Quality of Information Measures for Autonomous Decision-Making *

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Abstract

We present a methodology to detect changes in quality of information (QoI) of data received by an autonomous entity. QoI is defined as the inverse of the expected Kullback-Leibler distance between a reference probability distribution and the conditional distribution associated with the data. When the underlying dynamic process that generates the data is real-valued, the interacting multiple model Kalman filter (IMM-KF) can be used to compute QoI. For the case of discrete-event dynamics, we present an IMM Bayes filter to detect changes in QoI. Numerical examples are provided to illustrate the methodology.

1 Introduction

Consider a random variable x and two sensors labeled A and B providing information about x as shown in Figure 1. Intuitively, we say that a sensor provides high quality information if its distribution is close to the true distribution. For example, in Figure 1, sensor A provides higher quality information than sensor B even though sensor A has a higher variance. This intuitive notion of quality of information can be made precise as follows.

Let f, f_A and f_B be the probability density functions of x, sensor A and sensor B. Define quality of information (QoI) for sensor A as:

$$QoI\,(f, f_A) = \left[\int f(x) \log\left(\frac{f(x)}{f_A(x)}\right) dx\right]^{-1} \quad (1)$$

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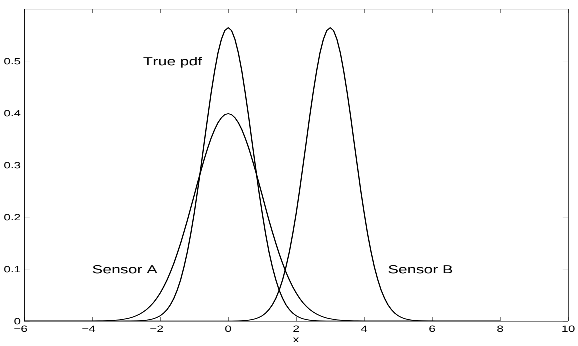

Figure 1: Sensor A gives information of better quality than Sensor B

i.e, the inverse of the Kullback-Leibler distance [4] from f to f_A, and similarly define $QoI\,(f, f_B)$ for sensor B. We say that *sensor A provides higher QoI than sensor B if and only if $QoI\,(f, f_A) \geq QoI\,(f, f_B)$*. The Kullback-Leibler distance, though not a metric, has some nice properties. It is always greater than or equal to zero and is equal to zero if and only if the two distributions being compared are equal. Moreover, from the definition of QoI, we have that $QoI\,(f, f_A) \geq QoI\,(f, f_B)$ if and only if

$$\int f(x) \log\left(\frac{f_A(x)}{f_B(x)}\right) dx \geq 0$$

where the integrand is the logarithm of likelihoods in favor of sensor A against sensor B, and the integral is the mean information for discrimination in favor of sensor A against sensor B (though not quite in the sense used by Kullback). Thus, many properties desirable in a measure of quality of information are captured by the definition (1). It should be noted that there is a large family of measures known as the Csiszar divergence measures [3], Kullback-Leibler being a special case, that can be used to define QoI measures.

This paper explores the use of QoI measures for autonomous decision making in a networked environment. Specifically, we consider a group of unmanned air vehicles (UAVs) that collect and process heterogenous data (eg. MTE/SAR for target recognition, INS/GPS for flight control), and communicate both real-valued and discrete-valued variables to accomplish mission objectives. As a means to guard against faults and malicious tampering of data, each UAV should determine the QoI of received data and, if the QoI is above a threshold, use the data to carry out its mission. This situation is different from the comparison between sensors made earlier. First, the QoI definition given by (1) cannot be used directly because the true probability distribution is unknown. Second, transmitted data such as UAV's position and target type are generated by non-stationary random processes as opposed to a random variable. Finally, the received data may be corrupted by faults at the source and by changes in the channel characteristics. The UAV may know the set of faults and channel models, but not necessarily the current fault or channel model. A consequence of these differences is that we can only provide a relative measure of QoI with which changes in QoI can be detected.

The paper is organized as follows. The next section gives details of QoI for dynamical systems. For the case of real-valued data and known Markov jump models, the interacting multiple model-Kalman filter (IMM-KF) may be used to compute QoI. For the case of discrete-valued data and known Markov jump models, we derive an IMM-Bayes filter to compute QoI. Section 3 presents numerical results to illustrate the defintion and computations. Section 4 presents conclusions.

2 Main Results

2.1 QoI Definition

Consider the dynamical system over $(\mathcal{W}, \mathcal{M}, \mathcal{X}, \mathcal{Y})$:

$$x(k+1) = f(x(k), w(k), m(k)) \tag{2a}$$

$$y(k) = h(x(k), w(k), m(k)) \tag{2b}$$

where $w(k)$, $m(k)$, $x(k)$ and $y(k)$ are the exogenous input, the system mode, the state vector and the measurement at time k respectively, $f : \mathcal{X} \times \mathcal{W} \times \mathcal{M} \to \mathcal{X}$ and $h : \mathcal{X} \times \mathcal{W} \times \mathcal{M} \to \mathcal{Y}$. We assume that the set of system modes $\mathcal{M}$ is a finite ordered set:

$$\mathcal{M} = \{m_1, m_2 \cdots, m_{n_m}\}$$

and that the system mode process $m = \{m(k)\}$ is a Markov chain with transition probability matrix $P = [p_{ij}]$ where p_{ij} is the probability of transitioning from mode j to mode i. We shall further assume that the exogenous input process is independent and identically distributed, and independent of the system mode process. Throughout this paper, we consider two cases: (i) $\mathcal{W}$=$\mathbb{R}^{n_w}$, $\mathcal{X}$=$\mathbb{R}^{n_x}$ and $\mathcal{Y}$=$\mathbb{R}^{n_y}$ and, (ii) $\mathcal{W}$, $\mathcal{X}$ and $\mathcal{Y}$ are non-empty finite sets. The first case corresponds to a standard discrete-time real-valued dynamical system, whereas the second case corresponds to an input-output automaton. When the system (2) is an input-output automaton, we order the sets $\mathcal{X}$, $\mathcal{W}$ and $\mathcal{Y}$:

$$\begin{aligned} \mathcal{X} &= \{x_1, x_2, \cdots, x_{n_x}\} \\ \mathcal{W} &= \{w_1, w_2, \cdots, w_{n_w}\} \\ \mathcal{Y} &= \{y_1, y_2, \cdots, y_{n_y}\} \end{aligned}$$

and denote by $p(x(k))$ the (column) vector whose jth entry is the probability of $x(k) = x_j$. Fix time k, a system mode $m(k) = m_i$ and an exogenous input $w(k) = w_j$. Then, the state transition indicated in (2a) can be written in matrix form as:

$$p(x(k+1)) = F_{m_i,w_j} p(x(k))$$

where F_{m_i,w_j} is a stochastic matrix. Similarly, the measurement equation (2b) can be written in matrix form.

We only give the definition for real-valued dynamical systems; the case of automata is analogous. Let $\mathcal{Y}_k$ denote the set of measurements upto and including time k and $p\left(x(k)\,|\mathcal{Y}_k\right)$ be the probability density of state x at time k conditioned on $\mathcal{Y}_k$. Let us also denote by S a subset of $\mathbb{R}^{n_x}$.

Definition 2.1 (QoI) *Consider the dynamical system (2). The expected quality of information supplied by y at time k relative to the density q_k on S is:*

$$\left(E\left[\int_S q_k(x) \log\left(\frac{q_k(x)}{p\left(x(k)\,|\mathcal{Y}_k\right)} \right) dx \right] \right)^{-1} \tag{3}$$

where the expectation is taken over all $\mathcal{Y}_k$.

The quantity being integrated over S in the above definition is a function of the measurement sample set $\mathcal{Y}_k$; it is a random variable. The integrated value is then inverted to get a stochastic QoI whose mean value is referred to as the expected QoI. Our notation for QoI does not explicitly show its dependence on S and q_k. We take S to be the entire state space and q_k to be a posterior density function in most cases. It should also be noted that the definition is valid for certain pairs of q_k and $p\left(x(k)\,|\mathcal{Y}_k\right)$ only.

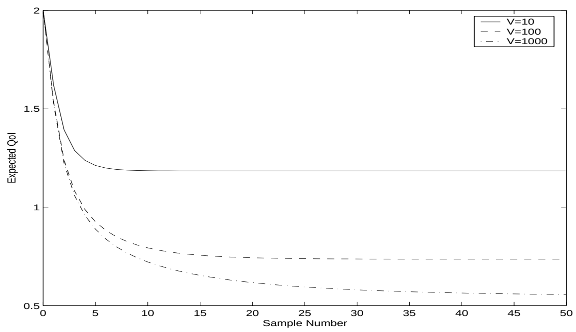

Figure 2: QoI supplied by measurement process for different noise levels compared to $V = 1$

As an illustration, consider the system:

$$x(k+1) = (99/100)x(k) + w(k)$$
$$y(k) = x(k) + v(k)$$

where w and v are scalar white noise processes with covariances 1 and V respectively. Intuitively, the QoI supplied by the measurement process y should degrade as the measurement noise covariance increases. To check if this is true, let $S = \mathbb{R}^{n_x}$ and q_k be the conditional state density function at time k with $V = 1$. Figure 2 shows expected QoI for measurement noise covariances of $V = 10, 100, 1000$ compared to $V = 1$. In this simple example, the conditional density functions can be calculated using a linear Kalman filter and the QoI formula can be evaluated analytically. As the figure shows, the QoI supplied by y degrades with measurement noise.

We shall now focus on the computation of $p\left(x(k)\,|\mathcal{Y}_k\right)$ which is the conditional state density function for (2). For linear dynamics with Markov mode jumps, the interacting multiple model Kalman filter (IMM-KF) [1, 2] is a recursive procedure for calculating $p\left(x(k)\,|\mathcal{Y}_k\right)$. The exact computation of $p\left(x(k)\,|\mathcal{Y}_k\right)$ for real-valued nonlinear dynamics is not practical and, often approximate schemes are employed. The next section gives an IMM-Bayes filter for the automaton case.

2.2 Interacting Multiple Model(IMM)-Bayes Filter for Automaton

The derivation is not included in the paper due to page limitations. It suffices to say that the IMM-Bayes algorithm given below follows from the total probability theorem and the Bayes theorem. For notational convenience, define:

$$X_{ik} = p\left(x(k)\,|\mathcal{Y}_k, m(k) = m_i\right)$$
$$X_{ik}^- = p\left(x(k)\,|\mathcal{Y}_{k-1}, m(k) = m_i\right)$$
$$Y_{ik} = p\left(y(k)\,|\mathcal{Y}_k, m(k) = m_i\right)$$
$$Y_{ik}^- = p\left(y(k)\,|\mathcal{Y}_{k-1}, m(k) = m_i\right)$$

Initialization: Initial distribution functions for state $p(x(0))$ and system mode $\lambda_0 = [\lambda_{i0}]$. Set $p\left(x(0)\,|\mathcal{Y}_0\right) = p(x(0))$ and $k = 1$.

Loop over time: Note that $p\left(x(k-1)\,|\mathcal{Y}_{k-1}\right)$, $\lambda_{i(k-1)}$ and y_k are known at time k. The following steps give $p\left(x(k)\,|\mathcal{Y}_k\right)$ and λik:

1. For each $1 \le i \le n_m$, run the Bayes filter

$$p(x(k)) = \left(\sum_{j=1}^{n_w} F_{m_i,w_j} X_{i(k-1)}\right) / n_w \quad (4a)$$

$$X_{ik} = \frac{1}{\gamma}\, p\left(y(k)\,|x(k)\right) p(x(k)), \quad (4b)$$

where γ is the normalization factor, for one step.

2. Form $HX_{ik}^- = Hp(x(k))$.

3. For each $1 \le i \le n_m$, update the mode probabilities $\lambda_{i(k-1)}$ to λ_{ik} using:

$$\lambda_{ik} = \frac{Y_{ik}^- \sum_{j=1}^{M} p_{ij}\lambda_{j(k-1)}}{\sum_{j=1}^{M}\left[Y_{ik}^- \left(\sum_{l=1}^{M} p_{jl}\lambda_{l(k-1)}\right)\right]}$$

where $p\left(y_k\,|\mathcal{Y}_{k-1}, m(k) = i\right)$ is the distribution of y_k conditioned on the past measurements and the mode i. It can be computed by applying the nonlinear transformation (2b) with $m(k) = i$ to the conditional state distribution function $p\left(x_k\,|\mathcal{Y}_{k-1}, m(k) = i\right)$.

4. Calculate the conditional distribution $p\left(x(k)\,|\mathcal{Y}_k\right)$ using:

$$p\left(x(k)\,|\mathcal{Y}_k\right) = \sum_{i=1}^{n_m} X_{ik}\, p\left(m(k) = m_i\,|\mathcal{Y}_k\right) \quad (5)$$

Set $k = k + 1$ and go to Step 1.

3 Numerical Examples

We present two examples arising in autonomous control of UAVs. The first example given in Section 3.1 deals with QoI of relative position updates received through a randomly-varying channel. Precision platform control as well as collision avoidance requires position updates of good quality. The second example given in Section 3.2 deals with a discrete-valued variable called Combat ID that is calculated on-board a vehicle and then transmitted to another. These examples demonstrate the range of applicability and limitations of our QoI methodology.

3.1 QoI of Received UAV Relative Position

Figure 3 shows UAV 1 transmitting its position through a channel to UAV 2. The data $y(k)$ received by UAV 2 satisfies:

$$x(k+1) = A_\lambda x(k) + B_\lambda \left(s(k) + n_t(k)\right)$$
$$y(k) = C_\lambda x(k) + D_\lambda n_r(k)$$

where $s(k)$ is the signal (UAV 1's position), n_t and n_r are transmitter and receiver noises, x denotes channel state and λ denotes mode switching process that takes values in $\{1, \cdots, M\}$. The channel dynamics in mode $\lambda = i$ linear time-invariant and is given by the state space matrices (A_i, B_i, C_i, D_i). The mode switching process λ is assumed to be Markov with a known transition probability matrix.

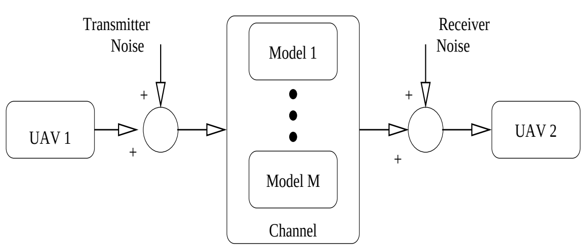

Figure 3: UAV 1 sends its position to UAV 2 through a noisy randomly-varying channel.

UAV 2 can reconstruct the signal using IMM-Kalman filter [1, 2] given the channel dynamics, signal statistics and noise statistics. Suppose that the channel undergoes a change to a case where the received signal has more noise than before. For example, set $D_2 = 10D_1$ (no other difference between modes) and consider a switch from mode 1 to mode 2. With the model switching, the UAV 1's position reconstructed by UAV 2 may have more noise and, when used for platform control, may cause control performance degradation. UAV 2 will need a procedure to quickly detect changes in QoI and determine if it should seek other sources of UAV 1's position. This is the motivation behind QoI definition. For numerical simulations, we assume three modes for channel, i.e, $\lambda \in \{1, 2, 3\}$. In each mode, the channel dynamics is given by an IIR stable transfer function of degree 3. The poles, zeros and dc-gain of the transfer function models are different. UAV 1 is assumed to move at constant speed and heading relative to UAV 2, and is at the same altitude as UAV 2.

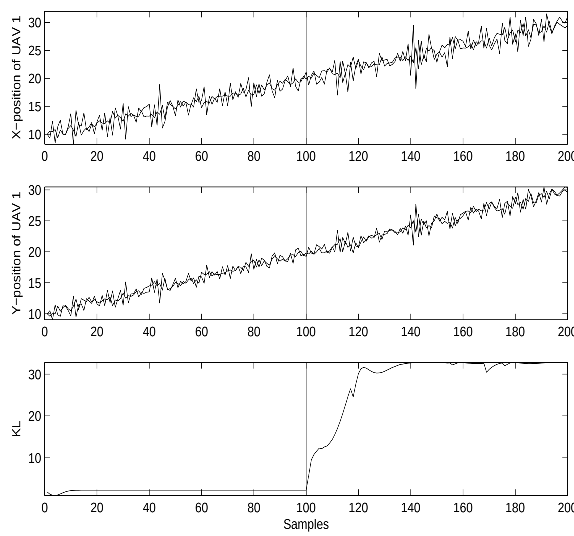

Figure 4: QoI change due to channel model switching at $t = 100$ second.

Figure 4 shows results of simulation where the channel dynamics undergoes an abrupt change at $t = 100$ second. The new mode has low signal to noise ratio. The plot on top shows the true x position and its estimate calculated by UAV 2 from the received position data. The middle plot shows y position and its estimate. The bottom plot is the expected Kullback-Liebler distance of the received data whose inverse is the expected QoI. The distance measure is calculated relative to the true density function of UAV 1's position. The plot shows that the channel change is quickly detected and the QoI of received data is lowered.

3.2 QoI of Received Combat ID

Consider a tactical ISR (intelligence, surveillance, reconnaissance) platform that collects MTE/SAR/EO-IR data as well as human intelligence data, fuses them and builds up its confidence level on a binary-valued Combat ID $\in \{E, F\}$. When the confidence level passes some threshold, either E or F is transmitted (a sequence of E's or a sequence of F's may be transmitted for reliability) to another entity for continuing the overall mission. After transmitting, the tactical ISR platform continues to collect data, updates Combat ID and transmits again. We may model this sequence of operations as as automaton:

$$\text{Combat ID}(k+1) = f\left(\text{Combat ID}(k), w(k)\right) \quad (6)$$

where f is a random transition map, w is a random process taking values in a finite set W. Here, w and f are abstract representations of data collection and fusion algorithms used to determine Combat ID. For numerical simulations, we take the set of possible data types to be:

$$\begin{aligned} W &= \{SAR_e,\ SAR_f,\ AOC_e,\ AOC_f\} \\ &= \{a, b, c, d\} \end{aligned}$$

where SAR and AOC denote respectively data collected by the platform using its sensors and data received from air operations command (AOC), and the subscripts indicate the state of Combat ID that they are likely to support. For simplicity, we use the letters a, b, c and d to denote the four data types.

We assume that the true value of Combat ID is F. Recall that (6) can be written equivalently in terms of Markov transition matrices. The normal operation of tactical ISR upon receiving SAR data is given by:

$$F_a = \begin{bmatrix} 1 & p \\ 0 & 1-p \end{bmatrix} \quad F_b = \begin{bmatrix} q & 0 \\ 1-q & 1 \end{bmatrix}$$

where the states are ordered as $\{E, F\}$ (i.e, the first row corresponds to transition to E and the second row corresponds to transition to F). For numerical simulations, we take $p = 0.1$ and $q = 0.05$. Similarly, upon receiving AOC data, the tactical ISR updates probability of Combat ID according to:

$$F_c = \begin{bmatrix} 1 & p \\ 0 & 1-p \end{bmatrix} \quad F_d = \begin{bmatrix} q & 0 \\ 1-q & 1 \end{bmatrix}$$

with $p = 0.05$ and $q = 0.05$. We will also consider an abnormal operation in which the sensed data processing changes. This abnormal operation is given by:

$$\hat{F}_a = \begin{bmatrix} 1 & 0.4 \\ 0 & 0.6 \end{bmatrix} \quad \hat{F}_b = \begin{bmatrix} 0.6 & 0 \\ 0.4 & 1 \end{bmatrix}$$

The SAR and AOC data types are generated stochastically.

Figure 5 shows the simulation of a tactical ISR (ISR 1) that remains in the normal mode of operation throughout the simulation. The plot on top left hand corner is the sequence of data received by the ISR; plot on top right is the probability of Combat ID computed from the data using ATR and fusion algorithms encoded by the stochastic automaton (6). The Combat ID is transmitted when (a) the probability is above a threshold level (0.85), and (b) if a transmission slot is available. We use a time division multiple access (TDMA) network for communication. The strike UAV which receives Combat ID from ISR 1 reconstructs the probability of Combat ID by running the IMM-Bayes filter (in this case a Bayes filter) given earlier. The results are in the bottom right plot.

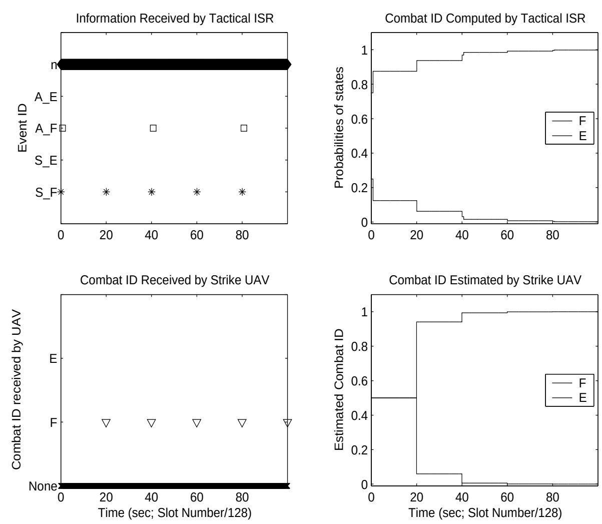

Figure 5: ISR 1 simulation

Figure 6 shows a second tactical ISR (ISR 2) that is also collecting data and transmitting Combat ID to the strike UAV. This ISR remains normal for sometime and then undergoes a change to an abnormal mode where the sensed data is not collected properly. The top right hand plot clearly shows the change in operational mode. Its effect on Combat ID transmission is no transmission since the probability thresh-

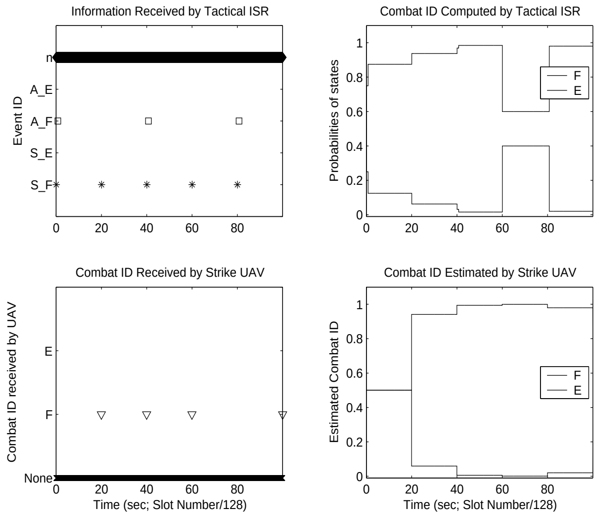

Figure 6: ISR 2 simulation

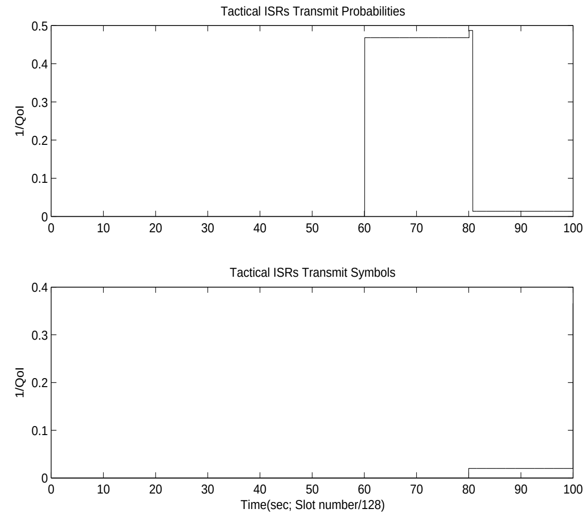

Figure 7: QoI of ISR 2 relative to ISR 1 for different transmission variables

old level is not reached. The strike UAV reconstructs probability of Combat ID using the IMM-Bayes filter with two modes of operation.

We computed QoI of ISR 2 relative to ISR 1 for two cases. In case 1, both ISRs transmit the probability of Combat ID so that the strike UAV can directly compute QoI. In case 2, both ISRs transmit the most likely Combat ID symbol (E or F) and the strike UAV reconstructs the probabilities using an IMM-Bayes filter. Figure 7 shows the QoIs computed for both cases. The top plot shows the case when probabilities are transmitted; while the bottom plot is for case 2. In both cases, QoI shows a change, but the detection is delayed due to the fact that no data is received (see the plots on bottom left in Figures 5-6). As we would expect, transmiting probability has advantages over transmiting symbols in terms of faster detection and stronger indication of QoI change.

4 Conclusions

This paper presented a methodology to define and detect changes in QoI of data received by an autonomous entity. QoI is defined as the inverse of the expected Kullback-Leibler distance between a reference probability distribution and the conditional distribution associated with the data. The interacting multiple model Bayes filter can be used for computing the expected QoI. We presented an IMM Bayes filter for the case of discrete-event dynamical system. Numerical examples for relative position data (real-valued) and Combat ID data (discrete-valued) show applicability of the methodology.

QoI is defined relative to a reference measure q_k. An ideal choice for q_k is such that a change in QoI occurs if and only if a system property (mode) changes. Such a choice cannot exist for dynamical systems as shown by the initial transient phase in Figure 2. We are only able to attribute the differences in expected QoI at steady state to the different noise covariances (modes). In other words, a change in QoI does not imply a change in system mode and there is a strictly positive probability of false alarms. It is easily argued using detectability concepts that there is also a strictly positive probability of missed-detection. A characterization of probabilities of false alarm and detection is important for reliable detection of changes in QoI.

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