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Structured Adaptive Model Inversion Control with Actuator Saturation Constraints Applied to Tracking Spacecraft Maneuvers

Monish D. Tandale, Kamesh Subbarao, John Valasek and Maruthi R. Akella

***Abstract*— This paper presents an adaptive control methodology that facilitates correct adaptation in the presence of actuator saturation constraints, model errors and initial condition errors. The central idea is to modify the reference trajectory on saturation, in such a way that the modified trajectory approximates the original reference closely, and can be tracked within saturation limits. Asymptotic stability of the tracking errors between the plant trajectories and the modified reference, and bounded learning of the adaptive parameters is guaranteed. Simulation results are presented for tracking of an attitude reorientation trajectory for a rigid spacecraft.**

I. INTRODUCTION

Structured Adaptive Model Inversion (SAMI) [1] is based on the concepts of Feedback Linearization [2], Dynamic Inversion, and Structured Model Reference Adaptive Control (SMRAC) [3][4][5]. In SAMI, dynamic inversion is used to solve for the control. The dynamic inversion is approximate, as the system parameters are not modeled accurately. An adaptive control structure is wrapped around the dynamic inverter to account for the uncertainties in the system parameters [6][7][8]. This controller is designed to drive the error between the output of the actual plant and the output of the reference trajectories to zero, with prescribed error dynamics. Most dynamic systems can be represented as two sets of differential equations, an exactly known kinematic level part, and a momentum level part with uncertain system parameters. The adaptation included in this framework can be limited to only the uncertain momentum level equations, thus restricting the adaptation only to a subset of the state-space, enabling efficient adaptation. SAMI has been shown to be effective for tracking spacecraft [9] and aggressive aircraft maneuvers [10]. The SAMI approach has been extended to handle actuator failures [11].

Adaptive control usually assumes full authority control, and lacks an adequate theoretical treatment for control in the presence of actuator saturation limits. Saturation becomes more critical for adaptive systems than non adaptive systems, since the adaptation is based on the tracking error. Assuming that the dynamics are modeled perfectly and only parametric uncertainties exist in the system, the tracking error has contributions due to the initial error conditions, parametric uncertainties, and saturation. The adaptation scheme should ideally adapt only the uncertain parameters. Hence the error driving the adaptation scheme should not include the error due to saturation. Including the error component due to saturation will cause incorrect adaptation.

Correct adaptation in the presence of saturation is ensured by using the concept of pseudo control hedging. The pseudo control hedging methodology has been successfully demonstrated by Johnson, E.N., Calise, A.J. et all in a neural network based direct adaptive control law [12][13][14]. The difference between the calculated and the applied control effort due to saturation results in a lack of acceleration produced in the plant as compared to the demanded reference acceleration. This is called the hedging signal. If the hedge is removed from the reference, the resulting modified reference can be tracked within saturation limits. The tracking error seen, will be only due to the initial error and the parametric uncertainty, and not due to saturation, hence the controller will adapt correctly.

This paper presents an application of Structured Adaptive Model inversion in alliance with Pseudo-Control Hedging, to the tracking of an attitude reorientation maneuver for a rigid spacecraft.

II. MATHEMATICAL FORMULATION

Consider a nonlinear dynamic system, which is affine in the control and can be split into a structured form as an exactly known kinematic differential equation and a momentum level equation with uncertain parameters.

$$\dot{\boldsymbol{\sigma}} = J(\boldsymbol{\sigma})\boldsymbol{\omega} \tag{1}$$

$$\dot{\boldsymbol{\omega}} = A\mathbf{g}(\boldsymbol{\sigma},\boldsymbol{\omega}) + B\mathbf{u} \tag{2}$$

where : $\boldsymbol{\sigma} \in \mathbb{R}^n$ is a vector of position level coordinates, $\boldsymbol{\omega} \in \mathbb{R}^n$ is vector of velocity level coordinates, $J \in \mathbb{R}^{n\times n}$ is a nonlinear transformation relating $\dot{\boldsymbol{\sigma}}$ and $\boldsymbol{\omega}$, $\mathbf{A} \in \mathbb{R}^{n\times p}$ is a matrix of system parameters, $\mathbf{g}(\boldsymbol{\sigma}, \boldsymbol{\omega}) \in \mathbb{R}^p$ is a vector of nonlinear functions of the system states, B $\in \mathbb{R}^{n\times m}$ is the control effectiveness matrix, and $\mathbf{u} \in \mathbb{R}^m$ is the control of vector inputs (under the assumption that the number of controls is at least equal to the number of velocity level states ($m \geq n$)). The control objective is to track

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M. Tandale is a Graduate Research Assistant with the Flight Simulation Laboratory, Aerospace Engineering Department, Texas A&M University, College Station, Texas 77843-3141. monish@neo.tamu.edu

K. Subbarao is an Assistant Professor with the Department of Mechanical & Aerospace Engineering, The University of Texas at Arlington, Arlington, Texas 76019-0018. subbarao@mae.uta.edu

J. Valasek is an Associate Professor and Director of the Flight Simulation Laboratory with the Aerospace Engineering Department, Texas A&M University, College Station, Texas 77843-3141. valasek@aero.tamu.edu, http://jungfrau.tamu.edu/valasek/

M. Akella is an Assistant Professor with Department of Aerospace Engineering & Engineering Mechanics at The University of Texas at Austin, Austin, Texas 78712-1085. makella@mail.utexas.edu

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prescribed reference trajectory in $\boldsymbol{\sigma_r}$, which is at least twice differentiable with respect to time. $\dot{\boldsymbol{\sigma}}_{\boldsymbol{r}}$ can be obtained by inverting Eqn.1, which requires that the mapping $J(\boldsymbol{\omega})$ be non singular.[1] $\dot{\boldsymbol{\omega}}_{\boldsymbol{r}}$ can be obtained by differentiating Eqn.1 with respect to time and using the definition in Eqn.1.

A. Calculation of the Control by Dynamic Inversion of the Dynamic Level Equation

Let the error in the position and the velocity level states be $\mathbf{s}$ and $\mathbf{x}$ respectively

$$\mathbf{s} = \boldsymbol{\sigma} - \boldsymbol{\sigma_r} \quad (3)$$
$$\mathbf{x} = \boldsymbol{\omega} - \boldsymbol{\omega_r} \quad (4)$$
$$\dot{\mathbf{x}} = A\mathrm{g} + B\mathbf{u} - \dot{\boldsymbol{\omega}}_{\boldsymbol{r}} \quad (5)$$

We want $x \to 0$ as $t \to \infty$. Adding and subtracting $A_h\mathbf{x}+\boldsymbol{\phi}$ on the right hand side

$$\begin{aligned}\dot{\mathbf{x}} &= A_h\mathbf{x} + \boldsymbol{\phi} + A\mathrm{g} + B\mathbf{u} \\ &\quad -(\dot{\boldsymbol{\omega}}_{\boldsymbol{r}} + A_h\mathbf{x} + \boldsymbol{\phi}) \end{aligned} \quad (6)$$

where A_h is Hurwitz matrix, which specifies how fast the velocity error stabilizes and $\boldsymbol{\phi}$ is a forcing function on the velocity error dynamics, which helps in achieving the tracking objective. This is discussed in detail later. Let

$$\boldsymbol{\psi} \triangleq \dot{\boldsymbol{\omega}}_{\boldsymbol{r}} + A_h\mathbf{x} + \boldsymbol{\phi} \quad (7)$$
$$\therefore \dot{\mathbf{x}} = A_h\mathbf{x} + \boldsymbol{\phi} + A\mathbf{g} + B\mathbf{u} - \boldsymbol{\psi} \quad (8)$$

Using dynamic inversion to solve for the control,

$$\mathbf{u} = B^{-1}(\boldsymbol{\psi} - A\mathbf{g}) \quad (9)$$

It is assumed that the number of controls is at least equal to the number of velocity level states being tracked so that the B^{-1} exists. For redundant actuation, where number of controls is greater than the number of states being tracked, the pseudo inverse can be used. The above control law prescribes $\dot{\mathbf{x}} = A_h\mathbf{x}+ \boldsymbol{\phi}$ dynamics to the velocity tracking error, which with the proper choice of $\boldsymbol{\phi}$, ensures that $x \to 0$ as $t \to \infty$.

B. Definition of the Adaptive Learning Parameters

The system parameters A and B are not known accurately, hence best guesses for A and B (A_{est} and B_{est}) will be used. Let

$$\mathbf{u}_{cal} = B_{est}^{-1}(\boldsymbol{\psi} - A_{est}\mathbf{g}) \quad (10)$$

A_{est} and B_{est} are the adaptive learning matrices which will be updated online. Note that the implementation of the control law requires the inverse of B_{est}. So instead of inverting B_{est} at every time instant, the control law is implemented with the adaptive estimate of B_{est}^{-1}. This is done with the following identity: $(d/dt)B_{est}^{-1} = -M(d/dt)B_{est}M$ where $M = B_{est}^{-1}$.

[1]For the spacecraft attitude tracking problem Modified Rodrigues Parameters (MRPs) will be adopted as the rigid body attitude measure. MRPs result in a minimal non-singular attitude description.

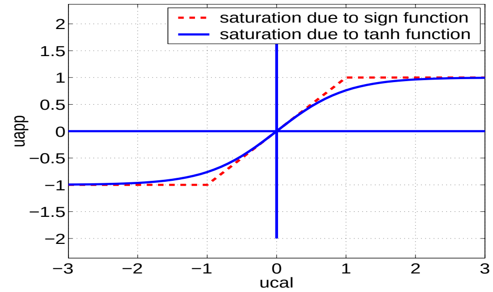

Fig. 2. Enforcing control saturation by tanh function

C. Enforcing Actuator Saturation Limits and Control Hedging

The calculated control is obtained from Eqn.10, but it has position as well as rate saturation limits. Consider the position saturation limits. The control that can be practically applied is

$$\mathbf{u}_{app} = \begin{cases}\mathbf{u}_{cal} & if|\mathbf{u}_{cal}| \leq \mathbf{u}_{max} \\ \mathbf{u}_{max}sign(\mathbf{u}_{cal}) & if|\mathbf{u}_{cal}| > \mathbf{u}_{max}\end{cases} \quad (11)$$

If we use the $sign$ function to limit the control position limits the relation between the applied control and the calculated control has a sharp corner when $\mid \mathbf{u}_{cal} \mid= \mathbf{u}_{max}$. To smoothen the discontinuity in the control rate a nonlinear mapping $(f : \mathbf{u}_{cal} \to [-\mathbf{u}_{max}, \mathbf{u}_{max}])$ is defined such that $f(x) = tanh(x)$.

$tanh(x)$ approaches ± 1 when $x \to \pm\infty$ and slope of $tanh(x) = 1$ at $x = 0$ and in the neighboring region. So, on appropriate scaling, $tanh$ can be used to represent saturation behavior.

$$\mathbf{u}_{app} = tanh(\mathbf{u}_{cal}/\mathbf{u}_{max}) * \mathbf{u}_{max} \quad (12)$$

To enforce the rate saturation limits a feedback controller structure is used, as shown in Figure 1. $\mathbf{u}_{app}$ is made to track $\mathbf{u}_{cal}$ with $\mathbf{u}_{rate}$ as the control and both $\mathbf{u}_{rate}$ and $\mathbf{u}_{app}$ are subjected to saturation limits using the $tanh$ function. Now, let $\boldsymbol{\delta}$ be the difference between the calculated control and the applied control

$$\boldsymbol{\delta} = \mathbf{u}_{cal} - \mathbf{u}_{app} \quad (13)$$

From Eqn.10 and Eqn.13,

$$\mathbf{u}_{app} = (B_{est})^{-1}(\boldsymbol{\psi} - A_{est}\mathbf{g}) - \boldsymbol{\delta} \quad (14)$$

Hence

$$\boldsymbol{\psi} = A_{est}\mathbf{g} + B_{est}\mathbf{u}_{app} + B_{est}\boldsymbol{\delta} \quad (15)$$

Substituting in Eqn.8 results in

$$\begin{aligned}\dot{\mathbf{x}} &= A_h\mathbf{x} + \boldsymbol{\phi} + A\mathbf{g} + B\mathbf{u}_{app} - \\ &\quad A_{est}\mathbf{g} - B_{est}\mathbf{u}_{app} - B_{est}\boldsymbol{\delta}\end{aligned} \quad (16)$$
$$\widetilde{A} \triangleq A - A_{est} \quad (17)$$
$$\widetilde{B} \triangleq B - B_{est} \quad (18)$$

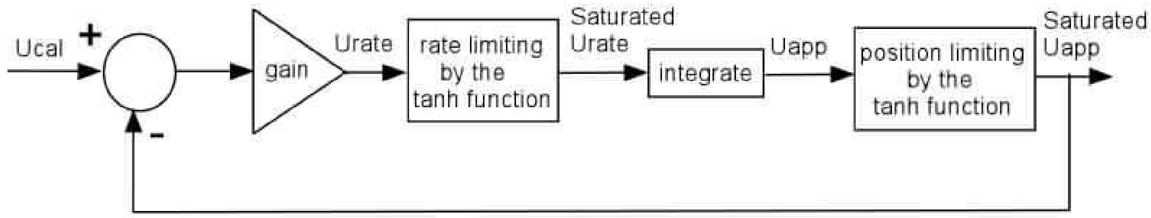

Fig. 1. Enforcing control saturation limits

Eqn.16 now becomes

$$\dot{\mathbf{x}} = A_h\mathbf{x} + \boldsymbol{\phi} + \widetilde{A}\mathbf{g} + \widetilde{B}\mathbf{u}_{app} - B_{est}\boldsymbol{\delta} \quad (19)$$

$$\begin{aligned} \dot{\boldsymbol{\omega}} - \dot{\boldsymbol{\omega}}_{\boldsymbol{r}} &= A_h(\boldsymbol{\omega} - \boldsymbol{\omega}_{\boldsymbol{m}} + \boldsymbol{\omega}_{\boldsymbol{m}} - \boldsymbol{\omega}_{\boldsymbol{r}}) + \boldsymbol{\phi} \\ &+ \widetilde{A}\mathbf{g} + \widetilde{B}\mathbf{u}_{app} - B_{est}\boldsymbol{\delta} \end{aligned} \quad (20)$$

$B_{est}\boldsymbol{\delta}$ represents the acceleration that could not be supplied because of the saturation limit and is termed as the hedging signal. Subtracting this hedging signal from the reference trajectory so that the resulting control obtained is within saturation limits.

$$\begin{aligned} &\dot{\boldsymbol{\omega}} - (\dot{\boldsymbol{\omega}}_{\boldsymbol{r}} - B_{est}\boldsymbol{\delta} + A_h(\boldsymbol{\omega}_{\boldsymbol{m}} - \boldsymbol{\omega}_{\boldsymbol{r}})) \\ &= A_h(\boldsymbol{\omega} - \boldsymbol{\omega}_{\boldsymbol{m}}) + \boldsymbol{\phi} + \widetilde{A}\mathbf{g} + \widetilde{B}\mathbf{u}_{app} \end{aligned} \quad (21)$$

$$\dot{\boldsymbol{\omega}}_{\boldsymbol{m}} = (\dot{\boldsymbol{\omega}}_{\boldsymbol{r}} - B_{est}\boldsymbol{\delta} + A_h(\boldsymbol{\omega}_{\boldsymbol{m}} - \boldsymbol{\omega}_{\boldsymbol{r}})) \quad (22)$$

where ($\boldsymbol{\omega}_{\boldsymbol{m}}$,$\dot{\boldsymbol{\omega}}_{\boldsymbol{m}}$) is the modified reference trajectory obtained due to the hedging. Consider the dynamics of the modified reference. Let $e \triangleq (\boldsymbol{\omega}_{\boldsymbol{m}}\text{-}\boldsymbol{\omega}_{\boldsymbol{r}})$ be the deviation between the modified reference trajectory and the original reference trajectory. Hence

$$\dot{e} = A_h e - B_{est}\delta \quad (23)$$

From Eqn.23 it can be seen that the hedge signal acts as a disturbance input to the modified reference error dynamics, while the term $A_h e$ is the stabilizing component. For any feasible desired trajectory the control should be unsaturated for some period of time and the hedge signal should be zero. This is because, demanding the system to track a desired trajectory for which the control saturates for the entire time duration is impractical. When the control goes out of saturation $\dot{e} = A_h e$ and hence the modified reference tends towards the original desired reference. We have shown via the above arguments that the modified reference will approach the desired reference, but no rigorous mathematical proof is provided. However it can be rigorously shown that the system tracks the modified reference. Henceforth, let (s, x) represent the deviations from the modified reference.

The modified momentum level tracking error is

$$\dot{\mathbf{x}} = A_h\mathbf{x} + \boldsymbol{\phi} + \widetilde{A}\mathbf{g} + \widetilde{B}\mathbf{u}_{app} \quad (24)$$

D. Incorporating position in the tracking error

Let the composite tracking error be defined as

$$\mathbf{y} \triangleq \dot{\mathbf{s}} + \lambda\mathbf{s} \quad (25)$$

where $\lambda \in \mathbf{R}^{n\times n}$ is a positive definite matrix. As $t \to \infty$, if $\mathbf{y} \to 0$, it is ensured that $s \to 0$ and $\dot{s} \to 0$. From Equations 1 and 3

$$\mathbf{y} = J\boldsymbol{\omega} - J_r\boldsymbol{\omega}_{\boldsymbol{r}} + \lambda\mathbf{s} \quad (26)$$

Adding and subtracting $J\boldsymbol{\omega}_{\boldsymbol{r}}$ from the right hand side and from the definition $\mathbf{x}$ = $\boldsymbol{\omega}$ - $\boldsymbol{\omega}_{\boldsymbol{r}}$

$$\mathbf{y} = J\mathbf{x} + J\boldsymbol{\omega}_{\boldsymbol{r}} - J_r\boldsymbol{\omega}_{\boldsymbol{r}} + \lambda\mathbf{s} \quad (27)$$

$$\begin{aligned} \dot{\mathbf{y}} &= J(A_h\mathbf{x} + \boldsymbol{\phi} + \widetilde{A}\mathbf{g} + \widetilde{B}\mathbf{u}_{app}) + \dot{J}\mathbf{x} \\ &+ (\dot{J} - \dot{J}_r)\boldsymbol{\omega}_{\boldsymbol{r}} + (J - J_r)\dot{\boldsymbol{\omega}}_{\boldsymbol{r}} + \lambda\dot{\mathbf{s}} \end{aligned} \quad (28)$$

Eqn.28 is obtained by differentiating Eqn.27 with respect to time and substituting for $\dot{x}$ from Eqn.24. The quantities $\widetilde{A}$ and $\widetilde{B}$ are not known, while all the other quantities are known. For the tracking error to stabilize, $\dot{\mathbf{y}} = A_h\mathbf{y}$ dynamics are prescribed to the known quantities. Hence

$$\begin{aligned} \boldsymbol{\phi} &= J^{-1}(A_h\mathbf{y} - \lambda\dot{\mathbf{s}} - \dot{J}\boldsymbol{\omega} + J_r\dot{\boldsymbol{\omega}}_{\boldsymbol{r}} + \dot{J}_r\boldsymbol{\omega}_{\boldsymbol{r}}) \\ &- \dot{\boldsymbol{\omega}}_{\boldsymbol{r}} - A_h\mathbf{x} \end{aligned} \quad (29)$$

The uncertain quantities will be handled with a Lyapunov analysis done later. Finally, substituting the value of $\boldsymbol{\phi}$ in Eqn.28

$$\dot{\mathbf{y}} = A_h\mathbf{y} + J(\widetilde{A}\mathbf{g} + \widetilde{B}\mathbf{u}_{app}) \quad (30)$$

E. Lyapunov Analysis and Update Laws for the Adaptive Learning Parameters

Now consider the error departure function as the candidate Lyapunov function. If P, W_1, W_2 are positive definite matrices,

$$V = \mathbf{y}^T P\mathbf{y} + Tr(\widetilde{A}^T W_1\widetilde{A} + \widetilde{B}^T W_2\widetilde{B}) \quad (31)$$

$$\begin{aligned} \dot{V} &= -\mathbf{y}^T Q\mathbf{y} + 2Tr(\widetilde{A}^T(J^T P\mathbf{y}\mathbf{g}^T + W_1\dot{\widetilde{A}}) \\ &+ 2Tr(\widetilde{B}^T(J^T P\mathbf{y}\mathbf{u}_{app}^T + W_2\dot{\widetilde{B}})) \end{aligned} \quad (32)$$

P is selected such that $PA_h + A_h^T P = -Q$, where Q is a positive definite matrix. Also using the identity: If M and N are row and column matrices respectively, then $MN = Tr(NM)$. Selecting $\dot{\widetilde{A}}$ such that the coefficient of $\widetilde{A}^T$ in Eqn.32 goes to zero. Retaining only the negative definite part $-\mathbf{y}^T Q\mathbf{y}$ and setting all other terms to zero,

$$\dot{\widetilde{A}} = -W_1^{-1}(J^T P\mathbf{y}\mathbf{g}^T) \quad (33)$$

From the definition of $\widetilde{A}$ and A is assumed to be constant, so that

$$\dot{A}_{est} = W_1^{-1}(J^T P \mathbf{y}\mathbf{g}^T) \tag{34}$$

$$Similarly \quad \dot{B}_{est} = W_2^{-1}(J^T P \mathbf{y}\mathbf{u}_{app}^T) \tag{35}$$

These are the update laws for the various adaptive parameters

III. Stability Analysis

$V = V(\mathbf{y}, \widetilde{A}, \widetilde{B})$, $V = 0$ when $\mathbf{y} = 0$, $\widetilde{A} = 0$ and $\widetilde{B} = 0$. (where 0 is a null vector or null matrix of appropriate dimensions). But the derivative $\dot{V} = \dot{V}(\mathbf{y})$ only. $\dot{V} = 0$, when $\mathbf{y} = 0$ irrespective of the values of $\widetilde{A}$ and $\widetilde{B}$. Hence $\dot{V}$ is negative semi definite. Thus the adaptive control law (Eqn.10) along with the update laws (Eqns.34, 35) ensure global stability.

From the properties of V and $\dot{V}$ stated above we conclude that $\mathbf{y} \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ and $\widetilde{A}$, $\widetilde{B} \in \mathcal{L}_\infty$. From our earlier discussion in section II-C, we know that, the modified reference will converge to the desired trajectory if the desired trajectory is feasible with respect to the control saturation constraints. By analyzing the various components of $\dot{\mathbf{y}}$, it can be concluded that $\dot{\mathbf{y}} \in \mathcal{L}_\infty$. From the Barbalat's lemma [16] we conclude that $\mathbf{y} \rightarrow 0$ as $t \rightarrow \infty$. Thus $\mathbf{s} \rightarrow 0$ and $\dot{\mathbf{s}} \rightarrow 0$ which $\Rightarrow \boldsymbol{\sigma} \rightarrow \boldsymbol{\sigma_m}$ and $\boldsymbol{\omega} \rightarrow \boldsymbol{\omega_m}$. Thus the states of the plant converge to the modified reference and perfect tracking can be achieved.

The tracking error between the plant trajectories and the modified reference is shown to be globally asymptotically stable for trajectories without singularities. However, the adaptively estimated parameters may not converge to the actual parameters of the system during the duration of the maneuver. Since it is assumed that parameters like A and B are constants, this formulation works only when the plant parameters are constant with respect to time.

IV. Numerical Example:

This numerical example simulates the tracking of an attitude reorientation trajectory for a rigid spacecraft. The spacecraft properties are obtained from [17] and the reference maneuver is selected similar to [9]. The reference maneuver is designed to orient a spacecraft at rest from 3-1-3 Euler Angles (-20,15,4 degrees) to the angles(40, 35, 40) with zero final angular velocity. The kinematic differential equation in terms of the MRPs is same as Eqn.1,

$$\dot{\boldsymbol{\sigma}} = J(\boldsymbol{\sigma})\boldsymbol{\omega} \tag{36}$$

where

$$J = \frac{1}{4}((1 - \boldsymbol{\sigma}^T\boldsymbol{\sigma})I_{3\times3} + 2\widetilde{\boldsymbol{\sigma}} + 2\boldsymbol{\sigma}\boldsymbol{\sigma}^T) \tag{37}$$

where $\boldsymbol{\sigma} \in \mathbb{R}^3$ is the vector of the MRPs and $\widetilde{\boldsymbol{\sigma}}$ is the cross product operator. The momentum level differential equation is as follows

$$I\dot{\boldsymbol{\omega}} = -\widetilde{\boldsymbol{\omega}}I\boldsymbol{\omega} + \mathbf{u} \tag{38}$$

where $\boldsymbol{\omega} \in \mathbb{R}^3$ is the angular velocity vector, I is the mass moment of inertia of the spacecraft and $\mathbf{u} \in \mathbb{R}^3$ is the vector of the control torques. Eqn.38 can be cast in the form similar to Eqn.2.

$$\dot{\boldsymbol{\omega}} = A\mathbf{g}(\boldsymbol{\sigma}, \boldsymbol{\omega}) + B\mathbf{u}$$

The simulation is done with large errors (approximately 75%) in the system parameters and initial condition, along with control saturation limits as listed in the Table I. Note: The units for the simulation parameters are: Inertia (kg-m^2), Angular velocities (deg/sec), Control Torque (Nm) and Torque Rate (Nm/s).

TABLE I

Simulation Parameters

Parameter	Actual Value	Guessed values
Inertia	$\begin{bmatrix} 30 & 10 & 5 \\ 10 & 20 & 3 \\ 5 & 3 & 15 \end{bmatrix}$	$\begin{bmatrix} 6 & 2.8 & 1.15 \\ 2 & 5.6 & 0.69 \\ 1 & 0.84 & 3.45 \end{bmatrix}$
States	Actual Value	Reference
$\sigma(t_0)$ $\omega(t_0)$	$[.02\ {-.006}\ {-.14}]^T$ $[0.2\ {-0.3}\ 0]^T$	$[.06\ {-.013}\ {-.07}]^T$ $[0\ 0\ 0]^T$
Control	Position Limit	Rate Limit
$\mathbf{u}$	$[0.8\ 0.8\ 0.8]^T$	$[0.8\ 0.8\ 0.8]^T$

The following cases were simulated:

1. No adaptation
2. No adaptation on saturation
3. Adaptation on saturation but without hedging, and
4. Adaptation on saturation with hedging.

Detailed simulation results are presented for the case when the adaptation is continued even after saturation and hedging is enforced. Due to the space constraint, only time histories of the performance error $|\boldsymbol{\sigma} - \boldsymbol{\sigma_r}|$ are presented for the other cases. From Figure 3, it can be seen that the hedging signal diverts the reference trajectory away from the desired trajectory towards the actual trajectory whenever the control saturates. The actual trajectories and the modified reference asymptotically converge to the desired trajectory. Figure 4 shows that the applied control is within position and rate saturation bounds. Also since the hyperbolic tangent $(tanh)$ is used to limit the control, the time history of the applied control is smooth at the point where the control hits the saturation limit. This avoids excessive control rates on the actuator.

From Figure IV we see that adaptive parameters settle to constant arbitrary values. So they are bounded but do not converge to their true values. Figure IV subplot 3 shows the time histories of the performance error $|\boldsymbol{\sigma}$ - $\boldsymbol{\sigma_r}|$ for a comparative evaluation of the various control strategies that

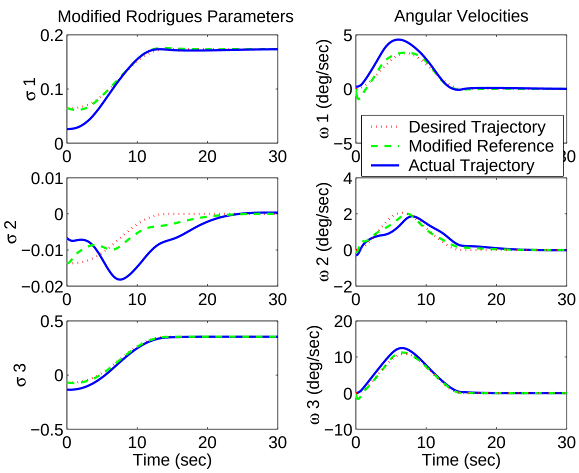

Fig. 3. Time Histories of the MRPs and Angular Velocities

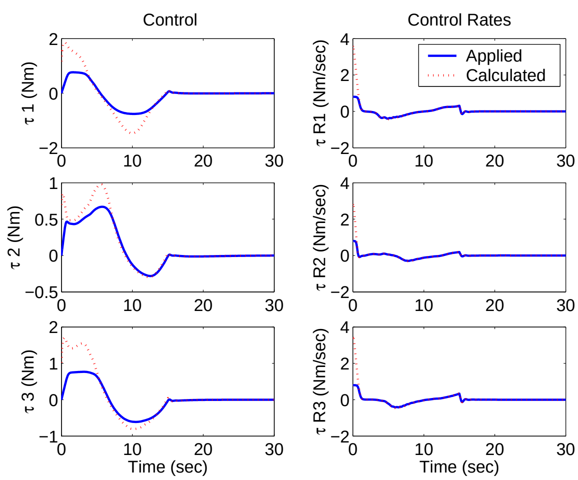

Fig. 4. Time Histories of Control Position and Rate

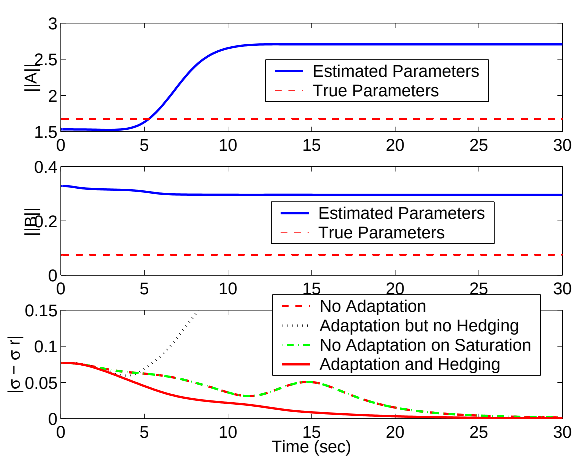

Fig. 5. Adaptive Parameters and Performance Error $|\sigma - \sigma_r|$

were simulated in the four test cases mentioned earlier. It can be seen that the actual trajectories do not converge to the reference in the case when ‘adaptation is continued without hedging’, due to wrong adaptation. In the cases when ‘adaptation is stopped on saturation’ and ‘no adaptation’, the plots of the performance index overlap as the control is saturated for almost the entire duration of the maneuver. Best performance is shown by the strategy of ‘continuing adaptation with hedging’ for the simulation presented in this paper.

V. Conclusions

This paper derived and validated a Modified Reference Structured Adaptive Model Inversion Control Law that facilitates correct adaptation in the presence of actuator saturation. This is achieved by modifying the reference so that saturation is avoided and the tracking error due to saturation does not influence the update of the learning parameters. The reference modification is heuristic and the control law guarantees asymptotic stability of the tracking errors between the plant trajectories and the modified reference. The adaptive learning parameters may not converge to the actual parameters within the duration of the maneuver. Based on the results presented in this paper, it is concluded that, in terms of convergence of the tracking errors, the modified reference control methodology shows improved performance over the other methods considered.

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