \omega^T\sqrt{\frac{1}{4(\eta\bar{b})^2}\omega^T R^{-1}\omega - l(\omega)}R^{-\frac{1}{2}}\nu$$

Considering the expression of $\bar{b}$ given by equation (17), we have that:

$$\dot{V} = -\frac{\sqrt{l}}{\eta}\sqrt{\omega^T R^{-1}\omega} + \sqrt{\frac{1}{\eta^2} - 1}\sqrt{l}\omega^T R^{-\frac{1}{2}}\nu \tag{28}$$

The above equation could be rewritten as:

$$\dot{V} = -\frac{\sqrt{l}}{\eta}\left[\sqrt{\omega^T R^{-1}\omega} - \sqrt{1-\eta^2}\omega^T R^{-\frac{1}{2}}\nu\right] \tag{29}$$

Since $\eta \in (0, 1]$ we have that:

$$\dot{V} \leq -\frac{\sqrt{l}}{\eta}\left[\sqrt{\omega^T R^{-1}\omega} - \omega^T R^{-\frac{1}{2}}\nu\right] \tag{30}$$

Noting that $\|\omega\|_R^2 = \omega^T R^{-1}\omega$ is a weighted norm , we have that:

$$\dot{V} \leq -\frac{\sqrt{l}}{\eta}\left[\|\omega\|_R - \|\omega\|_R \nu\right] \tag{31}$$

Thus, from Lemma 2, $\|\nu\| < 1$, which implies that $\dot{V} \leq 0$.

Since V is continuously differentiable, radially unbounded, positive de£nite and $\dot{V} \leq 0$ over the entire state space, by using the LaSalle Invariance Principle [8], one has that all trajectories converge to the largest invariant set $\bar{\Omega}$ in $\Omega = \{(e, \omega) : \dot{V} = 0\} = \{(e, \omega) : \omega = 0\}$. In the invariant set we have that $H\dot{\omega} = -k_p e_v = 0$ from (1). This implies that $\bar{\Omega} = \{(e, \omega) : e_v = 0, \omega = 0\}$. As $(e_0 = \pm 1, e_v = 0)$ represents the same physical orientation $(R_e = I)$, the identity error attitude R_e and zero angular velocity is a globally asymptotically stable equilibrium. ■

C. Attitude Control: Simulation Results

In order to illustrate the result presented in the Theorem 1, a simple example considered in [12], [13] is addressed here. The η and ν parameters of the passive satis£cing control are selected, as seen in [9], using a model prediction to make a prediction of the system state at the next time step and then to optimize over the selection parameters to minimize the predicted cost. Given initializing values of the η and ν parameters, η^i, ν^i, the cost of the system a short time in the future can be estimated at any ω as follows:

$$\begin{aligned} \omega(t+T) &= \omega(t) + T\dot{\omega}(t) \\ J(t+T) &= \omega^T(t+T)Q\omega(t+T) + k^T(\eta^i, \nu^i)Rk(\eta^i, \nu^i) \end{aligned}$$

where, T represents the time-step size, $J(\eta^i, \nu^i, \omega(t+T)$ is the predicted cost and ω is the system output. A simple gradient descent algorithm is used to £nd the η and ν values that minimize this cost function (see [9] for more details).

We consider the satellite set-point control problem of driving the attitude to a steady-state target attitude [14]. In (23) set the inertia matrix to be

$$H = \begin{pmatrix} 2 & .5 & 1 \\ .5 & 4 & 1 \\ 1 & 1 & 3 \end{pmatrix}$$

Consider the quaternion initial condition

$$q(0) = [0.8325,\ -0.2057,\ 0.3430,\ 0.3834]$$

and the desired orientation

$$q_d = [0.8339,\ 0.4353,\ 0.1252,\ -0.3192]$$

(a representation of A_d) and $k_p = 1$.

The Figures 5 and 6 present the performance of the proposed scheme for the above regulation problem. Fig. 5 shows the error quaternion and Fig. 6 the control signal, when the passive satis£cing control is applied in proportional only feedback system of Figure 4

Now the tracking trajectory problem is considered. The desired orientation, $A_d(t) = exp(\phi_d(t)k\times)$, is given by a rotation along an equivalent axis $k^T = [0.4896,\ 0.2032,\ 0.8480]$ of the initial attitude toward the

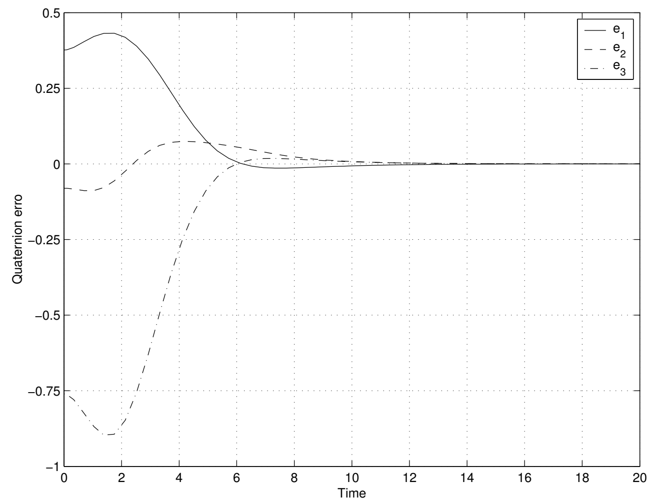

Fig. 5. Regulation Problem: Error Quaternion - Vector part.

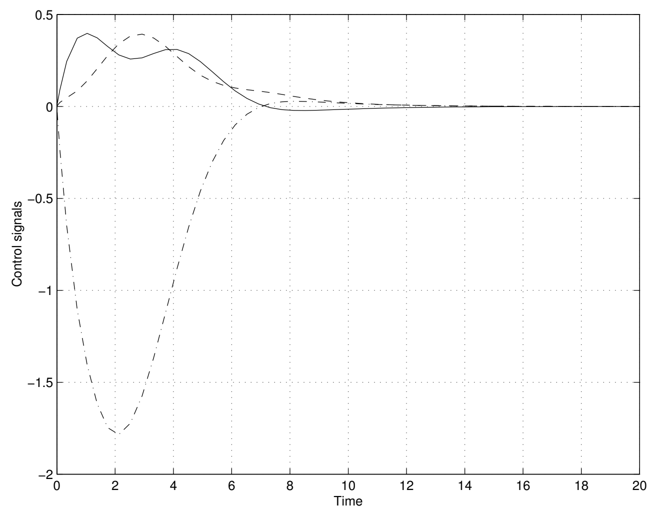

Fig. 6. Regulation Problem: Control Signal

desired attitute $I_{3\times 3}$. The desired rotational angle is given by

$$\phi_d(t) = \phi_f - (\phi_f - \phi_i)e^{-\alpha t^2}$$

with $\alpha = 0.5$, $\phi_i = 2.4648rad$ and $\phi_f = 0$. Thus, the initial condition is $q(0) = [0.332,\ 0.4618,\ 0.1917,\ 0.8]^T$.

Figures 7–8 illustrate again the nice performance obtained using the proposed scheme.

V. Conclusion

This paper considers the satis£cing control strategy for passive systems. An output feedback control, called passive satis£cing control, is obtained within this context. The advantage of this controller is that it does not rely on £nding a Control Lyapunov Function. The attitude control problem of a rigid body illustrate the application of the proposed strategy.

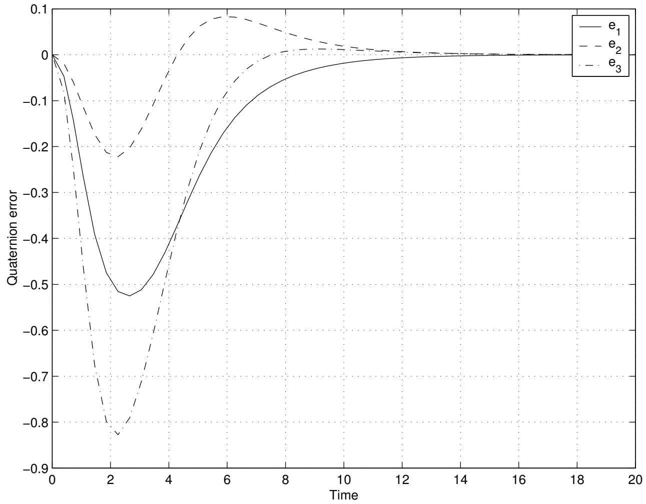

Fig. 7. Tracking Trajectory: Error Quaternion - Vector part.

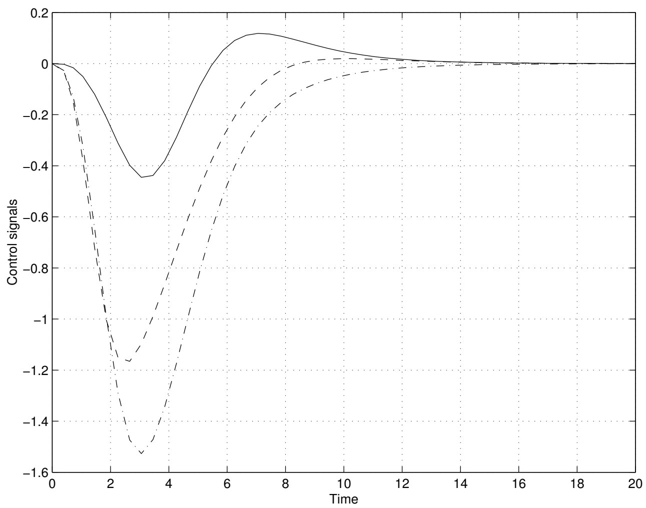

Fig. 8. Tracking Trajectory: Control Signal

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