Proceeding of the 2004 American Control Conference
Boston, Massachusetts June 30 - July 2, 2004

GMV and Restricted-Structure GMV Controller Performance Assessment – Multivariable Case

Pawel Majecki* and Michael J. Grimble

Abstract— **The application of control loop performance assessment and benchmarking techniques to multivariable industrial process control loops is considered. The results for assessing the performance of Multiple-Input, Multiple-Output (MIMO) control loops against the Generalized Minimum Variance (GMV) benchmark, using routine operating data and the knowledge of the interactor matrix, are presented. Then, assuming knowledge of the system model, the optimal controller is restricted to be of a low-order classical structure so that a more realistic benchmark is obtained. The technique may also be used to determine the best structure to use for a multivariable controller. This paper presents an extension of the existing results to the cases of multivariable data-driven GMV benchmarking and multivariable model-based RS-GMV benchmarking.**

I. INTRODUCTION

Minimum variance criteria have been used in stochastic performance assessment since the subject of control loop benchmarking was introduced by Harris [10]. The later research by Desborough and Harris [2] and Stanfelj et al. [16] built on this work, showing how time-series analysis can be used to estimate the minimum achievable variance of the controlled variable from routine operating data, and defining the "controller performance index" as the ratio of the minimum achievable variance to the actual variance. This early work was focused mostly on assessing SISO control loops against the Minimum Variance (MV) controller.

The "Generalized Minimum Variance" criterion (derived by Clarke and Hastings-James [1] and re-derived by Grimble [6] using an unconditional cost function) addressed some of the problems related with the MV control (aggressive control action, poor robustness) by considering a combination of the weighted error and control signals. The GMV benchmarking results for the scalar case were later presented in Grimble [8]. In the first part of this paper, these results are extended to multivariable systems, following the approach used by Huang and Shah [12] for the case of multivariable MV benchmarking. Essentially, we pose the GMV problem as a generalized MV problem and then make use of the results derived by these authors.

In practice, industrial processes are usually controlled by multi-loop PI/PID controllers, without taking into account interactions existing between loops. A question then arises how to adequately interpret the value of the calculated performance index: is it so low because the controller is poorly tuned or simply because it is not possible to obtain a better result with the existing controller structure? A natural solution may be to pre-specify the admissible controller structure in which case the optimal benchmark is actually achievable by the existing controller. The second part of the paper describes such a "restricted-structure" (RS) GMV benchmarking algorithm. It must be stressed here that while the RS benchmarking procedure provides a more meaningful performance assessment, it also requires full information of the process model – system identification is therefore an important step.

The paper is organized as follows: in Section 2, the system description and the GMV cost function are introduced; in Section 3, the optimal multivariable GMV controller is derived; Section 4 deals with the estimation of the controller performance index from routine operating data; Section 5 outlines the algorithm for restricted-structure GMV benchmarking; Section 6 includes a simulated example of the application of the presented benchmarking techniques to a simple multivariable plant; the paper closes with conclusions in Section 7.

II. STOCHASTIC SYSTEM DESCRIPTION AND GMV PERFORMANCE CRITERION

The discrete-time system of interest is a multivariable stochastic feedback control loop as shown in Fig. 1, and the plant, disturbance and reference left matrix fractions are defined as:

$$[W \quad W_d \quad W_r] = A^{-1}[B \quad C_d \quad E] \tag{1}$$

The multivariable plant W may in general have m inputs and r outputs and is controlled by an $m \times r$ linear controller $C_0 = C_{0n}C_{0d}^{-1}$. Both the reference and disturbance signals are modeled as outputs from linear dynamic systems excited by zero-mean independent white-noise vector sequences of unity covariance matrix.

A. Interactor matrix

The delay structure of the multivariable plant has a direct effect on the minimum achievable variance and is characterized by the so-called *interactor matrix* ([17], [3]).

Theorem 2.1: [12]. For any $r \times m$ proper, rational polynomial transfer-function matrix T, there exists a non-singular $r \times r$ polynomial matrix D, such that

(i) $\det[D(z)] = z^n$

(ii) $\lim_{z^{-1}\to 0} D(z) \cdot T(z^{-1}) = K,$ K finite and full rank

This work is supported by EPSRC Grant GR/R65800/01

* Corresponding author: p.majecki@strath.ac.uk

The authors are with Industrial Control Center, Department of Electrical Engineering University of Strathclyde, Glasgow, G1 1XQ, The United Kingdom

where n is the number of infinite zeros of T. The matrix D is defined as the interactor matrix and D^{-1} is called the inverse interactor matrix. □

The interpretation of the interactor is that it is a part of the transfer-function matrix that cannot be modified by state feedback [14] and therefore constitutes a fundamental performance limitation in the system. This is equivalent to the role that the time delay plays in scalar systems; however, the multivariable case is normally more complex due to interactions between loops.

In the simple case, the interactor may have a diagonal structure: $D = diag(z^{k_1}, z^{k_2}, ..., z^{k_r})$ where k_i is the minimum delay between all the inputs and output i. If all the delays are equal, then the simple interactor matrix is obtained: $D = z^k I_r$ and this is the direct equivalent of the scalar time delay. In general, however, the interactor can be a full matrix.

The knowledge of the interactor matrix is a prerequisite for the proposed controller performance assessment algorithm. In this paper, we assume that the interactor has already been calculated from the plant transfer-function matrix (e.g. using the algorithm given in [15]) or estimated from plant data [12].

The interactor matrix is not unique. A particularly useful form is the *unitary interactor matrix* which satisfies $D^T(z^{-1})D(z) = I$. The important property of a unitary matrix is that it does not change the spectral properties of a filtered signal, i.e. $\|Dx\|_2 = \|x\|_2$. In particular, the variance of the filtered signal remains the same as that of the original signal.

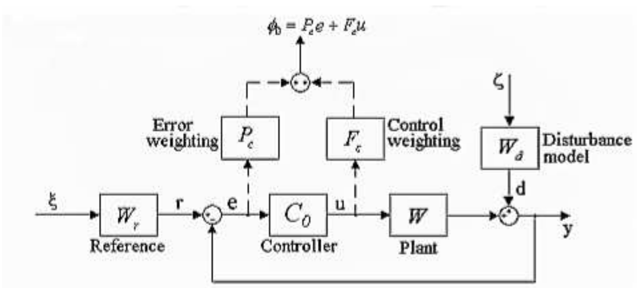

Fig. 1. Single Degree-of-Freedom Closed-Loop System

B. GMV cost function

By analogy with the ”Minimum Variance” control law, the GMV control algorithm can be defined as one that minimizes the following cost function:

$$J = E\left\{\phi_t^T \phi_t\right\} \tag{2}$$

where the “generalized” output signal ϕ_t

$$\phi_t = P_c e_t + F_c u_t \tag{3}$$

is a sum of dynamically weighted error and control signals (see Fig. 1). The multivariable dynamic weighting transfer-function matrices are defined as:

$$P_c = P_{cn} P_{cd}^{-1}, \quad F_c = F_{cn} F_{cd}^{-1} \tag{4}$$

These weightings can be used to frequency-shape the responses produced by the controller.

Unlike the related LQG control law, there is an additional restriction that must be imposed on the selection of the dynamic weightings – they cannot be chosen arbitrarily. The condition that must be satisfied is stated in Theorem (3.1) and ensures the stability of the closed-loop system.

C. Generalized plant

Our approach will be to reformulate the GMV problem as an MV problem for the generalized plant and subsequently reduce it from the general to the simple interactor problem. The expression for the controller error can be written as:

$$e_t = -D^{-1}\tilde{W} u_t + N \in_t \tag{5}$$

where D is the plant interactor matrix, $\tilde{W} = DW$ is the delay-free plant transfer-function matrix, $\in_t$ is a zero-mean, identity-covariance white-noise sequence and N represents the combined effect of all of the stochastic inputs.

The generalized output (3) can now be rewritten as follows:

$$\begin{aligned}\phi_t = P_c e_t + F_c u_t &= P_c(-D^{-1}\tilde{W} u_t + N \in_t) + F_c u_t \\ &= (F_c - P_c D^{-1}\tilde{W}) u_t + P_c N \in_t\end{aligned}$$

Introducing new symbols for the generalized plant: $W_G = D_G^{-1}\tilde{W}_G = F_c - P_c W$ and the disturbance model: $N_G = P_c N$, obtain

$$\phi_t = D_G^{-1}\tilde{W}_G u_t + N_G \in_t \tag{6}$$

which is in a form equivalent to (5). Note that D_G (the unitary interactor matrix of the generalized plant) may in general be different from the original interactor matrix D.

III. Optimal GMV controller and benchmark cost values

In order to simplify the problem, the analysis given in [12] is followed. Consider the filtered signal

$$\tilde{\phi}_t = z^{-k} D_G \phi_t = z^{-k}\tilde{W}_G u_t + \tilde{N}_G \in_t \tag{7}$$

where $\tilde{N}_G = z^{-k} D_G N_G$ and k is the order of D_G (the highest degree of its entries). Because of the unitary property of the interactor D_G, the spectrum of the signal $\tilde{\phi}_t$ is the same as that of ϕ_t and the original problem (2) is thus reduced to minimizing the variance of signal $\tilde{\phi}_t$.

The main step in the derivation of the GMV control law is to split the cost function (2) into two terms, one of which is independent of, and the other dependent on the controller. The control law then results by simply setting that second term to zero.

Splitting the disturbance term $\tilde{N}_G$ into unpredictable and predictable components using the Diophantine identity:

$$\tilde{N}_G = F + q^{-k} R, \tag{8}$$

equation (7) can now be rewritten as:

$$\tilde{\phi}_t = q^{-k}\left(\tilde{W}_G u_t + R \in_t\right) + F \in_t \tag{9}$$

Because the two terms on the right-hand side do not overlap in time, the expression for the optimal controller clearly follows by setting the term in brackets to zero. This gives the optimal control signal:

$$u_t^{opt} = -\tilde{W}_G^{-1} R \in_t \quad (10)$$

and the generalized output under GMV optimal control:

$$\tilde{\phi}_t^{opt} = F \in_t \quad (11)$$

In order to obtain an explicit expression for the controller, combine (10), (11) and (3) to get:

$$u_t^{opt} = -\left(FR^{-1}\tilde{W}_G + q^{-k}D_G F_c\right)^{-1} q^{-k} D_G P_c e_t \quad (12)$$

The minimum value of the cost function (minimum variance of the generalized output ϕ_t) follows from (11) as

$$J_{\min} = Var\left[F \in_t\right] = \sum_{i=0}^{k-1} trace\left(F^T F\right) \quad (13)$$

and depends only on the combined disturbance/stochastic inputs model and the interactor matrix of the generalized plant.

A. Optimal GMV controller – polynomial matrix description.

Equation (12) provides an expression for the optimal full-order GMV controller in terms of transfer-function matrices. However, for the restricted-structure controller design presented in section 5 we will also need a polynomial matrix version of this equation. This may be arranged following the introduction of a few factorizations and identities.

First, recalling the common denominator description in (1), we can write $\tilde{N}_G$ as a left matrix fraction $\tilde{N}_G = A^{-1}D_f$, where the strictly Schur polynomial matrix D_f is the solution of the following spectral factorization problem:

$$D_f D_f^* = EE^* + C_d C_d^* \quad (14)$$

The following left-to-right matrix fraction factorizations are needed in the solution: $P_{cd}^{-1}A^{-1}BF_{cd} = B_1 A_1^{-1}$, $D_f^{-1}AP_{cd} = A_2 D_2^{-1}$, $D_f^{-1}BF_{cd} = B_2 D_3^{-1}$.

The solution *(G,H,F)*, minimum row degree with respect to F, to the following coupled Diophantine equations is also required:

$$FA_2 + z^{-k}G = \tilde{P}_{cn}D_2 \quad (15)$$

$$FB_2 - z^{-k}H = \tilde{F}_{cn}D_3 \quad (16)$$

where $\tilde{P}_{cn} = z^{-k}D_G P_{cn}$ and $\tilde{F}_{cn} = z^{-k}D_G F_{cn}$. The following theorem gives the GMV optimal control law in terms of the polynomial matrices introduced above.

Theorem 3.1 (Multivariable GMV benchmark cost values): The GMV cost function (2) can be decomposed as follows:

$$J = J_{\min} + J_0 \quad (17)$$

where

$$J_{\min} = \frac{1}{2\pi j} \oint_{|z|=1} trace\{F^* F\} \frac{dz}{z} \quad (18)$$

$$J_0 = \frac{1}{2\pi j} \oint_{|z|=1} trace\{T_o^* T_o\} \frac{dz}{z} \quad (19)$$

and

$$T_o = (GD_2^{-1}P_{cd}^{-1}C_{0d} - HD_3^{-1}F_{cd}^{-1}C_{0n})(AC_{0d} + BC_{0n})^{-1}D_f \quad (20)$$

The "implied Diophantine equation" defining the characteristic polynomial matrix of the closed loop system:

$$GD_2^{-1}B_1 + HD_3^{-1}A_1 = D_G(P_{cn}B_1 - F_{cn}A_1) = D_c \quad (21)$$

A necessary condition for stability is therefore that the weighting matrices P_c and F_c be selected so that the polynomial matrix D_c is strictly Schur. □

Corollary 3.1 (Optimal GMV cost): Since J_{min} is independent of the controller parameters, the optimal GMV controller is obtained by simply setting T_0 to zero and the minimum value of the cost function becomes J_{min}. □

B. Selection of the weighting transfer-function matrices

As the properties of the benchmark controller depend on the choice of the dynamic weightings (4), some guidelines are needed to help in their selection. The weightings should ideally reflect the requirements imposed on the control system (regulatory performance, tracking/disturbance rejection, level of robustness) and these normally are process-dependent. Some rules of thumb, however, do exist and may be used as a starting point ([5], [7]).

In general, the frequency dependence of the weightings can be used to weight different frequency ranges in the error and control signals. The usual procedure is that the error weighting P_c normally includes an integral term, which leads to integral action in the controller, and the control weighting F_c is chosen as a constant, or as a lead term to ensure the controller rolls-off in high frequencies and does not amplify the measurement noise. An additional scalar may be used to balance the steady-state variances of the error and control signals.

While selecting the dynamic weightings, one has to be aware of the restriction stated in Theorem 3.1 - this stability condition must be satisfied when actually designing the GMV controller. Note that the benchmarking algorithm will still return a controller performance index even if the condition is not satisfied - this however will involve the assessment against an inadmissible controller, effectively underestimating the controller performance index.

IV. GMV CONTROLLER BENCHMARKING USING ROUTINE OPERATING DATA

Huang and Shah [12] used the Filtering and Correlation (FCOR) algorithm to estimate the minimum achievable output variance from routine operating data. Although only the minimum variance case has been considered by these authors, it is relatively straightforward to extend their results to the GMV case using the MV-GMV equivalence discussed in section 2.

The idea behind the FCOR algorithm is to estimate the coefficients of the matrix polynomial F and then use (13) directly. This can be done by cross-correlating the interactor-filtered generalized output $\tilde{\phi}_t$ with the estimated white noise input (signal $\in_t$), as outlined below.

FCOR algorithm (GMV case)

1) Filter the error and control signals to obtain the generalized output signal:

$$\phi_t = P_c e_t + F_c u_t$$

2) Estimate the interactor matrix of the generalized plant D_G and determine its order k
3) Filter ϕ_t through the interactor matrix D_G to obtain

$$\tilde{\phi}_t = q^{-k} D_G \phi_t$$

4) Estimate the noise vector sequence $\in_t$ (whitening process) – a common approach is to model the output signal as a VAR (Vector Auto Regressive) time series and then filter it to obtain the white noise 'innovations' sequence:

$$\in_t = A(q^{-1})\tilde{\phi}_t$$

5) Compute the cross-correlation between the output and the estimated noise:

$$r_{\tilde{\phi}a}(0) = E[\tilde{\phi}_t \in_t^T] = F_0 \Sigma_\in$$
$$\vdots$$
$$r_{\tilde{\phi}a}(k-1) = E[\tilde{\phi}_t \in_{t-k+1}^T] = F_{k-1}\Sigma_\in$$

where $\Sigma_\in = E\left[\in_t \in_t^T\right]$. The right-hand sides follow from equation (9), and the coefficients of F are thus determined.

6) Calculate the optimum covariance matrix:

$$\Sigma_{mv} = F_0 \Sigma_\in F_0^T + ... + F_{k-1}\Sigma_\in F_{k-1}^T$$

7) Compute the controller performance index:

$$\eta = \frac{trace\,(\Sigma_{mv})}{trace\left(\Sigma_{\tilde{\phi}}\right)} \qquad (22)$$

where $\Sigma_{\tilde{\phi}} = E\left[\tilde{\phi}_t \tilde{\phi}_t^T\right]$.

Another method of estimating the controller performance index, which is closely related to the multiple coefficient of determination R^2, was used by Desborough and Harris [2] in the scalar case and then was extended to multivariable case by Harris et al. [11]. This approach was also used by Grimble [5] in a scalar GMV benchmarking paper.

V. Restricted-structure multivariable GMV controller benchmarking

The GMV benchmark provides a more flexible performance measure than the MV benchmark, however the optimal controller is normally high order and the comparison with a PID controller may therefore be not realistic. This section addresses this question by explicitly restricting the controller structure to that of a filtered PID algorithm:

$$C_0(z^{-1}) = k_0 + \frac{k_1}{1-z^{-1}} + \frac{k_2(1-z^{-1})}{1-\tau_d z^{-1}} \qquad (23)$$

where τ_d is fixed. All elements of the controller transfer-function matrix are restricted to be of this form.

A. The GMV restricted-structure controller benchmark

The multivariable GMV theorem defines the optimal controller which minimizes the part of the cost function J_0 given in (19). The optimal full-order controller sets J_0 to zero – however, when the controller structure is restricted, the minimum value of (19) will generally be nonzero.

The multivariable controller can be represented in the matrix form as

$$C_0 = K_0 + K_1\frac{1}{1-z^{-1}} + K_2\frac{(1-z^{-1})}{1-\tau_d z^{-1}} \qquad (24)$$

where $K_n = \{k_{ij}^n; \quad i = 1..m; j = 1..r\}$, and equivalently in the right matrix fraction form as $C_0 = C_{0n}C_{0d}^{-1}$.

The basic idea is to use a parametric optimization algorithm to minimize the cost (19) with respect to the controller parameters; then (24) gives the formula for the optimal PID controller. Backsubstituting into the cost function and comparing with the value obtained for the existing controller results in the controller performance index.

B. Solution of the parametric optimization problem

The optimization algorithm is a direct generalization from the SISO case [5] and involves representing the integral (19) in the frequency domain:

$$\underset{C_0}{Min}\int_0^{2\pi} trace\{T_o(e^{j\omega})T_o(e^{-j\omega})\}d\omega \qquad (25)$$

This nonlinear optimization problem has to be solved numerically. One approach is to assume that the "denominator" matrix $(AC_{0d} + BC_{0n})$ is known from the previous iteration and then perform the least-squares minimization, approximating the integral in (25) with a summation. The optimization is with respect to the parameters appearing linearly in the "numerator" matrix, therefore such a problem can be easily solved. This step is iterated a number of times, and the whole procedure is known as a "successive approximation" algorithm. The details of the algorithm can be found in [9] and [13], and its basic steps are summarized in Fig. 2. A similar algorithm for the continuous-time LQG case has been presented in [4]. Our experience shows that the successive approximation algorithm is very fast at finding the minimum - usually it takes less than 7 iterations to converge to the optimal parameters, even if the cost value for the initial point is not bounded. However, as there is no convergence proof for this algorithm, we used it in combination with the Quasi-Newton gradient algorithm, which is guaranteed to converge although it requires a stabilizing initial condition.

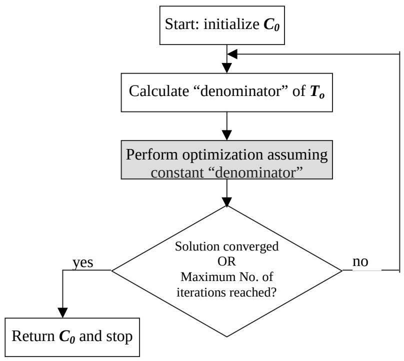

Fig. 2. Successive Approximation Algorithm

The nominal solution assumes that the controller has a full structure with all the elements of the matrix being PID controllers. However, restricting the structure (forcing some of the parameters to be zero) is relatively straightforward and does not lead to major modifications of the algorithm. The additional constraint is of course that the closed-loop system must be stable and this has to be verified e.g. by calculating the closed-loop system poles.

C. Calculation of the controller performance index

The previous section described the algorithm for computing the optimal restricted-structure controller. To benchmark the existing controller, there are a few steps remaining:

1) Calculate the minimum value of (19) corresponding to the optimal RS controller (J_0^{opt})
2) Determine (19) for the actual controller (J_0^{act})
3) Calculate the "Controller Performance Index":

$$\kappa = \frac{J_{\min} + J_0^{opt}}{J_{\min} + J_0^{act}} \qquad (26)$$

with J_{min} calculated from (18).

The controller performance index lies in the range $0 \ldots 1$, where "1" (optimum performance) is actually achievable within the existing controller structure.

An interesting question that arises is the potential benefit that may result from adding additional controllers into the original multi-loop system. When this benefit appears to be significant (relative to the estimated costs), then the tuning parameters are readily available from the optimization step.

VI. SIMULATED EXAMPLE

The example that we use for illustration comes from [12] and involves a 2×2 system with time delays. First, we show how to estimate the MV and GMV benchmark values from sampled data, and then demonstrate the potential of the restricted-structure design and benchmarking.

The process is described by equation (5), with the plant and disturbance transfer-function matrices given as:

$$W = \begin{bmatrix} \frac{z^{-1}}{1-0.4z^{-1}} & \frac{K_{12}z^{-2}}{1-0.1z^{-1}} \\ \frac{0.3z^{-1}}{1-0.1z^{-1}} & \frac{z^{-2}}{1-0.8z^{-1}} \end{bmatrix}$$

$$N = \begin{bmatrix} \frac{1}{1-0.5z^{-1}} & \frac{-0.6}{1-0.5z^{-1}} \\ \frac{0.5}{1-0.5z^{-1}} & \frac{1}{1-0.5z^{-1}} \end{bmatrix}$$

Both set-points are assumed to be zero (pure regulatory control). The parameter K_{12} determines the level of interaction between input 2 and output 1 and varies from 0 (no interaction) to 1 (large interaction). Irrespective of K_{12}, the plant has a general interactor matrix

$$D = \begin{bmatrix} -0.9578z & -0.2873z \\ 0.2873z^2 & -0.9578z^2 \end{bmatrix}$$

A. Estimation of the minimum variance from operating data

In this subsection we assume only the knowledge of the generalized plant interactor matrix. The existing controller is the multi-loop minimum variance controller C_0 calculated for the two single loops without considering the interactions.

Two choices of dynamic weighting functions have been considered in this example:

Case (1): Minimum variance weightings:

$$P_c = I_2, \quad F_c = \mathbf{0}$$

Case (2): GMV static weightings:

$$P_c = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad F_c = -D^{-1} \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix}.$$

Parameters R_1 and R_2 determine the relative importance attached to both control variances with respect to each other and to the error variances. In this example they have been fixed to $R_1 = 3$ and $R_2 = 5$. Also note that the weighting F_c has been premultipled by D^{-1} and in this case the interactor matrix D_G simply equals D.

The system has been simulated for different values of K_{12} and 5000 samples of the error and control signal have been collected in each case. Then the FCOR algorithm has been applied to the interactor-filtered generalized output and the benchmark cost evaluated. The controller performance index (CPI) has also been calculated using (22). The comparison of the error and control variances for the optimal MV and GMV controllers is presented in Table I, and the benchmarking results are shown in Figure 3.

Comments

- Although simplistic, the static GMV weightings do provide a means of defining a benchmark with balanced error and control variances. In this particular example, the plant becomes non-minimum phase for $K_{12} > 2$, resulting in the unbounded minimum variance control. The introduction of the control weighting makes the benchmark realizable.
- As expected, the performance of the multi-loop MV controller is close to 1 for small interactions between loops and decreases when these interactions increase.
- The existing controller, however, is not so good if the control variances are also of importance. This implies that detuning the controller may be needed in practice.

TABLE I

ERROR AND CONTROL VARIANCES

K_{12}	Controller	Var[e_1]	Var[e_2]	Var[u_1]	Var[u_2]
1	MV	1.387173	1.551406	0.798925	0.302076
	GMV	2.824503	2.113715	0.182249	0.014378
5	MV	1.389516	1.551405	Inf	Inf
	GMV	3.230315	1.914557	0.207795	0.004998
10	MV	1.40381	1.551394	Inf	Inf
	GMV	3.387025	1.878119	0.24094	0.002577

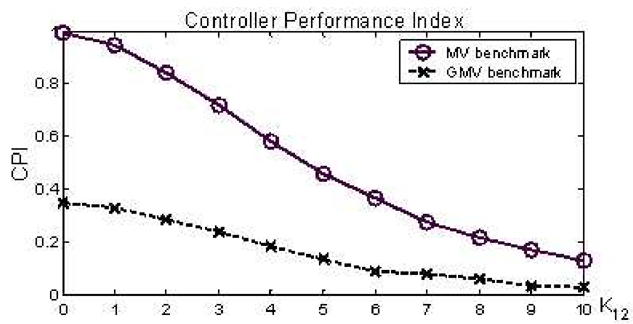

Fig. 3. MV and GMV Controller Performance Indices

B. Restricted-structure controller design and benchmarking

We will now illustrate the technique of restricted-structure design described in Section 5. The low-order controllers are assumed to be of the filtered PID structure with the derivative filter "time constant" $\tau_d = 0.5$. In order to compare "like with like", the existing controller will also be of the above type. It will be a multi-loop PID controller tuned using Ziegler-Nichols rules separately for two loops. This controller has been assessed against the full-order GMV controller, the optimal full-structure PID, optimal diagonal PID and two optimal triangular PID's. The GMV weightings have been the same as in the previous subsection. The values of the suboptimal term J_0 of the cost function have been calculated for all these different configurations and compared with the value obtained for the existing controller. The results are collected in Table 2. From the above results it is possible to tune the existing

TABLE II

RS BENCHMARKING RESULTS

Controller	Multi-loop PID	RS full	RS diag.
J_0	2.802634	0.015425	0.085873
Controller	RS diag. 2	RS upper trian.	RS lower trian.
J_0	0.502203	0.03569	0.065818

PID controllers "optimally" (in terms of the specified cost function) or predict how additional controllers would affect the performance of the system. This can be used as an indication of the profitability of the possible investment. In our case, it is clear that introducing feedback between output 1 and input 2 (rather than between output 2 with input 1) would bring greater improvement. This simple example thus illustrates the potential of the technique in analyzing structure, pairing the input-output variables and as a tuning guidance.

VII. CONCLUSIONS

Some aspects of multivariable controller performance assessment, using the GMV controller as a benchmark, have been presented. First, the multivariable optimal GMV controller has been derived using the concept of the generalized plant and its interactor matrix. The algorithm was then given for the data-driven controller performance assessment against the GMV benchmark. Finally, the restricted-structure version of the benchmarking algorithm has been presented. The RS algorithm given here is simple to apply, given the model of the system and its application for structure assessment, controller tuning and benchmarking have been indicated. A simple example has also been presented to illustrate the discussed results.

REFERENCES

[1] D.W. Clarke and R. Hastings-James. Design of digital controllers for randomly disturbed systems. *IEE Proceedings on Control Theory and Applications*, Vol.118:pp.1503–1506, 1971.
[2] L. Desborough and T. Harris. Performance assessment measures for univariate feedback control. *The Canadian Journal of Chemical Engineering*, Vol.70:pp.1186–1197, 1992.
[3] G. Goodwin and K. Sin. *Adaptive Filtering, Prediction and Control*. Prentice-Hall, 1984.
[4] D. Greenwood and M. Johnson. Multivariable controller benchmarking using lqg optimal control and a steel industry application. *Proceedings of the European Control Conference*, 2003.
[5] Michael J. Grimble. Restricted structure controller tuning and performance assessment. *IEE Proceedings on Control Theory and Applications*, Vol.149:pp.8–16, 2002.
[6] M.J. Grimble. Generalized minimum variance control law revisited. *Optimal Control Applications & Methods*, Vol.9:pp.63–77, 1988.
[7] M.J. Grimble. *Industrial Control Systems Design*. John Wiley, 2001.
[8] M.J. Grimble. Controller performance benchmarking and tuning using generalized minimum variance control. *Automatica*, Vol.38:pp.2111–2119, 2002.
[9] M.J. Grimble. Restricted structure control loop performance assessment for pid controllers and state-space systems. *Asian Journal of Control*, Vol.5(1):pp.39–57, 2003.
[10] T.J. Harris. Assessment of control loop performance. *The Canadian Journal of Chemical Engineering*, Vol.67:pp.856–861, 1989.
[11] T.J. Harris, F. Boudreau, and J.F. MacGregor. Performance assessment of multivariable feedback controllers. *Automatica*, Vol.32:pp.1505–1518, 1996.
[12] B. Huang and S.L. Shah. *Performance assessment of control loops: theory and applications*. Springer Verlag, Berlin, 1999.
[13] J. Maciejowski. *Multivariable Feedback Design*. Addison-Wesley, 1989.
[14] Y. Mutoh and R. Ortega. Interactor structure estimation for adaptive control of discrete-time multivariable nondecouplable systems. *Automatica*, Vol.29:pp.635–647, 1993.
[15] M.W. Rogozinski, A.P. Paplinski, and M.J. Gibbard. An algorithm for the calculation of a nilpotent interactor matrix for linear multivariable systems. *IEEE Transactions on Automatic Control*, Vol.32:pp.234–237, 1987.
[16] N. Stanfelj, T.E. Marlin, and J.F. MacGregor. Monitoring and diagnosing process control performance: the single-loop case. *Industrial & Engineering Chemistry Research*, Vol.32:pp.301–314, 1993.
[17] W.A. Wolovich and P.L. Falb. Invariants and canonical forms under dynamic compensation. *SIAM Journal of Control and Optimization*, Vol.14:pp.996–1008, 1976.