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Adaptive Control for Nonlinear Time-delay Systems with Low Triangular Structure

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ABSTRACT

Adaptive control of nonlinear time-delay system with low triangular structure is considered in this paper. Based on Lyapunov stability theory and Lyapunov-Krasovskiifunctionnal, an adaptive controller was proposed by the backstepping technique and parameter estimation method. The adaptive controller can make the closed-loop system uniformly asymptotically stable.

***Key words*:** Time-delay system, Adaptive control, Parameter estimation, Backstepping, Low triangular structure.

I. INTRODUCTION

It is well known that in many physical systems, biological systems, especially in industrial chemical process[1], both diverse nonlinearities and diverse time-delay phenomena coexist in the controlled object. Some results such as stability analysis[2], robustness analysis and disturbance decoupleing[3] have been obtained by generalizing the methods of dealing with linear time-delay systems and nonlinear systems, or by the famous Razumikhin-type theory—an important approach to investigate the delay systems. When dealing with time-delay systems, we often construct two kinds of controllers, which have their virtues and shortcomings respectively[4]. One is the memory controller[5], which is dependent on the past state, and the other is the memoryless controller, which is only connected with the current state.

It is not avoidable to include uncertain parameters and disturbance in practical systems due to modeling errors, linearization approximations, and so on. During the past recent years, the problem of robust stabilization of uncertain dynamical systems has received considerable attention of many researchers, and many solutions approaches have been developed. In [6], the authors tried to construct a memoryless controller to stabilize time-delay system with triangular structure, However, there are some fatal errors whether on assumptions or in reasoning process[7]. The conditions are decreased in [8] in which a memory controller was obtained and generalized to nonlinear system with the nested structure[9]. When the dynamical system with delayed state perturbation is dealt with, the upper bound of the delayed state perturbation norm is generally supposed to be known, and such bound is employed to construct some types of stabilizing state feedback controllers, or to develop some stability conditions. However, in the practical control problems, the bounds of the delayed state perturbations might not be exactly known. Therefore, for such a class of uncertain time-delay dynamical systems whose uncertainty bounds are partially known, adaptive control schemes should be introduced to update these unknown bounds. Adaptive output feedback practically stable controller designing scheme was presented in [10] for uncertain time-delay systems with unknown bounds for the uncertainties. Adaptive state feedback controller for uncertain systems with unknown multiple constant time-delays perturbation was studied in [11].

In this paper, an adaptive state feedback controller for the systems with low triangular structure is proposed by using backstepping technique and parameter estimation method[12] based on Lyapunov stability theory. The controller can make the closed-looped asymptotically stable.

II. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

Consider a class of dynamical systems described by the following differential equations:

$$\begin{aligned}\dot{\zeta}(t) &= F_0(\zeta(t-d_0),\zeta(t-d_1),\cdots,\zeta(t-d_m))\theta \\ &\quad + F_1(\zeta(t-d_0),\zeta(t-d_1),\cdots,\zeta(t-d_m), \\ &\qquad x_1(t-d_0),\cdots,x_1(t-d_m))x_1(t) \\ \dot{x}_1(t) &= G_1(x_1(t))x_2(t)+\phi_1(\zeta(t-d_0),\cdots,\zeta(t-d_m), \\ &\qquad x_1(t-d_0),\cdots,x_1(t-d_m))\theta \\ &\vdots \\ \dot{x}_i(t) &= G_i(x_1(t),\cdots,x_i(t))x_{i+1}+\phi_i(\zeta(t-d_0),\cdots,\end{aligned}$$

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$$\zeta(t-d_m),\bar{x}_i(t-d_0),\cdots,\bar{x}_i(t-d_m))\theta$$

$$\dot{x}_n(t)=G_n(x_1(t),\cdots,x_n(t))u+\phi_n(\zeta(t-d_0),\cdots,$$
$$\zeta(t-d_m),\bar{x}_n(t-d_0),\cdots,\bar{x}_n(t-d_m))\theta \quad (1)$$

where $t\in R$ is the time, $u(t)\in R$ denotes the input; $\zeta(t)=[\zeta_1(t),\zeta_2(t),\cdots,\zeta_r(t)]^T\in R^r$, $\bar{x}_i(t)=[x_1(t), x_2(t),\cdots,x_i(t)]^T\in R^i$ denote the current vector value of state variables; $d_i>0$, $i=1,\cdots,n$, are known constant delays, and $d_0=0$. θ is an unknown positive constant. $F_0(\cdot),G_i(\cdot),\phi_i(\cdot)$, $i=1,\cdots,n$, are some known smooth functions which satisfy $F_0(0,\cdots,0)=\phi_i(0,\cdots,0)=0$, $G_i(x_1(t),\cdots,x_i(t))\neq 0, i=1,\cdots,n$. We aim to design the adaptive state feedback controller $u(t)$ to stabilize the closed-loop system of (1).

Assumption For $F_0(\cdot)$, arbitrary $\zeta(t)$ and unknown constant θ, there exist the definite function $V_0(\zeta(t))$, nonnegative real numbers $c_{sj}, s=1,2,\cdots,r, j=0,\cdots,m,$ and positive β_0 satisfying the following equality

$$\frac{\partial V_0}{\partial\zeta}(\zeta(t))F_0(\zeta_s(t-d_0),\cdots,\zeta_s(t-d_m))\theta$$
$$+\sum_{S=1}^{r}\sum_{j=0}^{m}c_{sj}\{[\zeta_S(t)]^2-[\zeta_S(t-d_j)]^2\}$$
$$\leq-\beta_0\|\zeta(t)\|^2$$

(2)

III. MAIN RESULTS

Lemma: Consider the smooth function $E(x_1,x_2\cdots,x_n)$ satisfying $E(0)=0$. Then there exist smooth functions $e_i(x_1,x_2,\cdots,x_i), i=1,2,\cdots,r,$ satisfying the equation

$$E(x_1,x_2,\cdots,x_n)=\sum_{k=1}^{n}x_k e_k(x_1,\cdots,x_k)$$

Theorem: Consider the system (1) satisfying the assumption. Then under the following adaptive feedback controller $u(t)$ described by (3)

$$u(t)=\tilde{\varphi}_n(\zeta(t),\zeta(t-d_0^n),\cdots,\zeta(t-d_{m_n}^n),\bar{x}_n(t-d_0),$$
$$\bar{x}_n(t-d_1^n),\cdots,\bar{x}_n(t-d_{m_n}^n),\hat{\theta}_1;\cdots,\hat{\theta}_n))$$
$$\hat{\theta}_i=b_i(x_i-\varphi_{i-1}(\cdot))\phi_{i3}(\cdot), i=1,2,\cdots,n. \quad (3)$$

the closed-loop system of (1) is uniformly asymptotically stable in the presence of the unknown parameter θ. $\varphi_i(\cdot)$ is the virtual controller designed in step i, and $\varphi_0(\cdot)=0$.

Remark: It is fair to say that the backstepping technique has been a great success in nonlinear free time-delay robust control. It is not only applied to systems in lower triangular form, namely the strict feedback form but also applied to the systems in nested structure. In this paper, we will shown that may be applied successfully to time-delay systems with strict feedback form. Parameter estimation and tuning function are two classic methods in designing the adaptive controller. We will adopt the first method although it has certain shortcomings[12] .

Proof:

Step 1 Suppose $x_1(t)=z_1(t)$, and $x_1(t)$ is measurable, then the first two equations of the system (1) changed into

$$\dot{\zeta}(t)=F_0(\zeta(t-d_0),\zeta(t-d_1),\cdots,\zeta(t-d_m))\theta$$
$$+F_1(\zeta(t-d_0),\zeta(t-d_1),\cdots,\zeta(t-d_m),$$
$$z_1(t-d_0),\cdots z_1(t-d_m))z_1(t)$$
$$\dot{z}_1(t)=G_1(z_1(t))x_2(t)+\phi_1(\zeta(t-d_0),\cdots,\zeta(t-d_m),$$
$$z_1(t-d_0),\cdots z_1(t-d_m)\theta \quad (4)$$

Consider the following Lyapunov-Krasovskii function

$$V_1=V_0+\sum_{s=1}^{r}\sum_{j=0}^{m}\int_{\text{t-}d_j}^{\text{t}}\text{c}_{\text{sj}}[\zeta_s(\sigma)]^2\text{d}\sigma+\tfrac{1}{2}[z_1(t)]^2$$
$$+\sum_{s=1}^{r}\sum_{j=0}^{m}\int_{t\text{-}d_j}^{t}\text{c}_{\text{sj}}^1[\zeta_s(\sigma)]^2\text{d}\sigma+\frac{a_1}{2}(\theta-\hat{\theta}_1)^2$$
$$+\sum_{j=0}^{m}\int_{t-d_j}^{t}c_j^1[z_1(\sigma)]^2\text{d}\sigma \quad (5)$$

where $\text{c}_{\text{sj}}^1\geq 0$, $c_j^1\geq 0$, $a_1>0$ are some constants to be determined next . $\hat{\theta}_1$ is the first estimation of θ Then V_1 has the time derivative along the trajectories of systems (4) as following

$$\dot{V}_1=\frac{\partial V_0}{\partial\zeta}F_0\theta+\sum_{s=1}^{r}\sum_{j=0}^{m}c_{sj}\{[\zeta_s(t)]^2-[\zeta_s(t-d_j)]^2\}$$
$$+\sum_{j=0}^{m}c_j^1\{[z_1(t)]^2-[z_1(t-d_j)]^2\}$$
$$+z_1(\frac{\partial V_0}{\partial\zeta}F_1+G_1(z_1)x_2+\phi_1\hat{\theta}_1)$$
$$+\sum_{s=1}^{r}\sum_{j=0}^{m}c_{sj}^1\{[\zeta_s(t)]^2-[\zeta_s(t-d_j)]^2\}$$

$+(\theta-\hat{\theta}_1)(z_1\phi_1-a_1\dot{\hat{\theta}}_1)$

(6) $\frac{\partial V_0}{\partial \zeta}(0)F_1(0,\cdots,0)=0$. Because $\frac{\partial V_0}{\partial \zeta}F_1$ is a smooth function, therefore following from Lemma, there exist smooth functions

$$\bar{\phi}_1^{sj}(\zeta(t-d_0),\cdots,\zeta(t-d_m), z_1(t-d_0),\cdots,z_1(t-d_m))$$

$$\phi_1^{1j}(\zeta(t-d_0),\cdots,\zeta(t-d_m), z_1(t-d_0),\cdots,z_1(t-d_m))$$

satisfying

$$\frac{\partial V_0}{\partial \zeta}F_1=\sum_{s=1}^{r}\sum_{j=0}^{m}\bar{\phi}_1^{sj}\zeta_s(t-d_j)+\sum_{j=0}^{m}\phi_1^{1j}z_1(t-d_j)$$

From the inequality

$$2ab\le \alpha a^2+\beta b^2\ (\alpha>0,\beta>0,\alpha\beta=1),$$

there exist smooth functions

$$\bar{\alpha}_1^{sj}(\zeta(t-d_0),\cdots,\zeta(t-d_m), z_1(t-d_0),\cdots,z_1(t-d_m))$$

$$\alpha_1^{1j}(\zeta(t-d_0),\cdots,\zeta(t-d_m), z_1(t-d_0),\cdots,z_1(t-d_m))$$

satisfying

$$z_1\frac{\partial V_0}{\partial \zeta}F_1\le\sum_{s=1}^{r}\sum_{j=0}^{m}\{c_{sj}^1[\zeta_s(t-d_j)]^2+\bar{\alpha}_1^{sj}(z_1)^2\}+\sum_{j=0}^{m}\{c_j^1[z_1(t-d_j)]^2+\alpha_1^{1j}(z_1)^2\} \tag{7}$$

Let

$$\dot{\hat{\theta}}_1=\frac{1}{a_1}z_1\phi_1=b_1z_1\bar{\phi}_{13}\quad b_1=\frac{1}{a_1}\ge 0,\ \bar{\phi}_{13}=\phi_1 \tag{8}$$

From (2), (6), (7) and (8), we can obtain

$$\dot{V}_1\le-\beta_0\|\zeta\|^2+z_1(G_1x_2+\phi_1\hat{\theta}_1)+\sum_{s=1}^{r}\sum_{j=0}^{m}\bar{\alpha}_1^{sj}(z_1)^2+\sum_{j=0}^{m}\alpha_1^{1j}(z_1)^2+\sum_{s=1}^{r}\sum_{j=0}^{m}c_{sj}^1(\zeta_s)^2+\sum_{j=0}^{m}c_j^1(z_1)^2$$

Let c_{sj}^1, c_j^1 be sufficiently small nonnegative real numbers satisfying

$$-\beta_0\|\zeta\|^2+\sum_{s=1}^{r}\sum_{j=0}^{m}c_{sj}^1(\zeta_s)^2\le-\beta_1\|\zeta\|^2\quad((\beta_1>0).$$

Suppose

$$x_2(t)=\varphi_1(\zeta(t-d_0),\cdots,\zeta(t-d_m), z_1(t-d_0),\cdots,z_1(t-d_m)\ ,\ \hat{\theta}_1)$$

$$=\frac{1}{G_1}(-c_1^1z_1-\sum_{s=1}^{r}\sum_{j=0}^{m}\bar{\alpha}_1^{sj}z_1-\sum_{j=0}^{m}(c_j^1z_1+\alpha_1^{1j}z_1)-\phi\hat{\theta}_1)$$

Then

$$\dot{V}_1\le-\beta_1\|\zeta\|^2-c_1^1(z_1)^2\quad(c_1^1>0)$$

Step 2 Suppose

$$\zeta=\zeta,\ x_1=z_1,\ x_2=z_2+\varphi_1$$

the first three equations of system (1) will change into

$$\dot{\zeta}(t)=F_0(\zeta(t-d_0),\cdots,\zeta(t-d_m))\theta+F_1(\zeta(t-d_0),\cdots,\zeta(t-d_m), z_1(t-d_0),\cdots,z_1(t-d_m))z_1(t)$$

$$\dot{z}_1(t)=z_2(t)+\varphi_1(\zeta(t-d_0),\cdots,\zeta(t-d_m), z_1(t-d_0),\cdots,z_1(t-d_m),\hat{\theta}_1)$$

$$\begin{aligned}\dot{z}_2(t)=&G_2(x_1,x_2)x_3(t)+\bar{\phi}_{21}((\zeta(t-d_0^2),\cdots,\zeta(t-d_{m_2}^2),\bar{z}_2(t-d_0^2),\cdots,\bar{z}_2(t-d_{m_2}^2))\\&+\bar{\phi}_{22}(\zeta(t-d_0^2),\cdots,\zeta(t-d_{m_2}^2),\bar{z}_2(t-d_0^2),\cdots,\bar{z}_2(t-d_{m_2}^2),\hat{\theta}_1)\\&+\bar{\phi}_{23}(\zeta(t-d_0^2),\cdots,\zeta(t-d_{m_2}^2),\bar{z}_2(t-d_0^2),\cdots,\bar{z}_2(t-d_{m_2}^2),\hat{\theta}_1)\theta\end{aligned}$$

where $d_j^2>0$, $j=1,\cdots,m_2$, $\bar{\phi}_2(0,\cdots,0)=0$, $d_0^2=0$, $\bar{z}_2=[z_1,z_2]^T$

Consider the following Lyapunov-Krasovskii functional

$$V_2=V_1+\tfrac{1}{2}(z_2)^2+\sum_{s=1}^{r}\sum_{j=1}^{m_2}\int_{t-d_j^2}^{t}c_{sj}^2[\zeta_s(\sigma)]^2\mathrm{d}\sigma+\sum_{k=1}^{2}\sum_{j=0}^{m_2}\int_{t-d_j^2}^{t}c_j^k[z_k(\sigma)]^2\mathrm{d}\sigma+\frac{a_2}{2}(\theta-\hat{\theta}_2)^2 \tag{9}$$

where $c_{sj}^2\ge 0$, $c_j^k\ge 0, a_2>0$ are some constants to be determined next. $\hat{\theta}_2$ is the second estimation of θ.

For $\bar{\phi}_{21}$ is a smooth function and $\bar{\phi}_{21}(0,\cdots,0)=0$, there exist smooth functions

$$\bar{\phi}_{21}^{sj}(\zeta(t-d_0^2),\cdots,\zeta(t-d_{m_2}^2), \bar{z}_2(t-d_0^2),\cdots,\bar{z}_2(t-d_{m_2}^2))$$

$$\phi_{21}^{kj}(\zeta(t-d_0^2),\cdots,\zeta(t-d_{m_2}^2), \bar{z}_2(t-d_0^2),\cdots,\bar{z}_2(t-d_{m_2}^2))$$

satisfying

$$\overline{\phi}_{21}=\sum_{s=1}^{r}\sum_{j=0}^{m_2}\overline{\phi}_{21}^{sj}\zeta_s(t-d_j^2)+\sum_{k=1}^{2}\sum_{j=0}^{m_2}\phi_{21}^{kj}z_k(t-d_j^2)$$

There exist smooth functions

$$\overline{z}_2(t-d_0^2),\cdots,\overline{z}_2(t-d_{m_2}^2))$$

$$\alpha_2^{sj}(\zeta(t-d_0^2),\cdots,\zeta(t-d_{m_2}^2),\ \overline{z}_2(t-d_0^2),\cdots,\overline{z}_2(t-d_{m_2}^2))$$

satisfying

$$z_2\overline{\phi}_{21}\le\sum_{s=1}^{r}\sum_{j=0}^{m_2}\{c_{sj}^2[\zeta_s(t-d_j^2)]^2+\overline{\alpha}_2^{sj}(z_2)^2\}+\sum_{k=1}^{2}\sum_{j=0}^{m_2}\{c_j^k[z_k(t-d_j^2)]^2+\alpha_2^{kj}(z_2)^2\} \qquad (10)$$

Similar to the proof of step 1, from (9) and (10) we can obtain

$$\begin{aligned}\dot{V}_2&=\dot{V}_1+z_2G_2x_3+z_2\overline{\phi}_{21}+z_2\overline{\phi}_{22}+z_2\overline{\phi}_{23}\hat{\theta}_2\\&\quad+\sum_{s=1}^{r}\sum_{j=0}^{m_2}c_{sj}^2\{[\zeta_s(t)]^2-[\zeta_s(t-d_j)]^2\\&\quad+(\theta-\hat{\theta}_2)(z_2\overline{\phi}_{23}-a_2\dot{\hat{\theta}}_2\\&\quad+\sum_{j=0}^{m}c_j^k\{[z_k(t)]^2-[z_k(t-d_j)]^2\})\\&\le-\beta_1\|\zeta\|^2-b_1^1(z_1)^2+z_1z_2+z_2G_2x_3\\&\quad+z_2\overline{\phi}_{22}+z_2\overline{\phi}_{23}\hat{\theta}_2\ +\sum_{s=1}^{r}\sum_{j=0}^{m_2}c_{sj}^2[\zeta_s(t)]^2\\&\quad+\sum_{s=1}^{r}\sum_{j=0}^{m_2}\overline{\alpha}_2^{sj}(z_2)^2+(\theta-\hat{\theta}_2)(\overline{\phi}_{23}-a_2\dot{\hat{\theta}}_2)\\&\quad+\sum_{k=1}^{2}\sum_{j=0}^{m_2}\{c_j^k[z_k(t)]^2+\alpha_2^{kj}(z_2)^2\}\end{aligned}$$

Let

$$\dot{\hat{\theta}}_2=\frac{1}{a_2}z_2\overline{\phi}_{23}=b_2z_2\overline{\phi}_{23}\qquad(b_2=\frac{1}{a_2}\ge 0)$$

Let c_{sj}^2, c_j^1 be sufficiently small nonnegative real numbers to satisfy

$$-\beta_1\|\zeta\|^2-b_1^1(z_1)^2+\sum_{s=1}^{r}\sum_{j=0}^{m_2}c_{sj}^2[\zeta_s(t)]^2+\sum_{j=0}^{m_2}c_j^1[z_1(t)]^2\le-\beta_2\|\zeta\|^2-b_1^2(z_1)^2$$

where $\beta_2>0,\ b_1^2>0$. Suppose

$$\begin{aligned}x_3(t)&=\varphi_2(\zeta(t-d_0^2),\cdots,\zeta(t-d_{m_2}^2),\\&\qquad\overline{z}_2(t-d_0^2),\cdots,\overline{z}_2(t-d_{m_2}^2)\ ,\ \hat{\theta}_1\ ,\ \hat{\theta}_2)\\&=\frac{1}{G_2}(-b_2^2z_2+\sum_{s=1}^{r}\sum_{j=0}^{m_2}\overline{\alpha}_2^{sj}z_2+\sum_{j=0}^{m_2}c_j^2z_2\\&\quad+\sum_{k=1}^{2}\sum_{j=0}^{m_2}\alpha_2^{kj}z_2-z_1-\overline{\phi}_{22}-\overline{\phi}_{23}\hat{\theta}_2)\end{aligned}$$

then

$$\dot{V}_2\le-\beta_2\|\zeta\|^2-b_1^2(z_1)^2-b_2^2(z_2)^2\quad(b_2^2>0).$$

Continue the computation similarly to the step 2 till step n, we can obtain the following state transformation

$$\begin{aligned}&\zeta=\zeta\\&x_1=z_1\\&x_2(t)=z_2(t)+\varphi_1(\zeta(t-d_0),\cdots,\zeta(t-d_m),\\&\qquad z_1(t-d_0),\cdots,z_1(t-d_m)\ ,\ \hat{\theta}_1)\\&x_3(t)=z_3(t)+\varphi_2(\zeta(t-d_0^2),\cdots,\zeta(t-d_{m_2}^2),\\&\qquad\overline{z}_2(t-d_0^2),\cdots,\overline{z}_2(t-d_{m_2}^2)\ ,\ \hat{\theta}_1\ ,\ \hat{\theta}_2)\\&\qquad\vdots\\&x_{i+1}(t)=z_{i+1}(t)+\varphi_i(\zeta(t-d_0^i),\cdots,\zeta(t-d_{m_i}^i),\\&\qquad\overline{z}_i(t-d_0^i),\cdots,\overline{z}_i(t-d_{m_i}^i),\ \hat{\theta}_1,\cdots,\ \hat{\theta}_i)\\&\qquad\vdots\\&x_n(t)=z_n(t)+\varphi_{n-1}(\zeta(t-d_0^{n-1}),\cdots,\zeta(t-d_{m_{n-1}}^{n-1}),\\&\qquad\overline{z}_{n-1}(t-d_0^{n-1}),\cdots,\overline{z}_{n-1}(t-d_{m_{n-1}}^{n-1}),\hat{\theta}_1,\cdots,\hat{\theta}_n)\end{aligned}$$

and

$$\dot{\hat{\theta}}_i=b_i\overline{\phi}_{i3}(\zeta(t-d_0^i),\cdots,\zeta(t-d_{m_i}^i),\ \overline{z}_i(t-d_0^i),\cdots,\overline{z}_i(t-d_{m_i}^i)\ ,\ \hat{\theta}_1;\cdots,\ \hat{\theta}_{i-1})\quad i=1,2,\cdots n.$$

$$u(t)=\varphi_n(\zeta(t-d_0^n),\cdots,\zeta(t-d_{m_n}^n),\ \overline{z}_n(t-d_0^n),\cdots,\overline{z}_n(t-d_{m_n}^n)\ ,\ \hat{\theta}_1\ ;\cdots,\ \hat{\theta}_n)\qquad(11)$$

making the transformed system

$$\begin{aligned}&\dot{\zeta}=F_0\theta+F_1z_1\\&\dot{z}_1=z_2+\varphi_1+\phi_1\theta\\&\dot{z}_2=z_3+\overline{\phi}_{21}+\overline{\phi}_{22}+\overline{\phi}_{23}\theta\\&\quad\vdots\\&\dot{z}_n=G_nu+\overline{\phi}_{n1}+\overline{\phi}_{n2}+\overline{\phi}_{n3}\theta\end{aligned}\qquad(12)$$

have a Lyapunov-Krasovskii functional and its time derivative satisfies

$$\dot{V}_n \le -\beta_n \|\zeta\|^2 - \sum_{i=1}^{n} b_i^n (z_i)^2 \tag{13}$$

where $\bar{z}_i(t) = [z_1(t), z_2(t), \cdots, z_i(t)]^{\mathrm{T}} \in R^i$, $d_0^i = 0$, $i = 2, \cdots, n$, $\hat{\theta}_i$ is the i-th estimation of θ. 0thers are known real numbers or functions. From (13), we can know that the closed-loop system of (12) is asymptotically stable. $u(t)$ in (11) can be expressed as the following:

$$u(t) = \tilde{\varphi}_n(\zeta(t-d_0^n), \cdots, \zeta(t-d_{m_n}^n), \bar{x}_n(t-d_0^n), \cdots, \bar{x}_n(t-d_{m_n}^n), \hat{\theta}_1, \cdots, \hat{\theta}_n) \tag{14}$$

we can drawn conclusion that the closed loop system of (1) can be uniformly asymptotically stabilized under the adaptive controller $u(t)$ described by (14)

IV. CONCLUSION

The problem of adaptive control of nonlinear time-delay systems with triangular structure is dealt with in this paper. Based on Lyapunov stability theory and Lyapunov-Krasovskii functional, an adaptive controller was designed by backstepping and parameter estimation approach. The controller can guarantee the closed-loop dynamical system asymptotically stable under the unknown constant parameter. More emphasis should be put on the simplification of the complicated procedure later .

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