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Robust Adaptive Control for a Class of Nonlinear Uncertain Neutral Delay Systems

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***Abstract*— This paper focuses on the adaptive robust control for a class of nonlinear uncertain neutral delay systems. All uncertainties are assumed to be bounded by unknown constants, and an adaptive law constructed by making use of 1-norm of matrix is proposed to estimate these unknown constants. An adaptive controller is designed such that the solutions of resulting closed-loop system are uniformly ultimately bounded, moreover the original system state is asymptotically stable. Finally, a numerical example is given to illustrate the effectiveness of the proposed approach.**

I. INTRODUCTION

Dynamic systems often have time-delays in the processing state, input or related variables. In addition, there are many control systems having not only delay in the state but also delay in the state derivatives. Such dynamic systems are commonly referred to neutral delay systems [1,2]. The practical examples of neutral delay systems include the distributed networks containing lossless transmission lines, and population ecology and so on [3,4]. Because the delay effect is a common source of instability, it is of great importance to study these systems in theory as well as in practical application. During the past two decades, considerable attention has been paid to the research on the control of delay neutral systems, especially on the issue of stability and stabilization [5-9].

In general, the information of the upper bound of uncertainties is required. However, it is not an easy work to obtain such a knowledge in practice because of the complexity of the uncertainties. Therefore, adaptive control law should be introduced to estimate the unknown bounds [13,14]. In [13], dynamic systems containing uncertainties are addressed and the adaptive control law are presented. In [14], the problem of robust stabilization for dynamical systems with multiple delayed state perturbation is considered, and an adaptive law is proposed to estimate unknown gains. But to our best knowledge, for nonlinear uncertain neutral delay systems with the unknown bounds of uncertainties, the related results have not been reported yet in the control literature.

In this paper, we consider the problem of the adaptive robust control for a class of nonlinear uncertain neutral delay systems. Our purpose is to develop an adaptive controller such that the solutions of resulting closed-loop system are uniformly ultimately bounded, moreover the original system state is asymptotically stable. For the purpose, we first propose an adaptive law to estimate the unknown upper bounds , which is constructed by 1-norm of matrix and different from the method seen before. Then by making use of the updated value of these unknown bounds, we construct a memoryless state Feedback controller to solve the problem.

The organization of these paper is as follows. In section 2, some standard assumptions are introduced and the problem to be tackled is stated. Section 3 gives the main results. In section 4, a numerical example is given to illustrate the effectiveness of the proposed method.

In this paper, $\|x\|_1$ denotes 1-norm of vector $x \in R^n$, i.e.$\|x\|_1 = \sum_{i=1}^{n} |x_i|$; $\|A\|_1$ denotes the deduced matrix 1-norms. Use $\|\bullet\|$ to stand for either Euclidean norm of vectors or the deduced matrix 2-norm.

II. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

Consider the nonlinear uncertain neutral delay system described by

$$\begin{aligned} \dot{x}(t) &= (A+\Delta A)x(t) + (A_h + \Delta A_h)x(t-h) \\ &\quad + A_d\dot{x}(t-h) + e(t,x,x(t-h)) + Bu(t), \\ x(\theta) &= \varphi(\theta), \quad \theta \in [-h, 0], \end{aligned} \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the state vector, $u(t) \in \mathbf{R}^m$ is the control input.$e(t,x,x(t-h))$ is nonlinear uncertainty, A, A_h, $A_d$are all known constant matrices with appropriate dimensions.ΔA and ΔA_hdenote the unknown real-valued functions representing the time-varying parameter uncertainty of the matrices$A$ and A_h respectively. Scalar $h > 0$, $d > 0$ denote the state delay. Let $\tau = max\{h, d\}$. $\varphi(\theta) \in \mathbf{R}^n$ is a continuously differentiable vector-valued initial function on $[-\tau, 0]$.

In this paper, we need the following assumption:

(A1) The pair (A, B) is controllable.

(A2) The uncertain matrices and disturbanceΔA ,ΔA_h and $e(t,x,x(t-h))$ are continuously differentiable in x, and piecewisely continuous in t[10, 18].

(A3) There exist unknown continuous functions with appropriate dimensionA_1, A_2, A_3, [15-18]such that

$$\Delta A = BA_1, \Delta A_h = BA_2, e(t,x,x(t-h)) = BA_3.$$

Remark1: These so-called matching conditions restrict the applicability of the proposed control scheme. However, these class of systems have important applications, for example, robotic systems [17].

(A4) There exist unknown positive scalars g_0, g_1 and g_2 such that

$$\|A_1x + A_2x(t-h) + A_3\| \leq g_0 + g_1\|x\| + g_2\|x(t-h)\|.$$

Let $f = A_1x + A_2x(t-h) + A_3$, then system (**??**) can be rewritten as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_hx(t-h) + A_d\dot{x}(t-h) + Bf + Bu(t), \\ x(\theta) &= \varphi(\theta), \quad \theta \in [-h, 0], \end{aligned} \qquad (2)$$

The robust adaptive control problem is to design an adaptive controller such that the solutions of the closed-loop system (**??**) are uniformly ultimately bounded, particularly, the original system state is asymptotically stable.

III. MAIN RESULT

To study the problem of robust adaptive control, we first give Lemma 1.

Lemma 1 [19]. Let$A, M, E, F(t)$ and$P$ be real matrices of appropriate dimensions with $P > 0$ and $F(t)$ satisfies $F(t)^TF(t) \leq I$ Then the following results hold:
1 For any$\varepsilon > 0$,
$MF(t)E + E^TF(t)^TM^T \leq \varepsilon MM^T + \varepsilon^{-1}E^TE$
2. $AF + (AF)^T \leq APA^T + F^TP^{-1}F$.
We use adaptive control technique to get adaptive control gains $\hat{g}_0$, $\hat{g}_1$, $\hat{g}_2$.
Design the adaptive gains by

$$\begin{aligned} \dot{\hat{g}}_0 &= \|\wp^TPB\|_1, \\ \dot{\hat{g}}_1 &= \|\wp^TPB\|_1\|x\|_1, \\ \dot{\hat{g}}_2 &= \|\wp^TPB\|_1\|x(t-h)\|_1, \end{aligned} \qquad (3a)$$

where$\wp$ is difference operator and defined as

$$\wp(\varphi) = \varphi(0) - A_d\varphi(-d),$$

P is positive definite matrix .
The adaptive error of each gain is defined as follows

$$\tilde{g}_i = \hat{g}_i - g_i, (i = 0, 1, 2).$$

Thus

$$\dot{\tilde{g}}_i = \dot{\hat{g}}_i, (i = 0, 1, 2).$$

Let $u_{adp} = -(\hat{g}_0 + \hat{g}_1\|x\|_1 + \hat{g}_2\|x_h\|_1)\text{sign}(B^TP\wp)$,

$$u = -B^TPx + u_{adp}. \qquad (3b)$$

For convenient, we adopt the following notation:
$x_h = x(t-h), x_d = x(t-d), \wp(x(t) = \wp,\ f = f(t, x(t), x(t-h)), \bar{A} = A - BB^TP.$
Now, we give the main result of this paper.

Theorem 1. Consider neutral delay system (2)with the assumption (A1)-(A4).If there exist matrices$P > 0, Q > 0, S > 0$ and matrix N such that the LMI

$$\begin{bmatrix} \eta_0 & \eta_1 & PA_h & 0 \\ \eta_1^T & -M_1 & 0 & A_d^TPB \\ A_h^TP & 0 & -S & 0 \\ 0 & B^TPA_d & 0 & -I \end{bmatrix} < 0 \qquad (4)$$

holds, where

$$\begin{aligned} \eta_0 &= PA + A^TP + S + Q + NBB^TN^T - \\ &\quad NBB^TP - PBB^TN^T, \\ \eta_1 &= (S + Q + PA)A_d, \\ M_1 &= Q - A_d^T(S + Q)A_d. \end{aligned}$$

Then the solutions of closed -loop system (2) -(3) are uniformly ultimately bounded for any of the delays h and d. Moreover, the original system state converges to zero, that is $\lim_{t \longrightarrow \infty} x(t) = 0$.

Proof. it is easy to show from (4) that

$$M_1 = Q - A_d^T(S + Q)A_d > 0.$$

Therefore

$$A_d^TQA^Td - Q < 0,$$

thus the operator$\wp$ is stable.

Consider the following Lyapunov-Krasovskii candidate function

$$\begin{aligned} V(x_t, \tilde{g}_0, \tilde{g}_1, \tilde{g}_2) &= \wp^TP\wp + \textstyle\int_{-h}^0 x^T(t+\theta)Sx(t+\theta)d\theta \\ &\quad + \textstyle\int_{-d}^0 x^T(t+\theta)Qx(t+\theta)d\theta \\ &\quad + \tilde{g}_0^2 + \tilde{g}_1^2 + \tilde{g}_2^2, \end{aligned} \qquad (5)$$

where

$$x_t = x(t+\theta), \theta \in [-\tau, 0].$$

Differentiating $V(x_t, \tilde{g}_0, \tilde{g}_1, \tilde{g}_2)$ along the solution of (2) and (3) results in

$$\begin{aligned}
&\dot{V}(x_t, \tilde{g}_0, \tilde{g}_1, \tilde{g}_2) \\
&= 2\wp^T P\dot{\wp} + x^T(S+Q)x - x_h{}^T S x_h - x_d{}^T Q x_d \\
&\quad +2(\tilde{g}_0\dot{\tilde{g}}_0 + \tilde{g}_1\dot{\tilde{g}}_1 + \tilde{g}_2\dot{\tilde{g}}_2) \\
&= 2\wp^T P[Ax + A_h x_h - BB^T Px \\
&\quad -Bsign(B^T P\wp)(\hat{g}_0 + \hat{g}_1\|x\|_1 + \hat{g}_2\|x_h\|_1) + Bf] \\
&\quad +x^T(S+Q)x - x_h{}^T S x_h - x_d{}^T Q x_d \\
&\quad +2(\tilde{g}_0\dot{\tilde{g}}_0 + \tilde{g}_1\dot{\tilde{g}}_1 + \tilde{g}_2\dot{\tilde{g}}_2) \\
&\le 2\wp^T P(\bar{A}x + A_h x_h) \\
&\quad -2\|\wp^T PB\|_1(\hat{g}_0 + \hat{g}_1\|x\|_1 + \hat{g}_2\|x_h\|_1) \\
&\quad +2\|\wp^T PB\|(g_0 + g_1\|x\| + g_2\|x_h\|) \\
&\quad +x^T(S+Q)x - x_h{}^T S x_h - x_d{}^T Q x_d \\
&\quad +2[(\hat{g}_0 - g_0)\dot{\hat{g}}_0 + (\hat{g}_1 - g_1)\dot{\hat{g}}_1 \\
&\quad +(\hat{g}_2 - g_2)\dot{\hat{g}}_2] \\
&\le 2\wp^T P(\bar{A}x + A_h x_h) \\
&\quad -2\|\wp^T PB\|_1(\hat{g}_0 + \hat{g}_1\|x\|_1 + \hat{g}_2\|x_h\|_1) \\
&\quad +2\|\wp^T PB\|_1(g_0 + g_1\|x\|_1 + g_2\|x_h\|_1) \\
&\quad +x^T(S+Q)x - x_h{}^T S x_h - x_d{}^T Q x_d \\
&\quad +2[(\hat{g}_0 - g_0)\dot{\hat{g}}_0 + (\hat{g}_1 - g_1)\dot{\hat{g}}_1 \\
&\quad +(\hat{g}_2 - g_2)\dot{\hat{g}}_2] \\
&= 2\wp^T P[\bar{A}(x - A_d x_d) + \bar{A}A_d x_d + A_h x_h] \\
&\quad +(\wp + A_d x_d)^T(S+Q)(\wp + A_d x_d) \\
&\quad -x_h{}^T S x_h - x_d{}^T Q x_d \\
&= \wp^T(P\bar{A} + \bar{A}^T P)\wp + 2\wp^T P(\bar{A}A_d x_d + A_h x_h) \\
&\quad +\wp^T(S+Q)\wp + 2\wp^T(S+Q)A_d x_d \\
&\quad +x_d^T A_d^T(S+Q)x_d A_d - x_h{}^T S x_h - x_d{}^T Q x_d \\
&= \wp^T(P\bar{A} + \bar{A}^T P + S + Q)\wp \\
&\quad +\wp^T(S + Q + P\bar{A})A_d x_d \\
&\quad -x_h{}^T S x_h - x_d{}^T M_1 x_d + 2\wp^T PA_h x_h \\
&\le \wp^T(P\bar{A} + \bar{A}^T P + S + Q)\wp \\
&\quad +\wp^T(S + Q + P\bar{A})A_d M_1^{-1} A_d^T(S + Q + P\bar{A})^T\wp \\
&\quad +\wp^T PA_h S^{-1} A_h^T P\wp \\
&= \wp^T[P\bar{A} + \bar{A}^T P + S + Q + PA_h S^{-1} A_h^T P \\
&\quad +(S + Q + P\bar{A})A_d M_1^{-1} A_d^T(S + Q + P\bar{A})]^T\wp
\end{aligned}$$

Let

$$\begin{aligned}\Pi \;=\; & P\bar{A} + \bar{A}^T P + S + Q + PA_h S^{-1} A_h^T P \\ & +(S + Q + P\bar{A})A_d M_1^{-1} A_d^T(S + Q + P\bar{A})]^T\end{aligned} \quad (6)$$

By Schur complement, if the following matrix inequality

$$\begin{bmatrix} \Gamma_0 & \Gamma_1 & PA_h \\ \Gamma_1^T & -M_1 & 0 \\ A_h^T P & 0 & -S \end{bmatrix} < 0 \quad (7)$$

holds, where

$$\Gamma_0 = PA + A^T P - 2PBB^T P + S + Q,$$

$$\Gamma_1 = (S + Q + PA - PBB^T P)A_d,$$

then $\Pi < 0$. By Lemma 1, we have

$$\begin{bmatrix} 0 & -PBB^T PA_d \\ -A_d^T PBB^T P & 0 \end{bmatrix} \le \begin{bmatrix} PBB^T P & 0 \\ 0 & A_d^T PBB^T PA_d \end{bmatrix} \quad (8)$$

Considering (8), We know if the following inequality

$$\begin{bmatrix} \Gamma_2 & \Gamma_3 & PA_h \\ \Gamma_3^T & M_2 & 0 \\ A_h^T P & 0 & -S \end{bmatrix} < 0 \quad (9)$$

holds, where

$$\begin{aligned}
\Gamma_2 &= PA + AP - PBB^T P + S + Q, \\
\Gamma_3 &= (S + Q + PA)A_d, \\
M_2 &= -M_1 + A_d^T PBB^T PA_d
\end{aligned}$$

then (7) holds, and thus $\Pi < 0$. For any given matrix N,

$$(N - P)BB^T(N - P)^T \ge 0$$

is always true. Thus

$$-PBB^T P \le NBB^T N^T - NBB^T P - PBB^T N^T \quad (10)$$

Using Schur complement again, and taking (10) into account, we can easily obtain (9) from(4). So $\Pi < 0$. Thus

$$\dot{V}(x_t, \tilde{g}_0, \tilde{g}_1, \tilde{g}_2) \le \lambda_{max}(\Pi)\|\wp\|^2.$$

Noting the stability of the operator $\wp$ and the above inequality, we conclude the proof by using Th.7.1[2,PP297].

Remark 2: If $f(x(t), t) = 0$,then system(2)becomes

$$\begin{aligned}
\dot{x}(t) &= Ax(t) + A_h x(t-h) + A_d\dot{x}(t-h) + Bu(t), \\
x(\theta) &= \varphi(\theta), \quad \theta \in [-h, 0],
\end{aligned}$$

This system is exactly the one considered in Lemma 1 from [11]. Therefore Theorem 1 extends the stability result of [11].

Remark 3: From the above proof, we can see the robust adaptive control problem is solvable. However, the controller is discontinuous since it contains the sign function signδ. The direct application of such controller may give rise to undesirable chattering [20].To overcome this drawback, we replace signδ_i by saturation function $s_{i\delta}$to obtain a continuous control. The saturation function $s_{i\delta}$ is defined as

$$s_{i\delta} \equiv \text{sat}(\frac{\sigma_i}{\delta}) = \begin{cases} 1, & \sigma_i \geq \delta; \\ -1, & \sigma_i \leq -\delta; i = 1 \cdots, m \\ \frac{\sigma_i}{\delta}, & \|\sigma_i\| < \delta. \end{cases}$$

whereδ is a small positive constant number.

IV. SIMULATION

Consider the system (1) with parameters as follows

$$A = \begin{bmatrix} -1 & 1 \\ -2 & -3 \end{bmatrix}, A_h = \begin{bmatrix} 0 & -0.1 \\ 0.5 & 1 \end{bmatrix},$$
$$\Delta A = \begin{bmatrix} 0.1sin(2t) & -0.3sin(t) \\ -0.1sin(t) & 0.07sin(3t) \end{bmatrix},$$
$$\Delta A_h = \begin{bmatrix} -0.2sin(2t) & 0.1sin(3t) \\ 0.1sin(t) & -0.175sin(t) \end{bmatrix},$$
$$A_d = \begin{bmatrix} -0.05 & -0.027 \\ 0.01 & 0 \end{bmatrix},$$
$$e(t, x, x_h) = \begin{bmatrix} 0.2 + 0.6sin(3t) \\ 0.1 + 0.09sin(3t) \end{bmatrix}.$$

When$A_d = 0$, system (1) reduces to the similar system discussed in [21], which is a water-quality dynamic model of the River Nile. It is obvious that assumption (A1) - (A4) are satisfied. According to assumption (A3), ΔA, ΔA_h and $e(t, x, x_h)$ are matched with

$$A_1 = \begin{bmatrix} 0.1sin(2t) & -0.3sin(t) \\ -0.2sin(t) & 0.15sin(3t) \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -0.2sin(2t) & 0.1sin(3t) \\ 0.2sin(t) & -0.35sin(t) \end{bmatrix},$$
$$A_3 = \begin{bmatrix} 0.2 + 0.6sin(3t) \\ 0.1 + 0.09sin(3t) \end{bmatrix},$$

Solve the LMI (4), and we get$P = \begin{bmatrix} 1.8797 & -0.0336 \\ -0.0336 & 1.0804 \end{bmatrix}$, so adaptive controller is given by $u = -B^T Px + u_{adp}$, The modified sign function is $S_\delta = \text{sat}(\sigma/\delta)$ and $\delta = 0.005$. The closed-loop dynamic responses of computer simulation are given in fig. 1-3 with the initial conditions $x(0) = [5, -4]$. Fig. 1 shows that original system states approach to a small bounded region in finite time. Fig.2 depicts the input control signals. The adaptive gains are shown in Fig.3. They are all bounded.

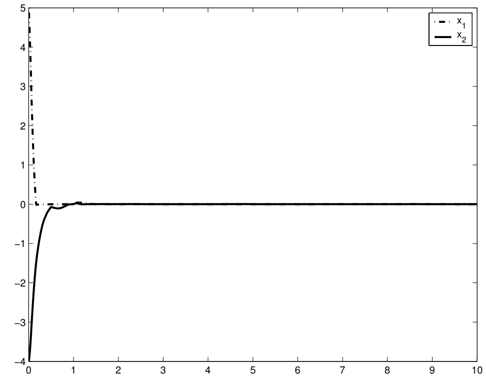

Fig. 1. The controlled state trajectories.

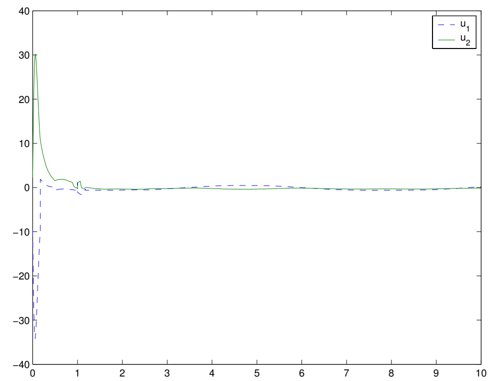

Fig. 2. The control signals

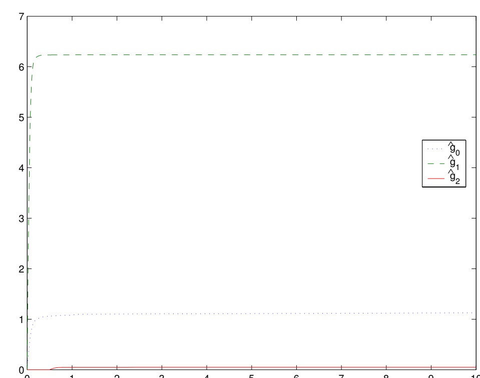

Fig. 3. The adaptive gains

V. CONCLUSION

This paper focuses on the adaptive robust control for a class of nonlinear uncertain neutral delay systems. An adaptive law is designed by 1-norm of matrix, which is different from those seen before. Under some necessary assumption, adaptive controller is designed such that the problem of robust adaptive control for a class of nonlinear uncertain neutral delay system have been solved.

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