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Control of a Dumbbell Spacecraft using Attitude and Shape Control Inputs Only

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***Abstract*— An elastic dumbbell spacecraft is assumed to consist of two identical mass particles that are connected by a long elastic link. The motion of the dumbbell spacecraft can be described by orbit, attitude and shape dynamics that arise due to gravitational forces, an elastic restoring force along the longitudinal axis of the spacecraft, and control forces that act to change the attitude and shape of the spacecraft. These control forces have the property that there is no net external force on the dumbbell spacecraft. Since the angular momentum of the spacecraft is necessarily conserved, Routh reduced equations of motion are developed that describe the reduced dynamics of the controlled spacecraft. The reduced equations of motion are developed; relative equilibria are determined, and simplified reduced equations, in a linear form, are obtained. These linear reduced equations demonstrate spacecraft controllability properties. It is shown that certain maneuvers, involving a change in orbit, can be accomplished using only attitude and shape control inputs. The proposed framework can be used to study this propulsion force approach to orbital maneuvers.**

I. Introduction

This paper formulates problems for underactuated control of a multibody spacecraft. A specific multibody spacecraft, namely a dumbbell spacecraft, is considered, and specific underactuation assumptions, namely that there is no net external control force, are made. These assumptions are made to make the formulation of the control problems tractable, while maintaining the important features, namely that there are coupled orbit, attitude and shape dynamics, of a multibody spacecraft.

An elastic dumbbell spacecraft is assumed to consist of two identical mass particles that are connected by a long elastic link. The motion of the dumbbell spacecraft is described by orbit, attitude, and shape dynamics that arise due to gravitational forces, an elastic restoring force that acts along the longitudinal axis of the dumbbell spacecraft, and control forces that act to change the attitude and shape dynamics only. For simplicity, it is assumed that the motion of the two mass particles that define the dumbbell spacecraft lie in a fixed orbital plane. An extension to the full three dimensional case is possible, but tedious.

Since it is assumed that there is no net external force on the dumbbell spacecraft, the angular momentum of the dumbbell spacecraft is necessarily conserved. The process of Routh reduction is followed to obtain reduced equations of motion for the orbit, attitude and shape dynamics of the dumbbell spacecraft. These reduced equations do incorporate attitude and shape control inputs. Relative equilibria are identified that correspond to circular orbits, and linear equations are determined that describe first order perturbations from a relative equilibrium.

For multibody spacecraft, such as an elastic dumbbell spacecraft, it is reasonably well known that the orbit, attitude and shape dynamics are coupled. This fact is inherent in prior literature that treats multibody spacecraft; see for example [1], [2], [3], [4], [5], [6], [7], [8]. Much of the literature is concerned with orbit and attitude dynamics only; see [4], [5], [6], [8]. Orbit and shape dynamics only are treated in [2], and [3]. Attitude and shape dynamics only have been treated in [7]. We have previously developed models for a dumbbell spacecraft in [1] that include orbit, attitude, and shape dynamics. This paper is related to [1] by considering the special case corresponding to the assumptions mentioned previously for which reduced equations of motion can be obtained.

The linear reduced equations for the dumbbell spacecraft are shown to be completely controllable. The implication of controllability is that certain kinds of orbital maneuvers can be achieved using only the attitude and shape control inputs. We mention several approaches for constructing control inputs that achieve desired orbit changes; one approach uses an impulse train, corresponding to high authority control inputs, while another approach uses low authority control inputs. It is interesting that the problem of attitude control to achieve an orbit change and the problem of shape control to achieve an orbit change have been treated in various publications mentioned previously over the last forty years. The former problem has been nicely treated in the excellent paper [8] using analytical methods; the latter problem has been treated for example in [2].

II. Routh Reduction of Dumbbell Spacecraft Dynamics

The position vector of the dumbbell spacecraft center of mass is defined in an inertial coordinate frame fixed to the center of a massive spherical body, e.g., the Earth. The variable r is the length of this position vector and $\phi \in \mathbf{S}^1$ represents the angle that the position vector of the dumbbell spacecraft makes with respect to an axis of the inertial frame. The attitude of the spacecraft is given by θ which is the angle between the longitudinal axis of the dumbbell spacecraft and the radial position vector of the dumbbell spacecraft. The shape of the dumbbell spacecraft is denote by q defined so that $2q$ is the length of the dumbbell spacecraft with $2(q-l)$ denoting the deformation of the dumbbell spacecraft along its longitudinal axis. That is $2l$ denotes the undeformed length of the spacecraft. Consequently, the configuration of the dumbbell spacecraft is represented by the coordinates (r, ϕ, θ, q), where (r, ϕ) denotes the position variables in $\mathbb{R}^2$ of the center of mass of the dumbbell spacecraft in polar coordinates, θ denotes the attitude variable of the dumbbell spacecraft, and q denotes the shape variable of the dumbbell spacecraft. As shown in Figure 1, there are control forces N and T on the dumbbell spacecraft. The control force N acts on each mass particle normal to the connecting link; this control force is referred to as the attitude control input. The control force T acts on each mass particle along the connecting link; this control force is referred to as the shape control input.

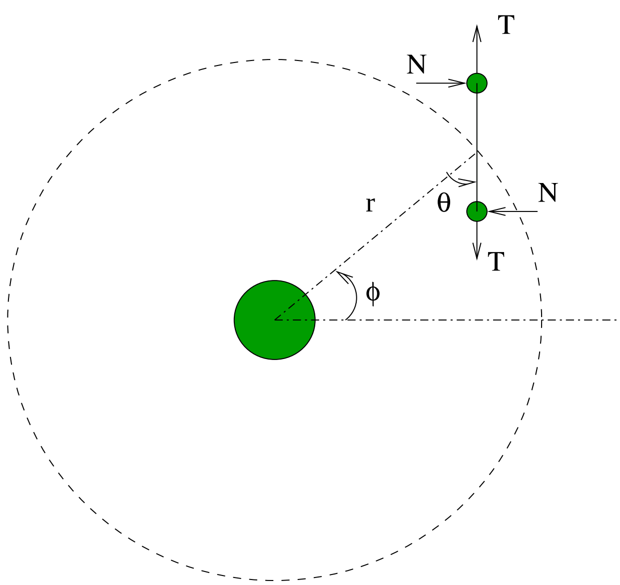

Fig. 1. Dumbbell spaceraft in planar orbit in central gravitational field.

Let the mass of each of the mass particles in the dumbbell spacecraft be m, and let k denote the elastic stiffness of the link joining the two masses. The Lagrangian then has the form

$$L(r,\theta,q,\dot{r},\dot{\phi},\dot{\theta},\dot{q}) = m(\dot{r}^2+\dot{q}^2+q^2\dot{\theta}^2+2q^2\dot{\theta}\dot{\phi} \\ +(r^2+q^2)\dot{\phi}^2) - V_g(r,\theta,q) - 2k(q-l)^2, \tag{1}$$

where the gravitational potential is

$$V_g(r,\theta,q) = -\frac{\mu m}{\sqrt{q^2+r^2-2qr\cos\theta}} \\ -\frac{\mu m}{\sqrt{q^2+r^2+2qr\cos\theta}}.$$

It is clear that the Lagrangian is independent of the angle variable ϕ. Since there is no net external force on the dumbbell spacecraft other than gravity and ϕ is a cyclic variable, the angular momentum

$$p = \frac{\partial L}{\partial \dot{\phi}} = 2m(q^2\dot{\theta} + (r^2+q^2)\dot{\phi}) \tag{2}$$

is conserved. As shown in [1], the spacecraft can not be linearly controllable near a relative equilibrium corresponding to a circular orbit with such actuation.

We carry out Routh reduction to obtain reduced equations of motion; we follow the development in [9], [10]. The classical Routhian is defined by setting the angular momentum p constant, performing a partial Legendre transformation in the variable ϕ, and substituting for $\dot{\phi}$ using (2):

$$R(r,\theta,q,\dot{r},\dot{\theta},\dot{q}) = L - p\dot{\phi} = m(\dot{r}^2+\dot{q}^2+q^2\dot{\theta}^2) \\ -\frac{(p-2mq^2\dot{\theta})^2}{4m(r^2+q^2)} - V_g(r,\theta,q) - 2k(q-l)^2. \tag{3}$$

The equations of motion of the reduced dynamics are obtained from the above Routhian by substituting the Routhian for the Lagrangian in the Lagrange-Routh equations. The radial equation of motion is given by

$$2m\ddot{r} - \frac{2mrq^4}{(r^2+q^2)^2}\dot{\theta}^2 - \frac{p^2 r}{2m(r^2+q^2)^2} + \frac{2rq^2p}{(r^2+q^2)^2}\dot{\theta} \\ +\frac{\mu m(r-q\cos\theta)}{(r^2+q^2-2qr\cos\theta)^{3/2}} \\ +\frac{\mu m(r+q\cos\theta)}{(r^2+q^2+2qr\cos\theta)^{3/2}} = 0. \tag{4}$$

The equation of motion for the attitude is given by

$$\frac{2mr^2q^2\ddot{\theta}}{r^2+q^2} + \frac{4mrq\dot{\theta}(q^3\dot{r}+r^3\dot{q})}{(r^2+q^2)^2} + \frac{2pqr(r\dot{q}-q\dot{r})}{(r^2+q^2)^2} \\ +\frac{\mu mqr\sin\theta}{(r^2+q^2-2qr\cos\theta)^{3/2}} \\ -\frac{\mu mqr\sin\theta}{(r^2+q^2+2qr\cos\theta)^{3/2}} = 2Nq. \tag{5}$$

where N is the attitude control input as shown in Figure 1. The equation of motion for the shape variable is

$$2m\ddot{q} - \frac{2mr^4q}{(r^2+q^2)^2}\dot{\theta}^2 - \frac{p^2q}{2m(r^2+q^2)^2} - \frac{2r^2qp}{(r^2+q^2)^2}\dot{\theta} \\ +\frac{\mu m(q-r\cos\theta)}{(r^2+q^2-2qr\cos\theta)^{3/2}} + \frac{\mu m(q+r\cos\theta)}{(r^2+q^2+2qr\cos\theta)^{3/2}} \\ +4k(q-l) = T, \tag{6}$$

where T is the shape control input as shown in Figure 1.

We can rewrite the reduced equations of motion in the form

$$\begin{aligned}\ddot{r}=&\frac{rq^4}{(r^2+q^2)^2}\dot{\theta}^2+\frac{p^2r}{4m^2(r^2+q^2)^2}-\frac{rq^2p}{m(r^2+q^2)^2}\dot{\theta}\\&-\frac{\mu(r-q\cos\theta)}{2(r^2+q^2-2qr\cos\theta)^{3/2}}\\&-\frac{\mu(r+q\cos\theta)}{2(r^2+q^2+2qr\cos\theta)^{3/2}},\end{aligned}\tag{7}$$

$$\begin{aligned}\ddot{\theta}=&-\frac{2\dot{\theta}(q^3\dot{r}+r^3\dot{q})}{rq(r^2+q^2)}-\frac{\mu(r^2+q^2)\sin\theta}{2qr(r^2+q^2-2qr\cos\theta)^{3/2}}\\&+\frac{\mu(r^2+q^2)\sin\theta}{2qr(r^2+q^2+2qr\cos\theta)^{3/2}}\\&-\frac{p(r\dot{q}-q\dot{r})}{mqr(r^2+q^2)}+\frac{r^2+q^2}{mr^2q^2}Nq,\end{aligned}\tag{8}$$

$$\begin{aligned}\ddot{q}=&\frac{r^4q}{(r^2+q^2)^2}\dot{\theta}^2+\frac{p^2q}{4m^2(r^2+q^2)^2}+\frac{r^2qp}{m(r^2+q^2)^2}\dot{\theta}\\&-\frac{2k}{m}(q-l)-\frac{\mu(q-r\cos\theta)}{2(r^2+q^2-2qr\cos\theta)^{3/2}}\\&-\frac{\mu(q+r\cos\theta)}{2(r^2+q^2+2qr\cos\theta)^{3/2}}+\frac{T}{2m}.\end{aligned}\tag{9}$$

At an equilibrium of the reduced equations of motion, the following algebraic equations are satisfied

$$\begin{aligned}&\frac{-p^2r_e}{2m(r_e^2+q_e^2)^2}+\frac{\mu m(r_e-q_e\cos\theta_e)}{(r_e^2+q_e^2-2q_er_e\cos\theta_e)^{3/2}}\\&+\frac{\mu m(r_e+q_e\cos\theta_e)}{(r_e^2+q_e^2+2q_er_e\cos\theta_e)^{3/2}}=0,\end{aligned}\tag{10}$$

$$\begin{aligned}&\frac{\mu mq_er_e\sin\theta_e}{(r_e^2+q_e^2-2q_er_e\cos\theta_e)^{3/2}}\\&-\frac{\mu mq_er_e\sin\theta_e}{(r_e^2+q_e^2+2q_er_e\cos\theta_e)^{3/2}}=0,\end{aligned}\tag{11}$$

$$\begin{aligned}&\frac{-p^2q_e}{2m(r_e^2+q_e^2)^2}+\frac{\mu m(q_e-r_e\cos\theta_e)}{(r_e^2+q_e^2-2q_er_e\cos\theta_e)^{3/2}}\\&+\frac{\mu m(q_e+r_e\cos\theta_e)}{(r_e^2+q_e^2+2q_er_e\cos\theta_e)^{3/2}}+4k(q_e-l)=0,\end{aligned}\tag{12}$$

where (r_e,θ_e,q_e) denote an equilibrium of the reduced equations. From equations (10)-(12), we see that there are two classes of equilibria for the reduced equations:

$$\begin{aligned}&\theta_e=n\pi,\ n\in\mathbb{Z},\ p^2=\frac{4\mu m^2(r_e^2+q_e^2)^3}{r_e(r_e^2-q_e^2)^2},\ \frac{p^2q_e}{2m(r_e^2+q_e^2)^2}\\&=\frac{\mu m}{(r_e+q_e)^2}-\frac{\mu m}{(r_e-q_e)^2}+4k(q_e-l),\ \text{and}\end{aligned}\tag{13}$$

$$\begin{aligned}&\theta_e=(n+\frac{1}{2})\pi,\ n\in\mathbb{Z},\quad p^2=4\mu m^2\sqrt{r_e^2+q_e^2},\\&\frac{p^2q_e}{2m(r_e^2+q_e^2)^2}=\frac{2\mu mq_e}{(r_e^2+q_e^2)^{3/2}}.\end{aligned}\tag{14}$$

These equilibria of the reduced equations define the relative equilibria of the original (unreduced) equations, as given in [1].

III. Linearized Reduced Dynamics and Linear Controllability Properties

We now linearize the equations of motion about the equilibria given by (13) and (14). Perturbations of the reduced configuration variables from their equilibrium values are denoted by $(\delta r,\delta\theta,\delta q)$.

Throughout this section, we assume q_e to be positive.

The linearized equations of motion about equilibria given by (13), with $\theta_e=0$, are first presented. The linear reduced radial, attitude and shape equations of motion are given by:

$$\begin{aligned}&2m\delta\ddot{r}+\frac{2pq_e^2r_e}{(r_e^2+q_e^2)^2}\delta\dot{\theta}+\Big(\frac{2p^2r_e^2}{m(r_e^2+q_e^2)^3}-\frac{p^2}{2m(r_e^2+q_e^2)^2}\\&-\frac{2\mu m}{(r_e-q_e)^3}-\frac{2\mu m}{(r_e+q_e)^3}\Big)\delta r+\Big(\frac{2p^2r_eq_e}{m(r_e^2+q_e^2)^3}\\&+\frac{2\mu m}{(r_e-q_e)^3}-\frac{2\mu m}{(r_e+q_e)^3}\Big)\delta q=0,\end{aligned}\tag{15}$$

$$\begin{aligned}&\frac{2mr_e^2q_e^2}{r_e^2+q_e^2}\delta\ddot{\theta}-\frac{2pq_e^2r_e}{(r_e^2+q_e^2)^2}\delta\dot{r}+\frac{2pr_e^2q_e}{(r_e^2+q_e^2)^2}\delta\dot{q}+\\&\Big(\frac{\mu mq_er_e}{(r_e-q_e)^3}-\frac{\mu mq_er_e}{(r_e+q_e)^3}\Big)\delta\theta=2Nq_e,\end{aligned}\tag{16}$$

$$\begin{aligned}&2m\delta\ddot{q}-\frac{2r_e^2q_ep}{(r_e^2+q_e^2)^2}\delta\dot{\theta}+\Big(\frac{2p^2q_e^2}{m(r_e^2+q_e^2)^3}-\frac{p^2}{2m(r_e^2+q_e^2)^2}\\&-\frac{2\mu m}{(r_e-q_e)^3}-\frac{2\mu m}{(r_e+q_e)^3}+4k\Big)\delta q+\Big(\frac{2p^2r_eq_e}{m(r_e^2+q_e^2)^3}\\&+\frac{2\mu m}{(r_e-q_e)^3}-\frac{2\mu m}{(r_e+q_e)^3}\Big)\delta r=T.\end{aligned}\tag{17}$$

The linearized equations of motion about equilibria given by (14), with $\theta_e=\frac{\pi}{2}$, are next presented. The linear reduced radial, attitude and shape equations of motion are given by:

$$\begin{aligned}&2m\delta\ddot{r}+\frac{2pq_e^2r_e}{(r_e^2+q_e^2)^2}\delta\dot{\theta}+\Big(\frac{2p^2r_e^2}{m(r_e^2+q_e^2)^3}-\frac{p^2}{2m(r_e^2+q_e^2)^2}\\&+\frac{2\mu m}{(r_e^2+q_e^2)^{3/2}}-\frac{6\mu mr_e^2}{(r_e^2+q_e^2)^{5/2}}\Big)\delta r\\&-\frac{6\mu mq_er_e}{(r_e^2+q_e^2)^{5/2}}\delta q=0,\end{aligned}\tag{18}$$

$$\begin{aligned}&\frac{2mr_e^2q_e^2}{r_e^2+q_e^2}\delta\ddot{\theta}-\frac{2pq_e^2r_e}{(r_e^2+q_e^2)^2}\delta\dot{r}+\frac{2pr_e^2q_e}{(r_e^2+q_e^2)^2}\delta\dot{q}\\&-\frac{6\mu mr_e^2q_e^2}{(r_e^2+q_e^2)^{5/2}}\delta\theta=2Nq_e,\end{aligned}\tag{19}$$

$$\begin{aligned}&2m\delta\ddot{q}-\frac{2r_e^2q_ep}{(r_e^2+q_e^2)^2}\delta\dot{\theta}+\Big(\frac{2p^2q_e^2}{m(r_e^2+q_e^2)^3}-\frac{p^2}{2m(r_e^2+q_e^2)^2}\\&+\frac{2\mu m}{(r_e^2+q_e^2)^{3/2}}-\frac{6\mu mq_e^2}{(r_e^2+q_e^2)^{5/2}}+4k\Big)\delta q\\&-\frac{6\mu mq_er_e}{(r_e^2+q_e^2)^{5/2}}\delta r=T.\end{aligned}\tag{20}$$

If we denote the vector of reduced configuration perturbations by $\delta x=[\delta r\ \delta\theta\ \delta q]^{\mathrm{T}}$, then these linearized equations of motion can be expressed as a linear, second order, vector differential equation of the form

$$M\delta\ddot{x} + C\delta\dot{x} + K\delta x = Bu, \tag{21}$$

where M is a symmetric positive definite inertia matrix, C is a skew-symmetric matrix representing gyroscopic terms, K is a symmetric stiffness matrix, and B is a control input matrix. For the dumbbell spacecraft, the forms of the matrices M, C, K, and B are given by

$$M = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & c_1 & 0 \\ -c_1 & 0 & c_2 \\ 0 & -c_2 & 0 \end{bmatrix},$$
$$K = \begin{bmatrix} k_1 & 0 & k_2 \\ 0 & k_3 & 0 \\ k_2 & 0 & k_4 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 2q_e & 0 \\ 0 & 1 \end{bmatrix}, \tag{22}$$

where $u = [N \quad T]^{\mathrm{T}}$ is the control input vector. The values of the constants in equation (22) are determined by the above linear equations (15)-(17) or equations (18)-(20). The structure of the linear reduced equations of motion is that of a *linear gyroscopic system*; compare with the form of the linearized unreduced equations of motion (see [1]). These linear reduced equations of motion have the general structure of a linear Hamiltonian system [9], [10], [11]; it is interesting to note that the process of Routh reduction maintains this Hamiltonian structure.

In assessing complete controllability of second order equations in the above form, we make use of necessary and sufficient conditions for complete controllability given in [12]. According to [12], the equation (21) is completely controllable if and only if the controllability rank condition

$$\operatorname{rank}[\lambda^2 M + \lambda C + K, \; B] = 3 \tag{23}$$

holds for all λ that satisfies $\det[\lambda^2 M + \lambda C + K] = 0$. This controllability result is now used to determine the controllability properties for the linear reduced equations of motion.

Proposition 1. *The linearized equations of motion for the reduced dynamics are completely controllable if the attitude and shape are both actuated.*

The controllability matrix of equation (23) has rank 3 for all values of λ.

This result means that it is possible to modify the orbit of the dumbbell spacecraft from a circular orbit to a family of nearby orbits by simultaneously using only attitude and shape control inputs as shown in Figure 1.

Proposition 2. *The linearized equations of motion for the reduced dynamics are completely controllable if only the shape is actuated.*

The controllability matrix has rank 3 for all values of λ since $k_3 \neq 0$.

Proposition 3. *The linearized equations of motion for the reduced dynamics are completely controllable if only the attitude is actuated.*

The linearized equations are completely controllable since $k_1 k_4 \neq k_2^2$, which implies that the controllability matrix has rank 3 for all values of λ.

The last two results suggest that even if only the attitude or only the shape degree of freedom is actuated, then the reduced equations of motion are linearly controllable in a neighborhood of either class of relative equilibria.

IV. Orbital, Attitude, and Shape Maneuvers using Attitude and Shape Control Inputs

Maneuver control problems for the linear reduced equations of motion can be defined by specification of suitable boundary conditions that consist of an initial reduced state (configuration and configurtion rate) and a final reduced state (configuration and configuation rate). A positive maneuver transfer time can also be specified. The initial state and the final state must be consistent in the sense that they correspond to the same angular momentum constant. The initial state and the final state must also be close (in the sense of a norm) in order to utiliize a common linear reduced set of equations. For such specification of initial state and final state and specification of a maneuver transfer time, the control problem (or motion planning problem) is to determine the control input(s), e.g. the attitude control input and/or the shape control input, that transfer the state of the linear reduced equation from the specified initial state to the specified final state in the specified transfer time.

On the basis of the controllability properties stated in the previous section, well-defined maneuver control problems are guaranteed to have a solution using both attitude and shape control inputs, using only attitude control inputs, or using only shape control inputs. There are several standard control methodologies that can be utilized to construct numerical solutions to such linear maneuver problems.

These maneuver problems can be formulated as fuel optimal control problems where the attitude and shape control inputs are assumed to be impulsive [13]. If both attitude and shape control inputs are available, then the maneuver control problems can be solved using a train of three impulses; if only an attitude control input (or only a shape control input) is available, then the maneuver control problems can be solved using a train of six impulses.

Alternatively, these maneuver control problems can be formulated as quadratic optimization problems. Standard tools are available for such problems. Well-known methods that make use of the controllability Grammian would seem to be especially suited.

A special class of maneuver problems for the reduced dynamics are the rest-to-rest maneuver problems. These maneuver problems, in the present context, involve a transfer from one relative equilibrium (of the unreduced dynamics) to another nearby relative equilibrium. Rest-to-rest maneuvers are possible only if the initial state and the final state correspond to the same angular momentum constant.

V. Conclusions

As shown in [1], the complete orbit, attitude and shape equations of motion are not completely controllable using attitude and shape control inputs in any meaningful sense; this fact is a consequence of the conservation of angular momentum of the dumbbell spacecraft. However, in this paper we have demonstrated that the reduced equations of motion for the dumbbell spacecraft are completely controllable at least locally near relative equilibria corresponding to circular orbits. This is a very important property since it suggests that the use of attitude and shape inputs can be used to achieve certain orbit maneuvers. Although controllability of this type has been implicitly recognized in prior publications, our contribution is to demonstrate this controllability property in a formal way based on the linear reduced equations of motion.

In addition to exposing the controllability of the linear reduced equations of motion for the dumbbell spacecraft, these equations can also be utilized to construct specific orbit, attitude, and shape maneuvers. We have made a few comments about how this might be accomplished, but this represents an area of continuing research.

Finally, we mention several possible extensions of the results in this paper. Here we have made the assumptions of planar orbit, attitude and shape dynamics for the dumbbell spacecraft, since this leads to relatively simple equations of motion. We have developed more general results for the general three dimensional case; these results will be reported elsewhere. This paper has also treated a very specific spacecraft, namely an elastic dumbbell spacecraft. We believe that similar results can be developed for more complex multibody spacecraft, including tethered spacecraft and space robots.

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