

# Load Governor for Fuel Cell Oxygen Starvation Protection: A Robust Nonlinear Reference Governor Approach

Jing Sun and Ilya Kolmanovsky

**Abstract**—The fuel cells oxygen starvation problem is addressed in this paper using a robust load governor. By regulating the current drawn from the fuel cell, the pointwise-in-time constraints on the oxygen excess ratio and on the oxygen mass inside the cathode are strictly enforced to protect the fuel cells from oxygen starvation. The load governor is designed using a nonlinear reference governor approach. Parameter uncertainties such as those due to imperfect controls of temperature and humidity are handled in the load governor design using a novel approach based on sensitivity functions. Simulation results are included to demonstrate the effectiveness of the proposed scheme. The results are compared with those of a linear filter which has been proposed in the prior literature to achieve similar goals.

## I. INTRODUCTION

Fuel cell research has recently received a great deal of attention because of its strategic importance in our nation's energy policy and its positive impact on the environment. Successful and wide-spread production applications of fuel cell systems in transportation and power generation industries will alleviate our society's dependence on fossil fuels and make the renewable resource a reality. However, challenging problems in the areas of material, manufacturing, fuel processing and handling, and control need to be resolved before the cars propelled with fuel cells can be driven on the road.

Major control problems for fuel cells are highlighted in [1], [2]. In this paper, we consider the load control of the fuel cells with the main focus on preventing oxygen starvation in this system. Power generation in fuel cells relies on the continuous supply of oxygen on the cathode side and hydrogen on the anode side, when proper temperature and humidity conditions are maintained. The amount of power the fuel cells system delivers is controlled by the amount of current drawn from the system, if the proper conditions for cell operation are maintained. When a large load is applied to the cells, the sudden increase in the current can cause the system to stall if the depleted oxygen cannot be replenished immediately and sufficiently. The cell starvation can lead to system stall or permanent cell damage.

To protect the fuel cells from overloading and starvation, especially during the transient period, one can supply excessive oxygen and hydrogen to the cells in the steady-state operation, thus increasing the reserve of available power in anticipation of the load increase. This strategy, however, is conservative, and it leads to increased parasitic losses,

decreased fuel and air utilization, and thus a compromised system performance. Another approach is to modify the load demand with a first order or other slew-type filter [3], so that the application of the load is delayed to give time for the air and fuel supplies to catch up. While a filter can mitigate the problem and is easy to implement, it cannot guarantee that cell starvation is eliminated. When the filter is tuned for the worst-case scenario, it often leads to a conservative design with slow response. Other approaches, such as the Model Predictive Control (MPC) [4], have also been proposed to address the issues.

In this paper, we formulate the fuel cells starvation protection as a constraint-enforcement problem. The reference governor [5], a mechanism effective in guaranteeing pointwise-in-time state and input constraints being satisfied, is explored for the load (current) governance. A load governor is designed to minimize the load tracking error while at the same time checking conditions for constraint violation. If constraint violation is predicted, the load will be reduced until all the constraints are satisfied. The computations involved in implementing the load governor include the simulations of the plant model over a finite time horizon and bi-sectional search for determining the optimal gain. Although the performance of the reference governor can be suboptimal as compared to more flexible MPC controllers, the computational implementation of the reference governor is much simpler, which is a significant advantage for chronometric and memory constrained automotive micro-controllers.

In dealing with the load control for the oxygen starvation protection problem, we first assume the temperature and humidity of the fuel cells are controlled around its setpoint with good accuracy. This allows us to use a nominal model for the load governor design and implementation. Given that the temperature and humidity controls represent some of the toughest challenges in fuel cell system management, this assumption most likely will be violated in real application. To mitigate the problems associated with model uncertainties, especially those caused by temperature and humidity fluctuation, we propose a robust load governor by taking into account explicitly the parametric uncertainties in our design. In particular, we incorporate two parameters in the model to represent uncertainties in (i) the supply manifold relative humidity; (ii) the vapor saturation pressure in the cathode due to temperature deviation. In addition to the computations involved with the nominal load governor, i.e., the simulation of the nominal plant model and bi-sectional search, the robust load governor uses sensitivity functions

Correspondence to: Professor Jing Sun, Naval Architecture and Marine Engineering Department, University of Michigan, Ann Arbor, MI 48109, U.S.A. Email: jingsun@umich.edu

for the linearized model and an estimate for the error bound for linearization to guarantee the robustness performance.

The paper is organized as follows: A control oriented model based on [6] will be briefly presented in Section II, followed by a discussion on the control design objectives. Section III describes the load governor, which is based on the reference governor concept, for the fuel cell current control. The robust load governor, aimed at guaranteeing constraint satisfaction under parametric uncertainties, is delineated in Section IV. Section V highlights the simulation results, where the performance of the robust load governor is compared with that of a regular reference governor and with the system using first order filters. Section VI concludes the paper with a brief summary.

For the sake of clarity, we limit the scope of this paper to preventing oxygen starvation by regulating the fuel cell current, with constraints imposed on the oxygen excess ratio and the amount of oxygen left in the cathode. However, the same scheme and design process can be extended to include additional constraints involving the conditions on the anode side. The robust load governor developed in this paper can also be extended to general cases which involve multiple uncertain parameters.

## II. CONTROL DESIGN MODEL AND PROBLEM FORMULATION

### A. A control-oriented model

This paper is primarily concerned with the oxygen starvation protection problem for Proton Exchange Membrane fuel cells (PEMFC), although the concepts and techniques are not restricted to this system. Several control-oriented models have been developed and explored for PEMFC control design and system optimization. In this paper, the design of the load governor is based on the model developed in [6], where the dynamics of the supply and return manifolds, the characteristics for the cathode and anode, and the fuel cell auxiliaries are identified. To concentrate on the oxygen starvation protection, a 4-state model for the air dynamics of the fuel cells model, simplified from [6], is considered:

$$\dot{p}_{sm} = \frac{\gamma R_a}{V_{sm}} (W_{in} T_{in} - W_{out} T_{sm}), \quad (1)$$

$$\dot{m}_{O_2,ca} = W_{O_2,in-ca} - W_{O_2,out-ca} - W_{O_2,rct}, \quad (2)$$

$$\dot{m}_{N_2,ca} = W_{N_2,in-ca} - W_{N_2,out-ca}, \quad (3)$$

$$\dot{m}_{v,ca} = W_{v,in-ca} - W_{v,out-ca} + W_{v,gen} + W_{v,mbr}, \quad (4)$$

where  $T, W, p, V$  denote the temperature, mass flow rate, pressure, and volume, respectively, and the subscript represents the variable taken at a specific location. For example,  $T_{sm}$  represents the supply manifold temperature, while  $W_{in}$  denotes the intake mass flow.  $m_{O_2,ca}, m_{N_2,ca}, m_{v,ca}$  denote the total mass of oxygen, nitrogen, and vapor in the cathode.  $W_{x,in-ca}, W_{x,out-ca}$  represent the mass flow of constituent  $x$  in and out of the cathode respectively.  $W_{O_2,rct}$  represents

the oxygen consumed due to reaction, which is dictated by the stack current  $I_{st}$  through the relation:

$$W_{O_2,rct} = M_{O_2} \frac{n I_{st}}{4F}, \quad (5)$$

where  $n$  is the number of cells in the stack and  $F = 96485$  is the Faraday number.  $W_{v,gen}, W_{v,mbr}$  are the rates at which water (vapor) is generated due to  $H_2/O_2$  reactions and transported across the membrane respectively; their expressions can be found in [6].

Assuming the flows from the supply-to-cathode and cathode-to-return manifolds are controlled by linear nozzles, the flow rates  $W_x$  of different constituents in and out of the cathode are calculated using their partial pressures or mole fractions and orifice model. For example, with the relative humidity and temperature in the supply manifold controlled at  $\phi_{sm}$  and  $T_{sm}$  respectively, the mass flow rates from the supply manifold to the cathode are given as:

$$W_{ca-in} = A_{sm2ca} (p_{sm} - p_{ca}),$$

$$W_{O_2,ca-in} = W_{ca-in} \frac{p_{O_2,sm} M_{O_2}}{p_{sm} M_{sm}},$$

where  $M_{O_2}, M_{sm}$  are the mole weight of oxygen and gas mixture in the supply manifold respectively, and  $p_{x,sm}$  is the partial pressure of the constituent  $x$  in the supply manifold. The partial pressure of the oxygen in the supply manifold can be expressed as

$$p_{O_2,sm} = 0.21(p_{sm} - \phi_{sm} p_{sat}(T_{sm})),$$

where  $p_{sat}$  is the vapor saturation pressure which is a function of the temperature. Similar expressions can be obtained for all other terms in the model (1)-(4).

In addition to the plant model, we also assume that a Proportional+Integral (PI) feedback control, in combination with feedforward scheduling, is used to regulate the oxygen excess ratio, defined as

$$\lambda_o = \frac{W_{O_2,in-ca}}{W_{O_2,rct}}, \quad (6)$$

to a setpoint. We thus augment the plant model with the controller state to include the following dynamics:

$$\begin{aligned} \dot{x}_c &= e_\lambda, \\ W_{in} &= K_{ff}(I_{st}) + K_p e_\lambda + K_i x_c, \end{aligned} \quad (7)$$

where  $e_\lambda = \lambda_o - \lambda_o^s$  is the oxygen excess ratio regulation error, and  $K_{ff}$  is the feedforward term that will be scheduled based on the stack current  $I_{st}$ .  $K_p, K_i$  are constant gains for the PI controller. The parameters  $K_{ff}, K_p, K_i$  are fixed from the outset for the load governor design. When these values change, only the underlying model used in the load governor implementation needs to be updated. All other algorithms and implementation for the load governor will remain the same, since they do not use the parameters in the model explicitly.

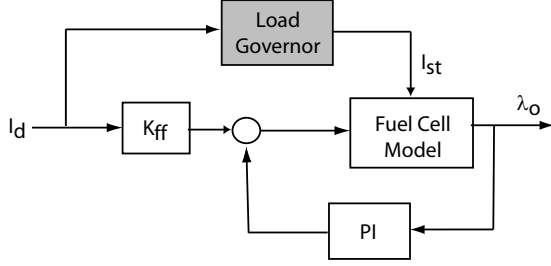


Fig. 1. Fuel cell control systems with the load governor.

### B. Design objectives

At steady state conditions, a fuel cell is operated at a fixed oxygen excess ratio  $\lambda_o^s$  where an overall optimal system efficiency is achieved [8]. The regulation of  $\lambda_o$  is assumed to be achieved by the controller (7). During the transient when a large load<sup>1</sup> is suddenly applied,  $\lambda_o$  will temporarily go below this setpoint, since the depleted oxygen cannot be immediately replenished due to manifold dynamics. Oxygen starvation is most likely to occur when  $\lambda_o$  falls too far below the setpoint for a sustained period of time. The risk of oxygen starvation is also increased when the oxygen mass in the cathode,  $m_{O_2,ca}$ , is too low.

Our objective is to design an add-on governor for the load application. Instead of applying the demanded current  $I_d$  immediately and directly to the fuel cell, we want to control the current which is drawn from the fuel cell so that:

- The applied load  $I_{st}$  tracks the demanded load  $I_d$  as closely as possible;
- The following constraints are satisfied for all time:

$$m_{O_2,ca} \geq m_{O_2,ca}^{min}, \quad \lambda_{O_2} \geq \lambda_{O_2}^{min}. \quad (8)$$

Since  $I_{st}$  will dictate how much oxygen will be depleted instantaneously, as shown in (5), the oxygen starvation can be effectively eliminated by preventing the load from drawing too much current from the cell stack.

### III. REFERENCE GOVERNOR FOR LOAD CONTROL

Reference governor is an add-on mechanism for enforcing constraints through a modified input [5]. It is a simple, yet effective scheme to avoid constraint violation for both linear and nonlinear systems. For the fuel cell oxygen starvation protection problem under consideration, the load governor serves as an interface between the commanded load and the applied load, as shown in Figure 1. It accepts input commands and modifies their value so that: (i) all the constraints are satisfied; (ii) the tracking error between the commands and actual inputs is minimized.

According to the reference governor concept (see [5], [7] and references therein), the stack current  $I_{st}$  to be drawn from the fuel cells over the time interval  $[kT, (k+1)T]$ ,

<sup>1</sup>In this paper, the terms “load” and “current” are synonymous.

where  $T$  is the sample period, can be generated based on the current request,  $I_d(kT)$ , as follows:

$$\begin{aligned} \bar{I}_{st}(kT) &= \bar{I}_{st}((k-1)T) + \beta(kT)(I_d(kT) - \bar{I}_{st}((k-1)T)), \\ I_{st}(\tau) &= \bar{I}_{st}(kT), \quad kT \leq \tau < (k+1)T. \end{aligned} \quad (9)$$

The parameter  $\beta(kT) \in [0, 1]$  in (9) is maximized at each sample time  $k$ , subject to the condition that maintaining  $I_{st}(\tau) = \bar{I}_{st}(kT)$  for  $\tau \geq kT$  guarantees that the constraints will be satisfied for all  $\tau \geq kT$ .

To determine the parameter  $\beta$  at each time instant  $kT$ , the fuel cell model, including the controller dynamics (7), is simulated forward in time over the interval of  $[kT, kT + T_s]$ , where  $T_s$  is the simulation horizon. If the constraints are violated during the simulation period,  $\beta(kT)$  will be reduced and simulation re-initiated. If all the constraints are satisfied for the simulated trajectory, the value of  $\beta(kT)$  will be increased to minimize the tracking error. The process will be repeated using the bi-sectional search until  $\beta$  converges.

**Remark 1:** Based on the general guidelines,  $T_s$  should be selected so that if  $I_{st}$  is maintained at a constant value after the time instant  $t$  and the constraints are satisfied over the time interval  $[t, t + T_s]$  then they will be also satisfied over the interval for any  $T_a > T_s$ . Frequently, it suffices to choose  $T_s$  to be 2-5 times the system time constant. Note that although larger  $T_s$  causes computational overhead to simulate the model, the dimensionality of the optimization problem to determine  $\beta$  does not change.

**Remark 2:** The implementation of the load governor requires simulating the plant model multiple times for each sample time. Compared to other constraint-enforcing algorithms, such as the model predictive control, this computational demand is much less stringent and can be more easily satisfied.

### IV. ROBUST LOAD GOVERNOR

The difficulties in precisely controlling the temperature and humidity of the fuel cell air delivery system are well appreciated by the fuel cell control community. When temperature and humidity deviate from their set-points, the objective of the constraint enforcement may not be achieved with the regular load governor developed in Section III. The uncertainties we face in the fuel cell model motivate us to seek improvement of the nominal load governor design to guarantee the robustness with respect to constraint satisfaction. We refer to the resulting scheme as a robust load governor.

If  $\theta \in \Theta$  is a vector of uncertain parameters and  $\Theta$  is a compact set, then in selecting  $\beta(kT)$  in (9), the conditions that constraints are satisfied for  $\tau \geq kT$  with  $I_{st}(\tau) = \bar{I}_{st}(kT)$  must be guaranteed for all  $\theta \in \Theta$ . There are intricacies in the reference governor design for systems with parametric uncertainties. They have to be considered to rigorously guarantee constraint enforcement and assure the desired response properties of the reference governor [5]. Firstly, no feasible  $\beta(kT) \in [0, 1]$  may exist at some time instants  $kT$  even if a feasible initial  $\beta(0)$  does exist.

In this situation  $\beta(kT)$  must be set to zero. Secondly, to theoretically guarantee the desirable response properties of the reference governor such as finite-time convergence of  $\bar{I}_{st}$  to  $I_d$  (so that the reference governor becomes inactive in finite time) the algorithm to compute  $\beta(kT)$  has to be slightly modified (see [5]). Thirdly, computational procedures are needed to check the constraints for all  $\theta \in \Theta$ .

We now describe an approximate procedure which can efficiently handle the constraint checking for general nonlinear systems. We will afterwards discuss its applications to the fuel cell oxygen starvation protection problem.

Let us consider a general nonlinear system represented by

$$\begin{aligned}\dot{x} &= f(x, r, \theta) \\ z &= g(x, r, \theta)\end{aligned}\quad (10)$$

where  $x, r, \theta$  are vectors of the states, reference inputs, and uncertain parameters respectively, while  $z$  is the vector of variables subject to pointwise-in-time constraints. To implement the robust load governor, we need a computational procedure to determine, for given  $x(0)$  and  $r$ , whether the constraints, written as  $z(t) \in \mathcal{C}_z$  for  $0 \leq t \leq T_s$ , are satisfied for all  $\theta \in \Theta$ . During the reference governor operation, this procedure is to be applied at each time instant  $kT$  with the current state of the system in place of  $x(0)$  and with  $r$  set to the proposed reference governor output.

Suppose  $\theta_0 \in \Theta$  and let  $x_n(t), z_n(t)$ , be the simulated trajectory of states and constrained variables, respectively, i.e.,

$$\begin{aligned}\dot{x}_n &= f(x_n, r, \theta_0) \\ z_n(t) &= g(x_n(t), r, \theta_0).\end{aligned}$$

For  $\theta \in \Theta, \theta \neq \theta_0$ ,  $z(t)$  can be approximated by  $z_f(t)$  which is defined by

$$z_f(t) = z_n(t) + (\delta_\theta^z(t))^T(\theta - \theta_0), \quad (11)$$

where  $\delta_\theta^z$  is the sensitivity function of  $z$  with respect to the parameter  $\theta$  around the nominal trajectory  $x_n, z_n$ , i.e.,  $\delta_\theta^z = \frac{\partial z}{\partial \theta} |_{\theta=\theta_0}$ . Assuming that  $f$  and  $g$  are sufficiently smooth, the sensitivity function can be computed as a solution to the following linearized matrix differential equation,

$$\begin{aligned}\dot{x}_\theta &= \frac{\partial f}{\partial x} \Big|_{(x=x_n, \theta=\theta_0)} x_\theta + \frac{\partial f}{\partial \theta} \Big|_{(x=x_n, \theta=\theta_0)} \\ \delta_\theta^z &= \frac{\partial g}{\partial x} \Big|_{(x=x_n, \theta=\theta_0)} x_\theta + \frac{\partial g}{\partial \theta} \Big|_{(x=x_n, \theta=\theta_0)} \\ x_\theta(0) &= 0.\end{aligned}\quad (12)$$

From (11) and based on Taylor's theorem (see also [9]), the condition  $z(t) \in \mathcal{C}_z$  for all  $\theta \in \Theta$  can be replaced by the following condition:

$$z_n(t) + (\delta_\theta^z(t))^T(\theta - \theta_0) + M\|\theta - \theta_0\|^2 \mathcal{B} \subset \mathcal{C}_z \quad \forall \theta \in \Theta, \quad (13)$$

where  $\mathcal{B}$  is the unit ball. If  $M > 0$  is sufficiently large and (13) holds for  $0 \leq t \leq T_s$  then it implies  $z(t) \in \mathcal{C}_z$  for  $0 \leq t \leq T_s$ . Since the left hand side of the inclusion (13) is quadratic in  $\theta$ , (13) can be easily verified with computations

provided  $\mathcal{C}_z$  and  $\Theta$  have simple representations (such as parallelotopes or ellipsoids). To even further simplify the computations the term  $M\|\theta - \theta_0\|^2 \mathcal{B}$  may be replaced by  $M \max_{\theta \in \Theta} \|\theta - \theta_0\|^2 \mathcal{B}$ , but this may lead to a more conservative reference governor performance.

The robust reference governor based on (13) is applied to the oxygen starvation protection problem with  $\theta = [\theta_1, \theta_2]^T, \theta_1 = \phi_{sm}, \theta_2 = p_{sat}$ . The relative humidity in the supply manifold,  $\phi_{sm}$  is assumed to have uncertainty up to 50%, while the vapor saturation pressure, which is highly dependent on the temperature, can vary up to  $25kPa$  (this corresponds to about  $10^\circ C$  change in the temperature if the stack is operating around  $80^\circ C$ ).

The implementation of the robust load governor requires simulating the original plant model with the nominal parameters (for  $\lambda_n(t)$  and  $m_{O_2n}(t)$ ) and the four sensitivity functions of  $\frac{\partial \lambda}{\partial \phi_{sm}}, \frac{\partial \lambda}{\partial p_{sat}}, \frac{\partial m_{O_2,ca}}{\partial \phi_{sm}}, \frac{\partial m_{O_2,ca}}{\partial p_{sat}}$  over the time horizon of  $T_s = 5sec$ . The sensitivity functions are generated around the nominal trajectory  $\lambda_n(t)$  and  $m_{O_2n}$  using the linearized model (12). The linearized model for the fuel cells system was obtained using automated symbolic differentiation programmed with *Matlab*. The error between the nonlinear and the linearized system is taken into account through the incorporation of the  $M$ -term in (13).

**Remark 3:** The constant  $M$  in (13) can be estimated analytically or numerically. Analytical estimation of  $M$  requires the evaluation of the second-order sensitivity functions, and it could be very tedious when multiple uncertain parameters are involved. In our implementation,  $M$  was treated as a calibratable parameter, and was tuned until the constraints were satisfied for the worst case.

**Remark 4:** If the set  $\Theta$  has a large diameter, it may be partitioned as  $\Theta = \bigcup_{i=1, \dots, N} \Theta^i, N \geq 1$ . Given  $\theta_0^i \in \Theta^i, i = 1, \dots, N$ , the condition (13) can be replaced by the following  $N$  conditions:

$$\begin{aligned}z_n^i(t) + (\delta_\theta^{z,i}(t))^T(\theta - \theta_0^i) + M^i\|\theta - \theta_0^i\|^2 \mathcal{B} \subset \mathcal{C}_z \\ \forall \theta \in \Theta^i.\end{aligned}\quad (14)$$

Here  $z_n^i(t)$  is the  $i$ th nominal trajectory of the constrained variables corresponding to  $\theta = \theta_0^i$ ,  $(\delta_\theta^{z,i}(t))$  is the sensitivity along the state trajectory corresponding to  $\theta = \theta_0^i$ , and  $M^i > 0$  is a sufficiently large constant. If the sets  $\Theta^i$  have small diameters then small  $M^i$  suffice so that the conservatism of the reference governor based on (13) can be mitigated. On the other hand, multiple on-line simulations of the nominal model and of the linearized system are needed, which increases the computational overhead.

**Remark 5:** We subsequently assume that the state of the fuel cells system is known. If, in reality, the state of the fuel cells system can only be estimated with a known error bound, (e.g., it is only known that  $x(0) \in X$ ), (13) can be modified to

$$\begin{aligned}z_n(t) + (\delta_\theta^z(t))^T(\theta - \theta_0) + (\delta_{x_0}^z(t))^T(x(0) - x_0) \\ + (M_1\|\theta - \theta_0\|^2 + M_2\|x - x_0\|^2) \mathcal{B} \subset \mathcal{C}_z, \quad \forall \theta \in \Theta,\end{aligned}\quad (15)$$

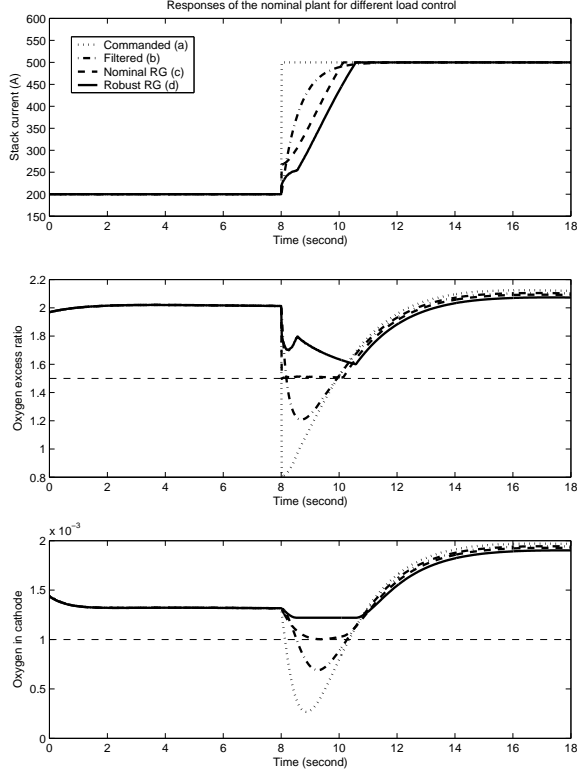


Fig. 2. Comparison of different load governors and their performance when applied to the nominal model.

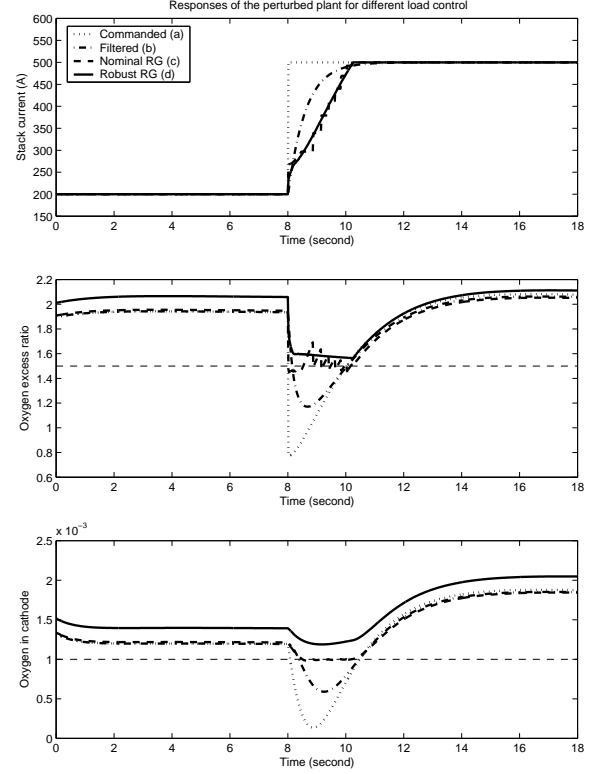


Fig. 3. Comparison of different load governors and their performance when applied to a perturbed model.

where  $(\delta_{x_0}^z(t))^T$  is the initial condition sensitivity function computed using matrix differential equations similar to (12), while  $M_1 > 0$  and  $M_2 > 0$  are sufficiently large and  $x_0 \in X$ .

## V. SIMULATION RESULTS AND COMPARISON

Both the nominal load governor (described in Section III) and the robust load governor (developed in Section IV) are now applied to the fuel cells model. The simulated steps represent a 150% change in the load demand. Simulation results are summarized and analyzed in this section, and compared to the conventional load filtering approaches.

Figure 2 shows the responses of the oxygen excess ratio and cathode oxygen mass for the system with the nominal parameter values, to (a) the commanded current; (b) the modified current with a 1st-order filter; (c) the modified current with a nominal load governor; and (d) the modified current with a robust load governor. The constraints for the oxygen excess ratio and cathode oxygen mass are satisfied by both load governors (c) and (d), while the same constraints are violated for the other two cases, when no load governor is used or when the first order filter is used. The time constant of  $0.6\text{sec}$  for the filter is selected based on the requirement of the load response time, obtained from a representative torque response of a comparable gasoline engine.

Figure 3 shows the results when the same load change

is applied to the system with perturbed parameters. In this case, both constraints are violated if no load governor is used. The nominal reference governor (c), which is designed for  $\theta_0$  without checking constraint violation conditions for other values of  $\theta \in \Theta$ , leads to intermediate steps during the transient when it finds no feasible solution for  $\beta$  and therefore  $\beta$  is set to 0. This results in the jittering trajectory as shown in Figure 3. Meanwhile, the constraints are slightly violated. The robust load governor (d), on the other hand, gives a smooth trajectory while satisfying all the constraints. The tracking performance for both the regular and the robust load governors, when applied to the perturbed system, is similar and therefore the tracking performance is not compromised in this case for achieving system robustness.

One could argue that the time constant for the first order filter can be tuned to satisfy the constraints if so desired. Figure 4 shows the results when the time constant is adjusted to meet the constraints for the nominal plant ( $\tau = 1.4\text{sec}$ ) and to the perturbed plant ( $\tau = 2.0\text{sec}$ ). The load response in both cases is compared to the reference governor approach. The results are also summarized in Table 1. Since the first order filter involves no feedback, the robustness of constraint enforcement property has to be satisfied by a conservative filter with slow time constant, which will necessarily lead to compromised tracking performance, as shown in Table 1.

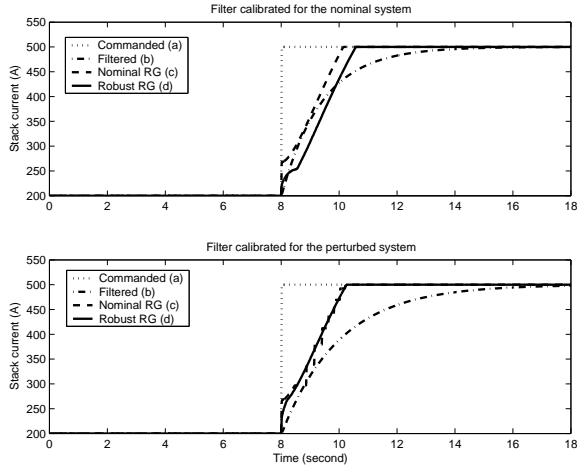


Fig. 4. Comparison of different load governors and their performance when applied to a perturbed model, when the filter time constant is calibrated to satisfy constraints for the nominal plant and the perturbed plant respectively.

TABLE I  
PERFORMANCE OF DIFFERENT LOAD GOVERNORS.

		Constraints satisfied?		Tracking performance	
		$P_0$	$P_\Delta$	$P_0$	$P_\Delta$
No governor		NO	NO	0	0
With filter	$\tau = 0.6$	NO	NO	1	1
	$\tau = 1.4$	Yes	NO	2.336	2.336
	$\tau = 2.0$	Yes	Yes	3.366	3.366
Nominal LG		Yes	NO	1.698	1.953
Robust LG		Yes	Yes	2.800	1.917

**Note:** (1)  $P_0$  and  $P_\Delta$  represents the nominal and perturbed plants respectively. (2) The tracking performance is measured by the integral of squares of the load tracking error, i.e.,  $\int_0^T (I_{st} - I_d)^2 dt$ . For comparison, the numbers shown in this table are normalized by the performance achieved by a first order filter with  $\tau = 0.6$ .

**Remark 6:** The reference governor can be also viewed as a nonlinear filter, whose time constant is adjusted as a function of system states. As such, the reference governor adjusts the input to the fuel cells system only when it is necessary, i.e., when constraint violation is predicted over the horizon. As a consequence of the state-feedback, the performance of the load governor will change as the underlying plant changes, as shown in Table 1. This is in contrast to the linear filter case when the same filtering mechanism is applied indiscriminately to all incoming commands. In particular, the response to even small load changes when no danger of constraint violation exists will be compromised. When the same constraint enforcing performance is achieved, the reference governor out-performs the first order filter in terms of load tracking, due to its built-in optimization function. This benefit is achieved at the cost of a moderate computational effort.

**Remark 7:** When on-board computational resources are limited and the application of real-time optimization is prohibitive, the load governor developed in this paper can also be used off-line as a calibration tool to help optimize other load control strategies, such as gain scheduled or time-

varying filters. From Figure 4, one can see that the response of the filter that satisfies the constraints should not rise faster than the reference governor for this application. This can be used as a guideline in selecting the filter time constant.

**Remark 8:** The load governor, like any other governance mechanism, will slow down the system response in order to deal with tight and active constraints. If the tracking performance is as imperative as the constraint enforcement requirement, then a compromise has to be made, or an additional power source, such as a battery or a super capacitor, has to be incorporated to meet both the load demand and the constraints. In this case, the reference governor can provide the information on the power deficit of the fuel cells systems, thus offer the guidelines for the sizing of the auxiliary power unit.

**Remark 9:** When the parametric uncertainties are deterministic, the conservativeness of the robust load governor can be reduced by combining the robust load governor with an on-line parameter identification.

## VI. CONCLUSION

In this paper, we applied the reference governor to the fuel cell oxygen starvation protection problems. The main advantage of the proposed scheme is that it enforces constraints with a minimum impact on system response time. The robust load governor, which takes into account the parametric uncertainties in the plant model, has shown robust performance with considerable parameter variation. The algorithm requires more computational resource for on-line implementation than a simple filter. On the other hand, it provides guaranteed constraint enforcement and improved load tracking performance.

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